論文 한국정밀공학회지 제 7 권 제 3 호 1990. 9
Journal of the Korean Society of Precision Engineering Vol. 7, No 3 Sep. 1990

비틀림진동 및 횡진동을 받는 기어시스템의 안정화에 관한 연구

A Study on the Stability of Geared Systems Subjected to Torsional and Lateral Instability

Abstract

Many high speed mechanical systems incorporate gearing for speed reduction. This study investigates the stability of mulit-rotor geared systems supported on oil film bearings taking into consideration the coupling between torsional and lateral dynamics. The emphasis of the study is on the analysis of the interaction between the combined torsional and whirl instabilities.

The feasility of inducing a lateral and the torsional instability to neutralize an anticipapted unstable condition is investigated. The possibility of suppressing the instabilities by controlling the parameters of the oil film bearings is also considered.

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NOMENCLATURE

M_i = mass of rotor i (i=1, 2, 3, 4)
J_i = mass moment of inertia of rotor i (i=1, 2, 3, 4)
x_i = displacement of rotor i along x direction (i=1, 2, 3, 4)
y_i = displacement of rotor i along y direction (i=1, 2, 3, 4)
θ_i = angular displacement of uniti w. r. t. blank z axis (i=1, 2, 3, 4)
d_t = displacement of tooth spring
d_g = gear whirl
e_i = position of unbalanced mass of unit i from the center of mass (i=1, 2, 3, 4)
ω_r = rotating frequency of shaft
ω_y = natural frequency of whirl along y direction
ω_{no} = nominal natural frequency of gear mesh mode
ω_n = uncoupled natural frequency
ω_{ex} = excitation frequency of torque
τ_{no} = period of gear mesh mode ($2\pi/\omega_{no}$)
τ_{ex} = period of torque excitation ($2\pi/\omega_{ex}$)
τ_y = period of whirl along y direction ($2\pi/\omega_y$)
k_{pqj} = stiffness coefficient of journal bearings of shaft j along p direction due to displacement along q direction (p=x, y; q=x, y; j=1, 2)
c_{pqj} = damping coefficient of journal bearings of shaft j along p direction due to displacement along q direction (p=x, y; q=x, y; j=1, 2)
k_{tj} = torsjonal stiffness of shaft j (j=1, 2)
I_j = area moment of inertia of shaft j w. r. t. x axis (j=1, 2)
A_{ij} = influence coefficient (i=1, 2, 3, 4; j=1, 2, 3, 4)
k_m = mesh stiffness
k_o = mesh damping
ω_m = mesh frequency
ϕ = pressure angle
r_i = radius of rotor i (i=1, 2, 3, 4)
D_j = diameter of shaft j (j=1, 2)
T_i = torque on rotor i (i=1, 2, 3, 4)
GR = qear ratio
W_t = width of tooth
E = Young's modulus
L_1 = length of shaft 1 between overhang mass and bearing close to it
L_2, L_3 = length of shaft 1 between gear and bearing
L_4 = length of shaft 2 between overhang mass and bearing close to it
L_5, L_6 = length of shaft 2 between gear and bearing
L = total length of shaft 1 ($=L_1+L_2+L_3$)
μ = relative oil viscosity
$\mu(T)$ = oil viscosity at temperature T

INTRODUCTION

Gear system dynamics is a problem which has received considerable attention due to its practical importance.

There are numerous sources of excitation in geared systems which include errors in manfacturing and assembly as well as the time-varying nature of mesh stiffness which is inherent to the transmission of motion between the mating teeth. These disturbance manifest themselves in torsional as well as lateral oscillation which can produce resonance and instabilities even under conditions of constant applied loads and speeds.

It has been common practice for the design procedure of a geared system to analyze the torsional dynamics and the lateral dynamics separately. In this way, the effect of gear coupling is only partially accounted for, and the coupling between torsional and lateral vibrations is neglected. As a result, the procedure fails to account for several important aspects such as the shift in critical speeds and change in mode shapes caused by the coupling, change in the instability threshold, the excitation of all rotors in the system caused by the unbalance in any of the rotors, and the damping in the torsional modes contributed by the lateral motion of the journals in their bearings.

This syudy is therefore initiated to investigate the combined effects of coupling the torsional vibration with the consideration of time-varying stifness and lateral dynamics of a multi-mass geared system.

SYSTEM SIMULATION

Physical Model

The systems dealt with in this work are typical geared systems. They represent a single reduction gear unit with and without overhang rotors.

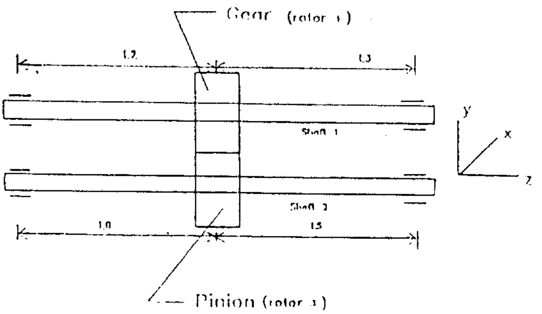

Fig. 1 Two Rotor Geared System

The systems, which are illustrated in Fig. 1 and Fig. 2, respectively, are composed of a gear pair with the high and low speed shafts simply supported on fluid film bearings. The shafs are assumed to be massless. Each of the gear wheels and the overhang masses is assumed to have three degrees of freedom representing the x and y movements of the center of mass and the angular oscillation about the center axis.

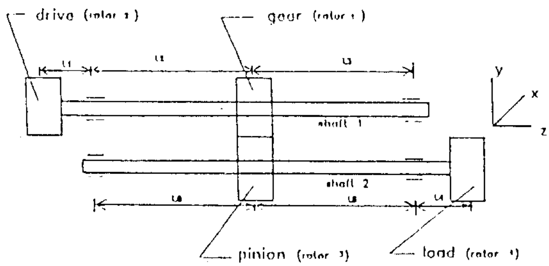

Fig. 2 Four Rotor Geared System

System Equations

The equations of motion can be derived in the same way for both systems. Therefore, the equations of motion for the four rotor geared system are obtained, and the two rotor system is considered to be a special case of the general formulation where the overhang masses are zero. Since the general system has twelve degress of freedom, a 12 × 12 matrix(Fig. 3) consisting of twelve equitions of twelve variables can be constructed as follows.

The coefficients in the matrix are obatined by assuming a small movement in each degree of freedom and evaluating the forces induced by this movment on the entire system. For example, for element 1(Fig 3), a small positive displacement x_1 is imposed on the system, and the resultant force on rotor 1 along the x direction is obtained to fill that element. The resultant force is evaluated from the reactions at the mesh, the shafts, and the bearings as a response to the movment. For the rest of the elements in the first column, a similar procedure is applied to obtain the forces on all four rotors. Also, the remaining elements in the matrix are determined by evaluating the forces due to the displacements x_2, x_3, ……, θ_4.

	x_1	y_1	θ_1	·	·	·	x_4	y_4	θ_4
$M_1 \frac{d^2x_1}{dt^2} =$	1	13	·	·	·		·	·	133
$M_1 \frac{d^2y_1}{dt^2} =$	2	14	·	·	·		·	·	134
$J_1 \frac{d^2\theta_1}{dt^2} =$	3	15	·	·	·		·	·	135
·	·	·	·	·	·		·	·	·
·	·	·	·	·	·		·	·	·
$J_4 \frac{d^2\theta_4}{dt^2} =$	12	24	·	·	·		·	·	144

Fig. 3 A 12 x 12 Matrix of Coefficients for the Equations of Motion

Force at the Mesh

Fig. 4 illustrates the geometry of pair of gears in mesh. The force induced on rotor 1 due to the displacement x_1 can be expressed as

$$\cos\phi[k_m(-\cos\phi)\ x_1+c_m(-\cos\phi)\ \frac{dx_1}{dt}] \quad (1)$$

where

k_m, the time varying stiffness of the mesh
$=k_0[1-A\cos(\omega_m t)]$ ……………… (2)
(k_0 : the nominal mesh stiffness)

c_m, the damping coefficient at the mesh

ω_m, the mesh frequency
= (rotating speed of shaft)
(number of teeth of gear) ………… (3)

A, constant representing the degree of fluctuation of stiffness
=.5 for spur gears …………………… (4)

ϕ, pressure angle
(assumed to be 20 degree) …………… (5)

Force at the Bearings

A diagrammatic illustration of the effect of the oil film bearing support, and the motion in the y-z plane are shown in Fig. 5.

The stiffness and the damping coefficients due to the bearing support is calculated based on the work by Seireg and Dandage[1].

The force on rotor 1 can be written as

$$-k_{xx1}\frac{L_1^2+L^2}{(L_1+L_2)^2}x_1-c_{xx1}\frac{L_1^2+L^2}{(L_1+L_2)^2}\frac{dx_1}{dt} \quad (6)$$

Force due to Bending

As illustrated in Fig. 6, the force due to the bending of the shaft[2] as a result of displacement x_1 can be expressed as

$$-\frac{6\ E\ I_1\ (L_2+L_3)}{L_2\ L_3[L_2^2+L_3^2-(L_2+L_3)^2]}x_1 \;\cdots\cdots \quad (7)$$

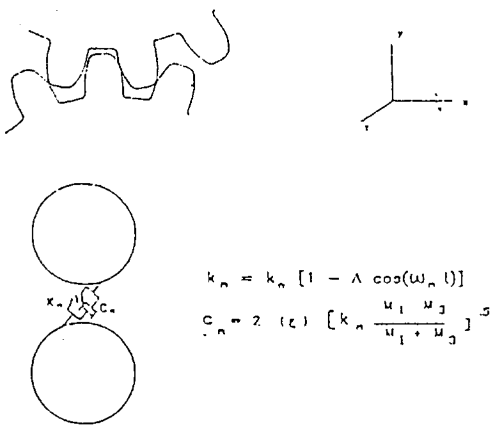

Fig. 4 Stiffness and Damping from Tooth Contact

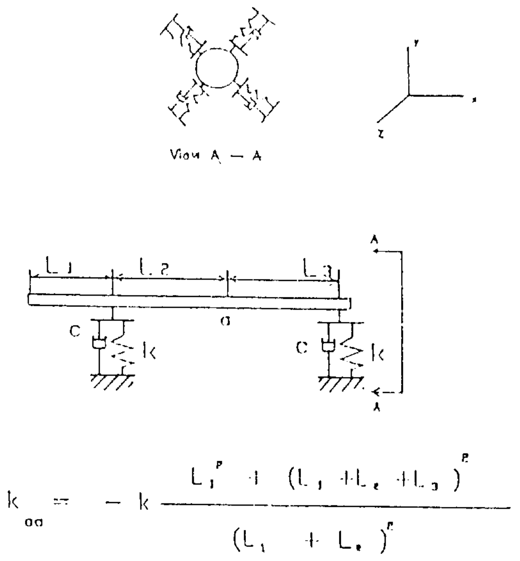

Fig. 5 Shaft Supported on Oil Film Bearings

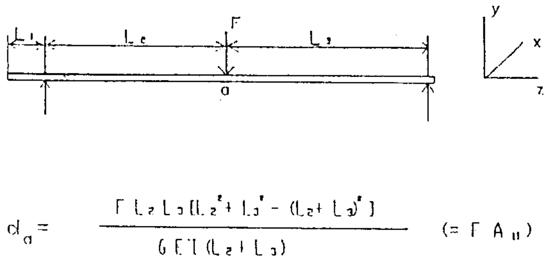

Fig. 6 Shaft Subjected to Bending

SYSTEM RESPONSE AND STABILITY CONSIDERATION

In geared system, instabilites can occur in the form of the lateral as well as torsional oscillations. The whiring and the torsional motion interact at the points of tooth conact. Accordingly, the amplitude build-ups can possibly be made to counteract and vanish.

Unstable situations are selected from the regions which represent practical operating speeds, and each case is stabilized by an additional excitation and by proper selection of the design and operating parameters including the bearing geometry, oils, geomotry of the gear, and the phase shift of the induced disturbances.

Although the time history of the deflection of tooth, d_t, and gear whirl, d_g, are selected to represent the torsional and the lateral dynamics of the system respectively, all the twelve main variables($x_1, x_2, \cdots, y_1, \cdots \theta_4$)are calculated simultaneously and are examined when needed. The values of d_t and d_g are computed as

$$d_t = r_1 (\theta_1 + \theta_3) \quad \cdots\cdots (8)$$

$$d_g = (x_1^2 + y_1^2)^{.5} \quad \cdots\cdots (9)$$

The following are the main system parameters used to investigate the system used in the reported study.

ϕ =20 degree

ω_1 =2 π N/60 rad/sec

ω_2 = ω_1 (GR) rad/sec

ω_m = ω_2 (# of teeth of pinion) rad/sec

D_1 =2.5″

D_2 =2.5″

d_b (bearing diameter) = D_1 + 2 c inch

1_b (bearing length) =2.5″

$L_2 = L_3 = L_5 = L_6 = 4.325''$

$M_1 = M_3 = .169$ s lug

r_1 =7″

GR = 1

W_t =1.5″

T = W_t (1000) 1b−in

$r_3 = r_1/GR$

$k_0 = 1.2(10)^6$ 1b/in

E =30 (10^6) 1b/in^2

Poissons's ratio=.292

The Selection of Bearing Parameters

The parameters of the oil film bearing for different situations are calculated as follows. In the case of stable system conditions, any bearing parameters which do not create damping are acceptable. However, in the case of system instability, the bearing parameters can be selected to supply the necessary amount of positive damping to the system. The system parameters are used to calculate the bearing parameters for the two rotor system. In the reported study, SAE 20 oil is selected as lubricant whose relative viscosity, μ, is 1.36x10 reyns. Oil inlet temperature is assumed to be 110°F, and iteration method[1] is used to calculate the temperature increase, and accordingly viscosities at those operating temperatures which are calculated from the relationship along with Sommerfeld numbers.

$$\mu(T) = \mu \ \exp[12 \ 1.5/(T+95)]$$

Then, the Sommerfeld numbers are used to calculate the stiffness and the dampimg coefficients for given systems in different operating conditions.

Table 1. Selection of Oil Film Bearings

Speed (rpm)	Radial Clearance(Inch)
4200	.002
23875	.004
35812	.005
47750	.006
95500	.006

Table 1 shows radial clearance of the fluid film bearing for each operating speed which is selected in an iterative way to reach the maximum damping for given condition[1].

The two mass pinion-gear system(Fig. 1) is considered here. The sfafts are originally supported on ball bearings. An unbalance is used to suppress the oscillation due to a torsional excitation.

It is conceptually possible to suppress a torsional oscillation by a lateral oscillation since both the lateral and the torsional vibrations of a geared system interact at the points of tooth contact during the mesh action. The examples discussed in this section are considered to illustrate the feasibility of utilizing the concept in the design of high speed gear systems where instabilities are expected to occur.

Stable System

In this case, if the amplitude of the response is allowable, then no additional treatment is necessary. In this section, however, the possibility of suppressing the response amplitude due to an existing torsional excitation on one of the gears by intoducing an unbalance on other gear is investigated.

System Response

The stable response of the system (Fig. 7) due to an oscillatory torque input is evaluated for the system(Fig. 1).

The computation is undertaken for the following conditions.

$T_1 = W_t(1000)$ 1b-in

$\omega_{ex} = 3750$ rad/sec

$\omega_r = 3750$ rad/sec

$\omega_{no} = 5012$ rad/sec

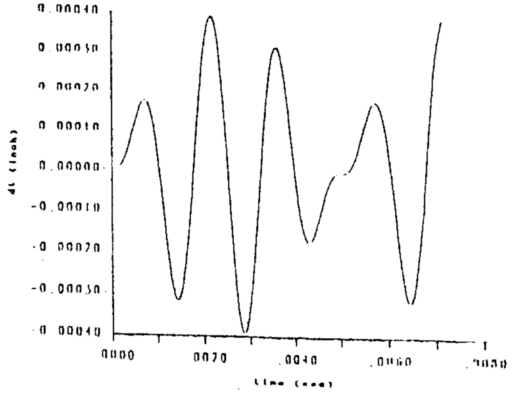

Fig. 7 Oscillatory Deflection of the Teeth at the Mesh due to a Torsional Excitation at Rotor t_{au} =.0017 t_{au}=.00125

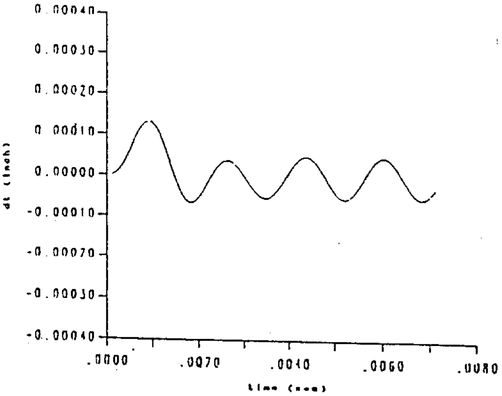

Fig. 8 Deflection of Gear Tooth at the Mesh due to the Two Excitations with Proper Phase Shift with the Effect of Oil Film Bearings t_{ex}=.0017, τ_{no}=.00125, τ_y=.00125

The Selection of a "Tuned" whirl and its Effect on the System Response

It is necessary in this case that the response due to the imposed unbalance has the same frequency as the response due to the torsional excitation.

In order for the response due to the unbalance to exhibit the same frequency as the torsional response, the rotating frequency of the shaft is selected to be the same as the torsional excitation frequency. Also, the amplitudes of the response due to the two different excitations should be of similar magnitude. Considering these factors, the following parameters of lateral vibrations are selected in an iterative way to obtain a tuned whirl.

e_1 =.00009/M_1 inch

ω_r=3750 rad/sec

This unbalance can be introduced in the system with 180 degrees of phase shift which is found to provide the most reduction in the amplitude of oscillation due to the torsional input. As shown in Fig. 8, the amplitude of the tooth spring deflection is suppressed by the tuned whirl, and the gear whirl exhibits decay instead of build-up.

System with Torsional Resonance

Resonance in this example is obtained by selecting the excitation frequency to be the same as the nominal natural frequency of the pinion-gear system i. e.,

$$\omega_{ex} = \omega_{no} = k_0 \frac{(J_1 + J_3)}{J_1 \ J_3} \quad \cdots\cdots (10)$$

Resonant Response

The resonant response of the system due to a torque excitation builds up linearly with time as would be expected.

The response shown in Fig. 9 is obtained for the following conditions.

e_3 =.00006/M_1 inch

ω_r=2500 rad/sec

ω_y=5029 rad/sec

It can be seen form the figure that the response build-up is at a frequency which is the same as the nominal natural frequency of the pinion-gear system.

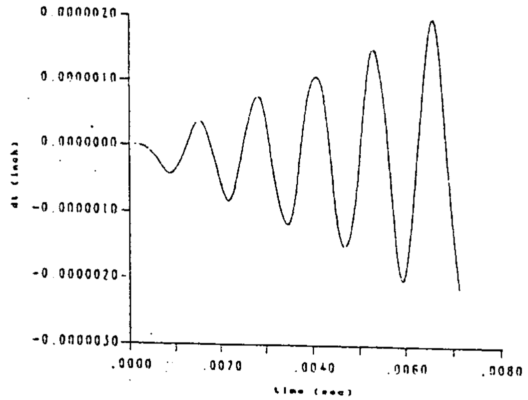

Fig. 9 Deflection of Gear Tooth at the Mesh due to a Torsional Excitation at Resonant Frequency at Rotor 1
τ_{ex}=.00125, τ_{no}=.00125

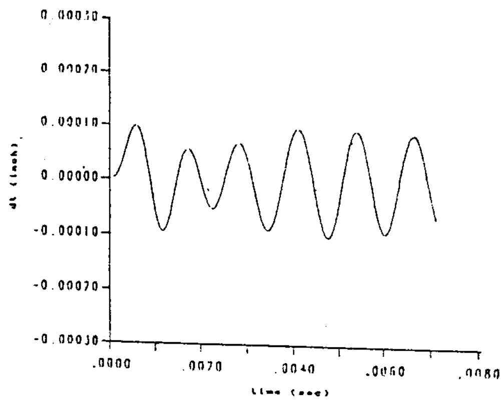

Fig. 10 Deflection of Gear Tooth at the Mesh due to the Two Excitations at Resonant Frequency with Proper Phase Shift with the Effect of Oil Film Bearings
τ_{ex}=.00125, τ_{no}=.00125, τ_{y}=.00125

The selection of the "Tuned" whirl and its Effect on the System Response

The resonant response of the same system due to the unbalance can be induced to have same form as that due to the fluctuating torque input when the torque exciting frequency is the same as the rotational frequency. The following inputs are used in the analysis.

e_3 = .000012/M_1 inch
ω_r = 5000 rad/sec
ω_y = 5029 rad/sec
ω_{ex} = ω_{no} = 5012 rad/sec

The oscillation due to an unbalance can be introduced to the system with a 61.9 degrees of phase lag which is determined iteratively to suppress the resonance due to the torgue input. As shown in Fig. 10, the amplitude of the tooth spring deflection is significantly reduced by the additional tuned input. Also, the gear whirl caused by oil film bearing and the unbalance excitation attains a stable behavior with a limit cycle.

Unstable System

An unstable situation, characterized by exponental build-up of the response amplitude, is expected under the following condition.

$$\frac{\omega_{ex}}{\omega_{no}} = 2\frac{\omega_{ex}}{\omega_m} \quad \cdots\cdots (11)$$

This condition is selected based on the data given in Fig. 7(a) of the work by Benton and Seireg[3].

Response of the Unstale System

The unstable response is obtained from condition which satisfies the relationship expresed in Eq. 11 between the excitation frequency, the nominal natural frequency of the pinion-gear system, and the mesh frequency.

T_1 = W_t (1000) 1b−in
ω_{ex}=2500 rad/sec

ω_{no} =5012 rad/sec
ω_{m} =10000 rad/sec
ω_{r} =2500 rad/sec

The Selection of the "Tuned" Whirl and its Effect on the system Response

The response of the system supported on the same oil film bearing due to the unbalance excitation is expected to have the same vibration frequency as the response due to torque input for the following conditions.

e_3 =.00006/M_1 inch
ω_r=2500 rad/sec
ω_y=5029 rad/seb

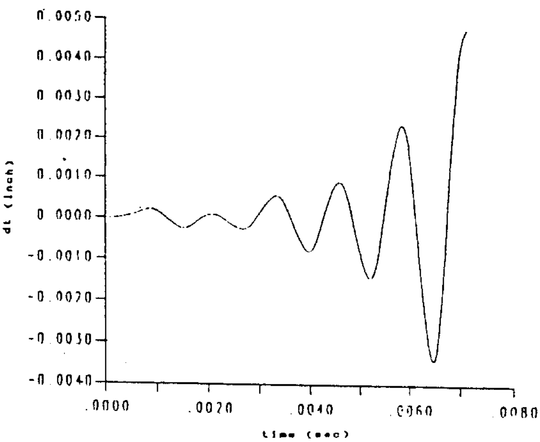

Fig. 11 Deflection of Gear Tooth at the Mesh due to a Torsional Excitation at Rotor 1 for Unstable Condition
τ_{ex}=.0025, τ_{no}=.00125

The additional oscillation created by the unbalance excitation is introduced in the system with a 32.8 degrees of phase lag which is determined iteratively.

The resultant tooth spring deflection is shown in Fig. 11, and Fig. 12 shows the gear whirl caused by the unbalance.

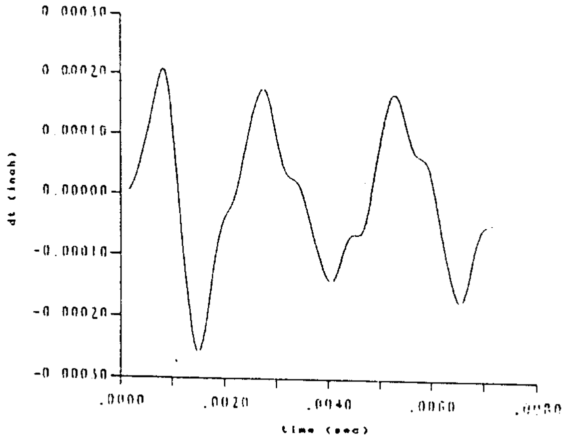

Fig. 12 Deflection of Gear Tooth at the Mesh due to the Two Excitations at Superharmonic Resonant Frequencies with Proper Phase Shift with the Effect of Oil Film Bearings
τ_{ex}=.0025, τ_{no}=.00125, τ_y=.00125

The investigated case shows that the potentially destructive effect from a torque excitation can be avoided by "tuned" whirl and the proper design of oil film bearing without causing a large amount of lateral gear movement. All the reported results have been obtained without considering the effect of the inherent damping in the mechanical system itself.

It can therefore be stated that it is posssible in many situations to rely on the appropriate coupling of torsional and lateral instabilities to neutralize the adverse effects which can result from either of them if they would occur separately.

APPLICATION TO GEARED SYSTEM WITH TWO OVERHANG MASSES

In here, a whirl excitation due to a

unbalanced overhang mass is introduced to a system to neutralize the effect of a torsional instability induced by a torque excitation on the other overhang mass. The objective is to investigate whether the same approach can be applied to complex system with more degrees of freedom than the two rotor system which has been considered so far.

Basically, the system analyzed in this section(Fig. 2) is the same as the one treated in previous sections except for the complexity (six more degrees of freedom) generated by introducing two additional rotors. The main distinction is that the effect of the excitation induced on the overhang rotors reaches the contact points on the teeth with some lag which depends on the dynamic parameters of the system.

In order to demonstrate the feasibiity of the approach, only the unstable case of exponential build-up of the response will be investigated.

The following are the main system parameters used to investigate the system used in the reported study.

ϕ =20 degree
N =4200 rpm
ω_1 $=2\pi$ N/60 rad/sec
ω_2 $=\omega_1$ (GR) rad/sec
ω_m $=\omega_2$ (# of teeth of pinion) rad/sec
D_1 =2.5″
D_2 =2.5″
d_b (bearing diameter) $=D_1+2$ c inch
1_b (bearing length) =2.5″
$L_1=L_4=$ 3″
$L_2=L_3=L_5=L_6=$ 16.2″
$M_1=M_3=$ .172 slug
$M_2=M_4=$ .172 slug
$r_1=r_1=$ 5″
GR= 1
W_t =3.0″
T =300 1b−in
$r_3=r_1/GR$
$r_4=r_3$
k_0 = 4865 lb/in
E $=30(10^6)$ $1b/in^2$
Poisson's ratio=.292

Unstable Response due to a Torque Excitation on an Overhang Mass

An unstable response(Fig. 13) with exponential build-up is obtained for the fillowing conditions which causes a superharmonic resonance of the pinion-gear system supported originally by ball bearings.

T_2 =300 1b−in
ω_{no} =316 rad/sec
ω_{ex} =316 rad/sec

In this case, the torque is applied on an overhang mass and transmitted to the pinion-gear system through the shaft.

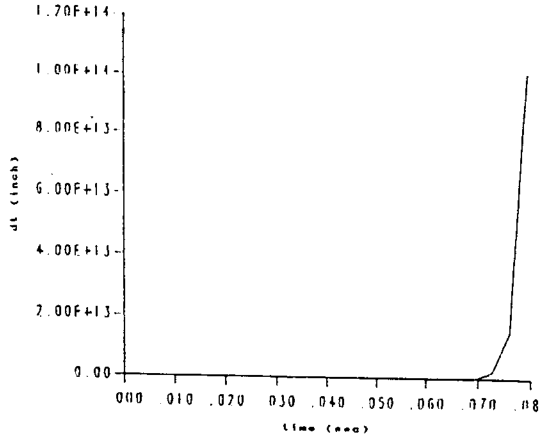

Fig. 13 Deflection of Gear Tooth at the Mesh due to a Torsional Excitation at Rotor 2 for Unstable Condition
τ_{ox}=.02, τ_{no}=.02

Selection of the "Tuned" Whirl and its Effect on the System Response

As in the previous cases, a whirl response with the same frequency is necessary to suppress the instability due to the torque. In this chapter the tuned response is obtained from the proper amount of eccentricity on the other overhang mass with a 150 degrees of phase lag which is obtained in an iterative way.

$e_4 = .08/M_4$ inch

$\omega_r = 440$ rad/sec

The effect of the unbalance introduced in an overhang mass as well as the effect of the torque on the other overhng mass is transmtted with phase lag to the tooth mesh where the lateral and the torsinal oscillations interact.

As in the previous case of the two rotor geared system, the parameters are modified to match the frquencies of the system response to the torque input and the unbalance excitations.

Using the two excitations along with the proper amount of the phase shift, the system shows very low response amplitude(Fig. 14).

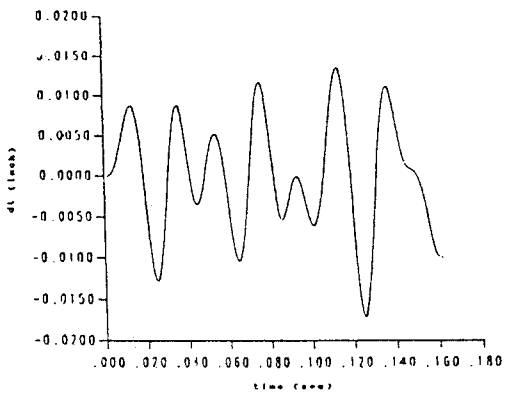

Fig. 14 Deflection of Gear Tooth at the Mesh due to the Two Excitations with Proper Phase Phase Shift with the Effect of oil Film Bearings $\tau_{ex}=.02$, $\tau_{no}=.02$

It can therefore be seen that the additional tuned whirl induced by the unbalanced overhang mass and the proper design of the oil film bearings can eliminate the potentially destructive instability due to the torque excitation applied on the other overhang mass.

CONCLUSIONS AND RECOMMENDATIONS

This study demonstrates the feasibility of utilizing lateral and tortional instability properties in multi-rotor geared system to neutralize the potentially destructive response if they occur separately. The use of oil film bearing to stabilize and suppress the amplitude of response has been illustrated by case studies. A procedure is given the designer in eliminating the instability potential in the design stage by utilizing other instabilities and damping in the bearings.

It can be concluded from this study that "tuned" instabilities can be induced in geared systems to provide safe operation at critical speeds where resonances and instabilities may occur. It is therefore possible to design a multi degree of freedom geared system where the build-up of the forces at the mesh can be avoided by appropriate selection of the system parameters. This is a practical approach since both the lateral and torsional oscillations in gear systems interact at the points of tooth contact.

REFERENCES

1. Seireg, A., and Dandage, S., "Empirical Design Procedure for the Termohydrodynamic Behavior of Journal Bearings," J. of Lubrication Technology,

Trans. of ASME, Paper NO. 81-Lub-19, 1982.

2. Budynas, R., Advanced Strength and Applied Stress Analysis, McGraw-Hill Book Company, Inc., New York, 1977.
3. Benton, M., and Seireg. A., "Simulation of Resonances and Instability Conditions in Pinion-Gear System," J. of Mechnical Design, Trans. of ASME, Vol. 100, No. 1, Jan. 1978, pp. 26-31
4. Azar, R., and Crossley, F., "Digital Simulation of Impact Phenomenon in Spur Gear Systems," Trans. of ASME, Aug. 1977, pp. 792-798.
5. Azar, R., and Crosley, F., "An Experimental Investigation of Impact Phenomenon in Spur Gear System," Proceedings of the Fourth World Fourth Congress on the Theory of Machines and Mechanisms, 1975, pp. 157-161.
6. Tse, F., Morse, I., and Hinkle, R., Mechnical Vibrations Theory and Applications, Ailyn and Boston, Inc., Boston, 1978.