A Study on Continuum Modeling of Large Platelike Lattice Structures

거대한 평판형 격자구조물의 연속체 모델링에 관한 연구

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요 약

거대한 평판형 격자구조물을 연속체 평판으로 모델링하기 위한 보다 용이한 기법을 에너지동등 개념에 근거하여 제시하였다. 단위격자가 갖는 탄성변형에너지와 운동에너지를 구할때 기존의 유한요소 행렬을 이용하였다. 연속체 평판의 등가물성치는 연속체평판과 격자평판에 해당하는 축소된 강성 및 질량행렬들을 직접 비교하여 구하였다. 본 연구에서 제안된 모델링기법이 기존의 잘 알려진 기법들에 못지 않는 좋은 결과를 보여주고 있음을 예제해석을 통해 확인하였다.

ABSTRACT

A rational and straightforward method is introduced for developing continuum models of large platelike periodic lattice structures based on energy equivalence, The procedure for developing continuum models involves using existing finite element matrices in calculating strain and kinetic energies of a repeating cell. The equivalent continuum plate properties are obtained from the direct comparison of the reduced stiffness and mass matrices for continuum and lattice plates. Numerical results prove that the method developed in this paper shows very good agreement with other well-recognized methods.

INTRODUCTION

Due to the requirements for low cost, lignt weight and high stiffness, as well as ease of packaging, transporting and assembling in space, truss-type lattice structure has been the dominant

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이 논문에 대한 토론은 1991년 3월 30일까지 본 학회에 보내주시면 1991년 9월호에 그 결과를 게재하겠습니다.

form for proposed large space structures(LSS). Such a structure is composed of many identical cells or elements that are connected end-to-end to form a specially periodic(repetitive) array.

Structural and dynamic characteristics of LSS must be predicted accurately during the initial design phase since they cannot be tested full scale in their operational environments prior to flight. Conventional finite element method for the LSS having dimensions on the order of 10^2-10^3m may require a significant amount of storage capacity and computing time, to obtain reliable solutions because of high structural flexiblity and large size, especially in the dynamic analysis. Thus, special techniques to cope with the very large number of elements and nodes within a complex structure are desired.

Alternative methods have been developed for simplified structural modeling of the lattice structures composed of repeating cells. Of these methods, the simplification of a periodic lattice structure by the equivalent continuum model is known to provide a very promising and practical solution method for overall vibration modes and structual response. The key to continuum modeling involves the determination of appropriate relationships between the geometric and material properties of the lattice and continuum models. Hence, the continuum modeling itself may not be unique and can fall in one of several distinct categories. In order to avoid an excessively long bibliography, the reader is reffered to Refs. 1−3.

The continuum models based on "energy equivalence" have shown to give satisfactory results when the wavelength of a vibration mode spans many repeating cells of a lattice structure. Here, "energy equivalence" means that the lattice and the continuum contain equal kinetic and strain energies when both are subjected to the same displacement and velocity fields. The accuracy and ease of application of the energy equivalence techniques in general depend on the way how the strain and kinetic energies are sucessfully calculated. Thus, the author proposed a new energy equvalence technique and applied it to the large beamlike lattice structures (simply, lattice beams) in his previous work[4]. The new technique involves the idea of using conventional finite element matrices in the calculation of strain and kinetic energies stored in a representative repeating cell.

The objectives of the present paper are (1) to propose a simple, rational method for developing continuum models for large platelike lattice structures (simply, lattice plates) and (2) to demonstrate the accuracy of the method by means of numerical tests. In fact, it is an extension of the continuum method developed by the author in Ref. 4.

DEVELOPMENT OF DYNAMIC COMTINUUM MODELS

1. GENERAL

This paper develops a procedure for formulating an equivalent homogenous anisotropic continuum plate (simply, continuum plate) representation of a lattice plate composed of many repeating cells. For continuum modeling, two basic assumptions on the lattice plate are made : (1) It behaves grossly as a continuum plate especially in lower mode vibrations. (2) The ratio of inplane dimensions to thickness is very high(more than 20 in general) and (3) the deflection is small compared to the thickness. Then, the lattice plate can be represented by the well-kenown classical thin plate[5], where the effects of transverse shear deformation and rotatory inertia are neglected. However, the continuum modeling procedure introduced herein may be readily extended to include the shear and rotatory effects which are often important for the dynamic analysis of trusses and lattice structures. According to the anisotropic plate theory[5, 6], the bending-extension coupling vanish if the continuum plate is homogenous or symmetric with respect to its midplane. Then, neglecting the midsurface stretching produced by the inplane forces, only the bending problem of continuum plate can be considered independently.

The continuum modeling based on energy equivalence needs the calculation of the mass matrices for a finite element can be derived directly from the strain and kinetic energies calculated after making a reasonable hypothesis for the displacement fields. The finite element matrices for axial-bar and beam elements are available from many text books. Therefore, it seems reasonable and convenient to utilize the element matrices in calculating the strain and kinetic energies of a representative repeating cell. The idea was the first proposed by the author and applied to the lattice beams, and now is extended to the lattice plates. The transition from the lattice plate to the continuum plate is done by the following procedure:

1) A representative repeating cell is isolated from the original lattice plate.

2) Appropriate continuum degrees-of-freedom (DOF) is introduced for easy calculation of the equivalent continuum plate properties.

3) Tranaformation matrices are formulated by relating the nodal DOF of lattice nodes to the continuum DOF.

4) The strain and kinetic energies stored in each lattice element are calculated in terms of continuum DOF by using well-defined existing finite element matrices. By summing all element energies, the total energies stored in a repeating cell are obtained as functions of continuum DOF.

5) Equivalent continuum stiffness and mass matrices are derived from the total energy expressions.

6) The finite element stiffness and mass matrices for the homogenous anisotropic continuum plate elements are derived to be reduced as functions of continuum DOF. The matrices include the equivalent continuum plate rigidities and inertia which are to be determined.

7) Equate the stiffness and mass matrices of 5) with those of 6) to determine the equivalent continuum plate rigidities and inertia of lattice plate.

In the present paper, we consider rectangular lattice plates having different types of repeating cells, as shown in Fig.1 and 2. The repeating cells are composed of several different lattice elements: top and bottom surface bars, vertical bars and diagonal bars. Generally the repeating cells are so constructed that all of the joints are located one of top, bottom and middle surfaces.

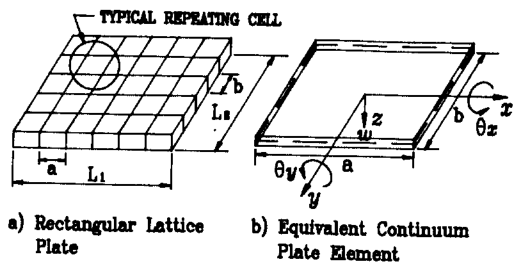

Fig. 1 Equivalent continuum plate element for a typical repeating cell

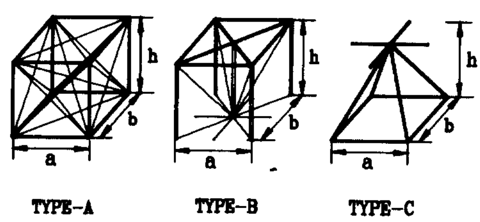

Fig. 2 Typical repeating cells used in the present study

Fig.3 shows the distinct three surfaces where joints are located. As done in conventional finite element analysis, the joints are considered as nodal points(nodes) where nodal DOF are defined. In Fig. 3, u and v are midsurface stretchings in the x-and y-directions, θ_x and θ_y are rotations about the x-and y-axes defined by

$$\theta_x = \frac{\partial w}{\partial y}, \quad \theta_y = -\frac{\partial w}{\partial y} \tag{1}$$

The nodal DOF at each node can be expressed in the form of displacement vector as shown in Fig.3. The nodal displacements vector at a node of (x_i, y_i, z_i) is defined by

$$\{\delta_i\} = \{u_i \ v_i \ w_i\}^T \tag{2}$$

Three rotational DOF can be readily added to $\{\delta_i\}$ for non-hinged joints. Neglecting midsurface stretchings u and v, the displacement vector at (x_i, y_i, o) is now defined by

$$\{\bar{\delta}_i\} = \{\bar{w}_i \ \bar{\theta}_{xi} \ \bar{\theta}_{yi}\}^T \tag{3}$$

Under the assumption of small amplitude

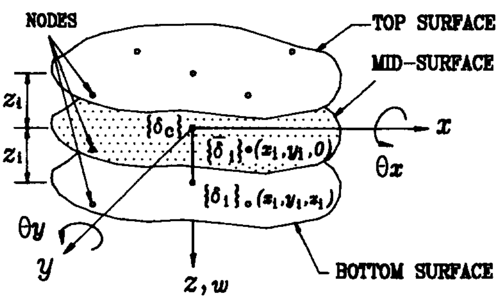

Fig. 3 Nodal DOF and sign convension

Table 1. Transformation matrices

$[A_i]$	$[B_i]$
$\begin{bmatrix} 0 & 0 & z_i \\ 0 & -x_i & 0 \\ 1 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & y_i & -x_i & -\frac{1}{2}x_i^2 & -\frac{1}{2}y_i^2 & -\frac{1}{2}x_iy_i \\ 0 & 1 & 0 & 0 & -y_i & -\frac{1}{2}x_i \\ 0 & 0 & 1 & x_i & 0 & \frac{1}{2}y_i \end{bmatrix}$

vibration, approximated relation between $\{\delta_i\}$ and $\{\bar{\delta}_i\}$ is found as follows;

$$\{\delta_i\} = [A_i] \ \{\bar{\delta}_i\} \tag{4}$$

Where $[A_i]$ is the transformation matrix given in Table 1. We now introduce the continuum DOF in the vector form of

$$\{\delta_c\} = \{w \ \theta_x \ \theta_y \ \kappa_x \ \kappa_y \ \kappa_{xy}\}^T \tag{5}$$

which is defined at the center of midsurface. Note that κ_x, κ_y and κ_{xy} are the curvatures defined by

$$\kappa_x = -\frac{\partial^2 w}{\partial x^2}, \quad \kappa_y = -\frac{\partial^2 w}{\partial y^2}, \quad \kappa_{xy} = -2\frac{\partial^2 w}{\partial x \partial y} \tag{6}$$

By the use of Taylor's series expansion, $\{\delta_i\}$ can be related to $\{\delta_c\}$ in the form of

$$\{\delta_i\} = [B_i]\{\delta_c\} \tag{7}$$

The transformation matrix $[B_i]$ is given in Table 1. Combining Eqs.(4) and (7), we obtain

$$\{\delta_i\} = [A_i] \ [B_i] \ \{\delta_c\} = [R_i] \ \{\delta_c\} \tag{8}$$

Thus, nodal DOF at each node can be represented as functions of continuum DOF from Eq.(8).

2. EQUIVALENT CONTINUUM MATRICES FOR LATTICE PLATES

Figure 4 shows a typical lattice element between two nodes, i and j. If we know the coordinates of the two nodes, the length and direction cosines of the element can be calculated to construct a coordinate transformation matrix $[S_e]$ which transforms the local coordinates to the global coordinates. The stiffness and mass matrices for the element with repect to global coordinates are then obtained from

$$[\bar{k}_e] = [S_e]^T \ [k_e] \ [S_e], \quad [\bar{m}_e] = [S_e]^T \ [m_e] \ [S_e] \tag{9}$$

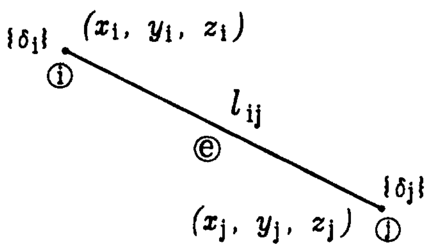

Fig. 4 Typical lattice element

where $[k_e]$ and $[m_e]$ are the finite element matrices with respect to local coordinates. Using the element matrices of Eq.(9), the strain and kinetic energies stored in an element are given by

$$V_e=\frac{1}{2}\{d_e\}^T[\bar{k}_e]\{d_e\},\ T_e=\frac{1}{2}\{\dot{d}_e\}^T[\bar{m}_e]\{\dot{d}_e\} \quad (10)$$

where

$$\{d_e\}=\{\delta_i\ \ \delta_j\}^T \quad (11)$$

Substitute Eq.(8) into Eq.(11) to obtain

$$\{d_e\}=\begin{bmatrix} R_i \\ R_j \end{bmatrix}\{\delta_c\}\equiv[R_e]\{\delta_c\} \quad (12)$$

Which is now expressed as the function of continuum displacement vector. Summing all of element energies, total energies stored in a repeating cell are calculated from

$$V=\sum_e V_e=\frac{1}{2}\{\delta_c\}^T[K_L]\{\delta_c\}$$

$$T=\sum_e T_e=\frac{1}{2}\{\dot{\delta}_c\}^T[M_L]\{\dot{\delta}_c\} \quad (13)$$

Where $[K_L]$ and $[M_L]$ are the equivalent continuum stiffness and mass matrices of lattice plate defined by

$$[K_L]=\sum_e([R_e]^T[\bar{K}_e][R_e]),$$
$$[M_L]=\sum_e([R_e]^T[\bar{m}_e][R_e]) \quad (14)$$

The matrices above are 6×6 symmetric matrices.

3. FINITE ELEMENT MATRICES FOR HOMOGENEOUS ANISOTROPIC PLATES

A rectangular finite element for homogeneous anisotropic plate is shown in Fig. 5. Assuming the midsurface of the element is placed on the midsurface of lattice plate, the nodal displacement vetor $\{\bar{\delta}_i\}$ with 3 nodal DOF is defined at each node. The nodal displacement vectors are now combined to construct a displacement vector in the form

$$\{\Delta\}=\{\bar{\delta}_1\ \bar{\delta}_2\ \bar{\delta}_3\ \bar{\delta}_4\}^T \quad (15)$$

By the use of Eq.(7), Eq.(15) can be rewritten as follows:

$$\{\Delta\}=\begin{bmatrix}[B_1]\\ \vdots \\ [B_4]\end{bmatrix}\{\delta_c\}\equiv[B]\{\delta_c\} \quad (16)$$

Where small amplitude vibration is implicitly assumed. Based on classical thin plate theory, the strain and kinetic energy of a finite element is obtained from

$$V=\frac{1}{2}\int\int\{\kappa\}^T[D]\{\kappa\}^T dxdy \quad (17)$$

$$T=\frac{1}{2}\int\int\rho\,\dot{w}^2dxdy$$

Where $\{\kappa\}=\{\kappa_x,\ \kappa_y,\ \kappa_{xy}\}^T$, $\rho(x,y)$ the mass density per unit area, w the displacement function and $[D]$ is the structural rigidity matrix of anisotropic plate given by

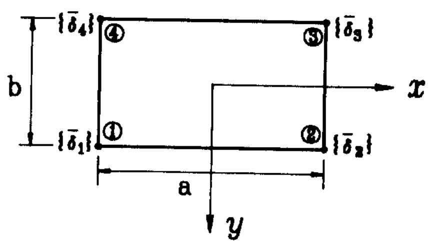

Fig. 5 Finite element of anisotropic plate

$$[D]=\begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{11} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \quad (18)$$

The displacement function within a finite element is assumed by the twelve-term polymomial as

$$\begin{aligned} \mathrm{w}= & a_1+a_2x+a_3y+a_4x^2+a_5xy+a_6y^2+a_7x^3 \\ & +a_8x^2y+a_9xy^2+a_{10}y^3+\ a_{11}x^3y+a_{12}xy^3 \quad (19) \end{aligned}$$

Upon introduction of Eqs.(1), (3) and (19) into Eq.(15), twelve unknown contants $a_1 \sim a_{12}$ can be represented as the function of $\{\Delta\}$. Then, the dispacement function can be written in concise form of

$$\mathrm{w}=\lfloor L \rfloor \ \{\Delta\}$$

in which $\lfloor L \rfloor$, 1×12 matrix, is a function of x and y, and depends upon the nodal coordinates. Details are referred to Ref.7. Substitution of Eqs. (20) and (6) into Eqs.(17) leads to the following energy expressions

$$V=\frac{1}{2}\{\Delta\}^T[K]\{\Delta\},\quad T=\frac{1}{2}\{\dot{\Delta}\}^T[M]\{\dot{\Delta}\} \quad (21)$$

from which the stiffness and mass matrices for the homogenous anisotroptic plate are formulated as follows:

$$\begin{aligned} [K] = & \frac{1}{15ab}\,[R]\{D_{11}[k_1]+D_{12}[k_2]+D_{12}[K_3]+D_{66}[k_4] \\ & +\ D_{16}[k_5]+D_{26}[k_6]\}\ [R] \\ [M]= & \frac{\rho ab}{25200}\,[R][m][R] \quad (22) \end{aligned}$$

It is found that the matrices $[R]$, $[k_1]$, $[k_2]$, $[k_3]$, $[k_4]$ and $[m]$ of Eq.(22) are identical to those for the orthotropic plate[8], except for newly added terms $[k_5]$ and $[k_6]$. Based on the sign convention defined in Fig.4, $[k_5]$ and $[k_6]$ are given in Table 2. Inserting Eq.(16) into Eqs.(21), the energies are expressed as functions of continuum

Table 2. Matrices $[k_5]$ and $[k_6]$ in Eq.(22)

$$k_5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -30 & 0 & 0 & 30 \\ & 0 & -5/2 & 0 & 0 & 5/2 & 0 & 0 & -5/2 & 0 & 0 & 5/2 \\ & & 15 & 0 & 5/2 & 0 & 30 & -5/2 & 15 & -30 & 5/2 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 30 & 0 & 0 & -30 \\ & & & & 0 & -5/2 & 0 & 0 & 5/2 & 0 & 0 & -5/2 \\ & & & & & -15 & -30 & 5/2 & 0 & 30 & -5/2 & -15 \\ & & & & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & 0 & -5/2 & 0 & 0 & 5/2 \\ & & & & & & & & 15 & 0 & 5/2 & 0 \\ & \text{SYMMETRIC} & & & & & & & & 0 & 0 & 0 \\ & & & & & & & & & & 0 & -5/2 \\ & & & & & & & & & & & -15 \end{bmatrix}$$

$$k_6 = \begin{bmatrix} 0 & 0 & 0 & 0 & -30 & 0 & 0 & 30 & 0 & 0 & 0 & 0 \\ & 15 & -5/2 & 30 & 0 & 5/2 & -30 & 15 & -5/2 & 0 & 0 & 5/2 \\ & & 0 & 0 & 5/2 & 0 & 0 & -5/2 & 0 & 0 & 5/2 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -30 & 0 \\ & & & & -15 & -5/2 & 0 & 0 & 5/2 & 30 & -15 & -5/2 \\ & & & & & 0 & 0 & 5/2 & 0 & 0 & -5/2 & 0 \\ & & & & & & 0 & 0 & 0 & 0 & 30 & 0 \\ & & & & & & & 15 & -5/2 & -30 & 0 & 5/2 \\ & & \text{SYMMETRIC} & & & & & & 0 & 0 & 5/2 & 0 \\ & & & & & & & & & 0 & 0 & 0 \\ & & & & & & & & & & -15 & -5/2 \\ & & & & & & & & & & & 0 \end{bmatrix}$$

DOF. Concisely,

$$V=\frac{1}{2}\{\delta_c\}^T[K_A]\{\delta_c\},\quad T=\frac{1}{2}\{\dot{\delta}_c\}^T[M_A]\{\dot{\delta}_c\} \quad (23)$$

In above, $[K_A]$ and $[K_B]$ are reduced 6×6 symmetic matrices, shown in Table 3 in partitioned forms.

Table 3. Reduced finite−element matrices $[K_A$ and $[M_A]$ in Eq.(23)

$$[K_A]=ab\begin{bmatrix} 0 & 0 \\ 0 & D \end{bmatrix} \qquad [M_A]=\frac{\rho ab}{25200}\begin{bmatrix} M_1 & M_2 \\ M_2^T & M_3 \end{bmatrix} \qquad [D]=\mathrm{Eq.}(18)$$

$$[M_1]=\begin{bmatrix} 25200 & 0 & 0 \\ 0 & 2100b^2 & 0 \\ 0 & 0 & 2100a^2 \end{bmatrix} \quad [M_2]=\begin{bmatrix} -1050a^2 & -1050b^2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad [M_3]=\frac{a^2b^2}{4}\begin{bmatrix} \frac{315a^2}{b^2} & 175 & 0 \\ 175 & \frac{315b^2}{a^2} & 0 \\ 0 & 0 & 175 \end{bmatrix}$$

4. EQUIVALENT CONTINUUM STRUCTURAL PROPERTIES

It is quite interesting to find that $[K_A]$ of Eq. (23) has values only in the right-bottom submatrix. Many numerical excercises have shown that only right-bottom submatrix of $[K_L]$ of Eq. (14) has values while the other submatrices vanish. This may be the advantage of the continuum DOF $\{\delta_c\}$ introduced in Eqs.(7) and (16), which also makes easy comparison between two matrices. As described in the procedure of continuum modeling, the equivalent continuum structural properties are obtained by equating Eqs. (14) to Eqs.(21). It follows that

$$[K_L]=[K_A], \quad [M_L]=[M_A] \tag{24}$$

For a thin anisotropic plate model considered herein, six plate rigidities and uniform mass density per unit area are required. They are approximated from Eq.(24), as follows:

$$D_{11}=\frac{1}{ab}K_L(4,\ 4),\quad D_{12}=\frac{1}{ab}K_L(4,\ 5)$$

$$D_{16}=\frac{1}{ab}K_L(4,\ 6),\quad D_{22}=\frac{1}{ab}K_L(5,\ 5)$$

$$D_{26}=\frac{1}{ab}K_L(5,\ 6),\quad D_{66}=\frac{1}{ab}K_L(6,\ 6)$$

$$\rho=\frac{1}{ab}M_L(1,\ 1) \tag{25}$$

where numbers in the parentheses above indicate the coordinates of matrix elements.

NUMERICAL TESTS AND DISCUSSIONS

In order to evaluate the proposed continuum method by means of numerical tests, three types of repeating cells shown in Fig. 2 are considered. Among a small number of researches on the cntunuum modeling of lattice plates, Noor[2] Sun[10] and Flower[11] considered lattice plate models of Type-A, Type-B, and Type-C in Fig. 2, respectively.

Noor developed the continuum models by expanding the nodal displacement of a lattice member in Taylor's series and by equating the strain and kinetic energies of the lattice and continuum plates. Sun related the deformation characteristics of a repeating cell to those of a continuum element by the use of static analysis. Finally, Flower used the discrete field approach to obtain the governing difference equations of the lattice plate and converted them into approximate differential equations. In the present study, their numerical results are compared to the results by the present continuum method. For easy comparisons, the same geometric and material properties of lattice elements as they used in Ref. 2, 10 and 11 are used in the present study. Details are shown in Table 4. Note that ρ is the mass density per unit volume, E Young's modulus, A cross-sectional area, a, b and h are dimensions of repeating cell in the $x-$, $y-$ and $z-$directions, respectively. L_x and L_y are dimensions of lattice plate in the $x-$ and $y-$directions. Each lattice model is also assumed to be subject to the same boundary conditions as in Refs. 2, 10 and 11. That is, Type-A is clamped along one edge and free on the other edges and Type-B and Type-C are simply supported on all edges.

Table 5 compares equivalent continuum plate properties for three lattice models. The present continuum method shows very good agreement with others. Thereby, the numerical results prove that the continuum method developed in this paper may provide satisfactory continuum models for large flexible platelike lattice structures in space.

Table 4. Geometric and material properties of lattice elements

LATTICE PLATES	TYPE-A	TYPE-B	TYPE-C
$\bar{\rho}$(kg/m³)	2768	2768	2768
E(N/m²)	71.7×10^9	71.7×10^9	71.7×10^9
a,b,h(m)	7.5,7.5,7.5	10.6,10.6,7.5	7.5,7.5,7.5
L_x, L_y(m)	75, 75	84.8, 84.8	75, 75
A (m²) —	40×10^{-6}	80×10^{-6}	80×10^{-6}
—	5×10^{-6}	50×10^{-6}	
═		10×10^{-6}	

Table 5. Equivalent continuum plate properties

LATTICE PLATES	TYPE-A		TYPE-B		TYPE-C	
METHOD	PRESENT	SUN[10]	PRESENT	NOOR[2]	PRESENT	FLOWER[11]
D_{11}(N-m)	2.246×10^7	2.246×10^7	2.364×10^7	2.366×10^7	2.151×10^7	2.151×10^7
D_{22}(N-m)	2.246×10^7	2.246×10^7	2.364×10^7	2.366×10^7	2.151×10^7	2.151×10^7
D_{12}(N-m)	9.506×10^5	9.500×10^5	6.175×10^6	6.179×10^6	0	0
D_{16}(N-m)	0	0	0	0	0	0
D_{26}(N-m)	0	0	0	0	0	0
D_{66}(N-m)	9.506×10^5	9.500×10^5	6.175×10^6	6.179×10^6	0	0
ρ(kg/m²)	0.221	0.221	0.17782	0.17794	0.2627	0.2627

CONCLUSION

Based on energy equivalence, now continuum modeling method is developed for large platelike periodic lattice plates. The key to continuum modeling involves the use of existing well-defined finite element matrices in calculating the strain and kinetic energies of a representative lattice cell. Numerical results illustrate that the present continuum method gives very reliable and competitive equivalent structural properties compared to other methods.

ACKNOWLEDGEMENTS

This paper was supported in part by RESEARCH FUND for JUNIOR SCHOLARS, korea Research Foundation.

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(접수일자 : 1990. 10. 29)