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The Wealth and Substitution Effect on the Demand for Children with Nonhomogeneous Production Function for the Quality of Children

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Ⅰ. Introduction

The aim of this paper is twofold. The one is to reconsider Willis' Economic Theory of Fertility Behavior with a little more general production function for child-quality and independently varying relative shadow price between the quality and quantity of children. The other is to present a graphic analysis of various implications which are included in the above model.[1)]

Since G. Becker had suggested the economic frame of fertility analysis in 1960, known as the New Home Economics,[2)] the relationship between fertility and income level had been tried to be explained within that model. Willis made the relationship more clear by introducing wife's opportunity cost of having children. For simply testable implications, he developed his model with several assumptions, one of which was that the production functions for child-quality are linearly homogeneous. Most of other studies of the quality and

1) R.J.Willis, "A New Approach to the Economic Theory of Fertility Behavior", *Journal of Political Economy*, vol. 81 Supplement (March/April, 1973) pp. s14-69

2) G.S.Becker, "An Economic Analysis of Fertility." In *Demographic and Economic Change in Developed Countries*, University-National Bureau Conference Series 11, Princeton, N.J. : Princeton Univ. Press 1960

quantity interaction in the demand for children assumed this homogeneous production for child quality.[3] However, this assumption is clearly only one case of general cases, and it is necessary to see what implications would be included in the Willis' model with nonhomogeneous production function for child-quality.

For this purpose, I will follow some of the Willis' derivation of demand functions for child-quantity, child-quality, and other non-child home-product just with nonhomogeneous production function for child-quality in section Ⅱ. Next, I will interpret the implications of the properties of the above demand functions, income effect and substitution effect, graphically in Ⅲ and Ⅳ respectively. Finally I will summarize what I have done with Willis' model and some implications for further study.

Ⅱ. Demand Functions for the Quality and Quantity of Child, and Other Home-Product with Nonhomogeneous Production Function for Child-Quality

Willis derived demand functions for the quantity and quality of children, subject to full wealth constraint including wife's shadow price of childbearing and childrearing with the following three assumptions :

1. the production functions for child quality are linearly homogeneous and identical,
2. there is no joint production of child quality, and
3. parents choose an equal level of child quality for each child born.[4]

These made it possible the child product is just the product of the number of children by their quality($C = NQ$), which is very simple and useful for empirical application. However it would not include all possible properties of the demand for child. I relaxed the first and third assumptions and expressed child is the general function of the number and quality, $C = C(N, Q)$.[5]

The demand functions for N, Q, S are derived by maximizing utility function $U = U(N, Q, S)$ subject to the full wealth constraint $P_cC(N, Q) + P_sS = I$ as follows.

Maximizing the Lagrangian expression, $L = U(N, Q, S) + \lambda(P_cC(N, Q) + P_sS - I)$, where λ is Lagrangian multiplier, the following first order conditions are obtained.

$$
(1) \quad
\begin{aligned}
& U_n + \lambda P_c C_n = 0 \\
& U_q + \lambda P_q C_q = 0 \\
& U_s + \lambda P_s = 0 \\
& P_c C(N, Q) + P_s S = I
\end{aligned}
$$

3) G.S. Becker, and H.G.Lewis, "On the interaction between the Quality and Quantity of Children." *Journal of Political Economy*, vol. 81 Supplement (March April) pp.s 279 288

Becker, G.S. and Nigel Tomes, "Child Endowments and the Quantity and Quality of Children." *Journal of Political Economy*, vol. 84 no. 4 Supplement (March/April, 1976), pp.s 143 162

4) R.J.Willis, *Ibid*, pp. s 20 21

5) D.N.De Tray expressed the function in this form but he did not emphasized that.: D.N. De Tray, "Child quality and the Demand for Children", *Journal of Political Economy*, vol. 81, Supplement(March/April, 1973)

where U_n, U_q, U_s are the partial derivatives with respect to N, Q, S, and C_n, C_q are the derivatives of C(N, Q) with respect to N, Q.

Demand functions are the solutions of simultaneous equations (1). To see the properties of the demand funtions, it is most general to take the total differentials of (1) and then we get the following set of equations in matrix form.

$$(2) \begin{bmatrix} U_{nn}+\lambda P_c C_{nn}, & U_{nq}+\lambda_c P C_{nq}, & U_{ns}, & P_c C_n \\ U_{qn}+\lambda P_c C_{qn}, & U_{qq}+\lambda P_c C_{qq}, & U_{nq}, & P_c C_q \\ U_{sn} & & U_{ss} & P_s \\ P_c C_n & & P_s & 0 \end{bmatrix} \begin{bmatrix} dN \\ dQ \\ dS \\ d\lambda \end{bmatrix} = \begin{pmatrix} -\lambda C_n & 0 & 0 \\ -\lambda C_q & 0 & 0 \\ 0 & -\lambda & 0 \\ -C(N,Q) & -S & 1 \end{pmatrix} \cdot \begin{pmatrix} DP_c \\ DP_s \\ DI \end{pmatrix}$$

Among the second-order conditions for utility maximization are $D<0$, D_{11}, D_{22}, $D_{33}>0$, where D is the determinant of the bordered Hessian matrix on the left in (2) and D_{11}, D_{22}, D_{33} are the cofactors of the elements of the principal diagonal. Holding P_s constant and solving for dN, dQ, dS, the followings are obtained.

$$(3)\quad dN = \frac{1}{D}[-\lambda(C_nD_{11}-C_qD_{21})dP_c - D_{41}(dI-C(N,Q)dP_c)]$$

$$(4)\quad dQ = \frac{1}{D}[-\lambda(-C_nD_{12}+C_qD_{22})dP_c + D_{42}(dI-C(N,Q)dP_c)]$$

$$(5)\quad dS = \frac{1}{D}[-\lambda(C_nD_{13}-C_qD_{23})dP_c - D43(dI-C(N,Q)dP_c)]$$

Where D_{ij} is the cofactor of ij-th element of the bordered Hessian matrix on the left in (2) and again C_n, C_q are the partial derivatives of C(N, Q) with respect to N and Q, Since $C = C(N, Q)$ and $dC = C_n dN + C_q dQ$, dC can be obtained as follows.

$$(6)\quad dC = \frac{1}{D}[-\lambda(C_n(C_nD_{11}-C_qD_{21}) + C_q(-C_nD_{12}+C_qD_{22})]dP_c + [-C_nD_{41}+C_qD_{42}][dI-C(N,Q)dP_c]]$$

The wealth effect can be obtained by holding $dP_c=0$ in (3)-(6) as followings.

$$(7)\quad dN/dI = -D_{41}/D$$

$$(8)\quad dQ/dI = D_{42}/D$$

$$(9)\quad dS/dI = -D_{43}/D$$

$$(10)\quad dC/dI = -C_nD_{41}+C_qD_{42}$$

Compensated substitution effects are obtained from (3)-(6) by holding $dI = C(N, Q)dP_c$:

$$(11)\quad dN/dP_c = \frac{1}{D}[-\lambda(C_nD_{11}-C_qD_{21})],$$

$$E_n = \frac{-\lambda P_c}{D}[\frac{C_n}{N}D_{11}-\frac{C_q}{N}D_{21}]$$

$$(12)\quad dQ/dP_c = \frac{1}{D}[-\lambda(-C_nD_{12}-C_qD_{22})],$$

$$E_q = \frac{-\lambda P_c}{D}[\frac{-C_n}{Q}D_{12}-\frac{C_q}{Q}D_{22}]$$

$$(13)\quad dS/dP_c = \frac{1}{D}[-\lambda(C_nD_{13}-C_qD_{23})],$$

$$E_s = \frac{D_{33}}{D}(-\frac{\lambda P_s}{S}-)>0$$

$$(14)\quad dC/dP_c = \frac{1}{D}[-\lambda(C^2_nD_{11}-2C_nC_qD_{21}+C^2_qD_{22})],$$

$$E_c = \frac{D_{33}}{D}[-\frac{\lambda P^2_s}{CP_c}]<0$$

where E_n, E_q, E_s, E_c are the elasticities of N, Q, S, C with respect to P_c, E_s and E_c can be identified to be positive and negative respectively from the second-order condition with the substitution $C_nD_{13}\ \ C_qD_{23} = (-P_s/P_c\ D_{33}$ and $C^2_nD_{11}\ \ 2C_nC_qD_{12} + N^2D_{22}\ \ (P_s/P_c)^2D_{33})$

III. Graphic Analysis of Wealth Effect on the Demand for Children

For the grahpic analysis I will add one assumption that the shadow price of child-quality and quantity can vary independently, which might be more reasonable with C = C (N, Q). From (7), (8), (9), and (10), we have three possible cases of wealth effect upon the total child services and three possible cases of the effect of total child-services on the number(quantity) of children demanded. These nine cases of wealth effect on the quantity of child can be drawn in a set of two diagrams, C S diagram and N - Q diagram, which are shown in Figure 1.

The first pair of diagrams, which is the case of $-C_nD_{41} + D_qD_{42} = 0$ in (10), show that even when the wealth effect on C is zero, the child-quantity can be changed by the effect of wealth change on C_n/C_q which is equal to the slope of the constraint in maximizing C = C (N, Q) in N - Q space.[6] The left diagram of the first pair shows that the child-quantity N_0 can go to N_1 (when $D_{41} < 0$), N_2 (when $D_{41} > 0$), and stay N_0 (when $D_{41} = 0$). The second pair of diagrams shows that, when the wealth effect on C is positive which is the case of $-C_nD_{41} + C_qD_{42} > 0$, the wealth effect on child-quantity could be positive, zero, and negative. Similarly the third pair of diagrams shows that, when the wealth effect on C is negative which is the case of $-C_nD_{41} + C_qD_{42} < 0$, the wealth effect on the child-quantity may be negative, zero, and even positive.

All these nine cases of possible wealth effect on the number of children demanded can be imaginable only with the assumption of independently varying relative shadow price of child-quantity(P_n/P_q).

IV. Graphic Analysis of Compensated Substitution Effect on the Demand for Child

From (14) in II, we know that the compensated substitution effect is negative, which means that the change in shadow price of child service brings about the change of the amount of child services in counter direction. This can be drawn like Figure 2.A. As P_c goes up to P'_c, the compensated demand for child services reduces to C_1 from C_0. However, this does not always mean lower child quantity because again the relative shadow price of child quantity can be affected by the change in the shadow price of total child services. Figure 2. B shows three possible cases of the substitution effect on the number of children. Each of these cases is corresponding with the sign of $C_nD_{11} - C_qD_{21}$ in (11).

6) Since $C = C(N, Q)$, not just NQ, the first-order condition of maximizing $C = C(N, Q)$ subject to $P_cC = P_nN + P_qQ$ gives $P_n/P_q = C_n/C_q$.

Figure 1.

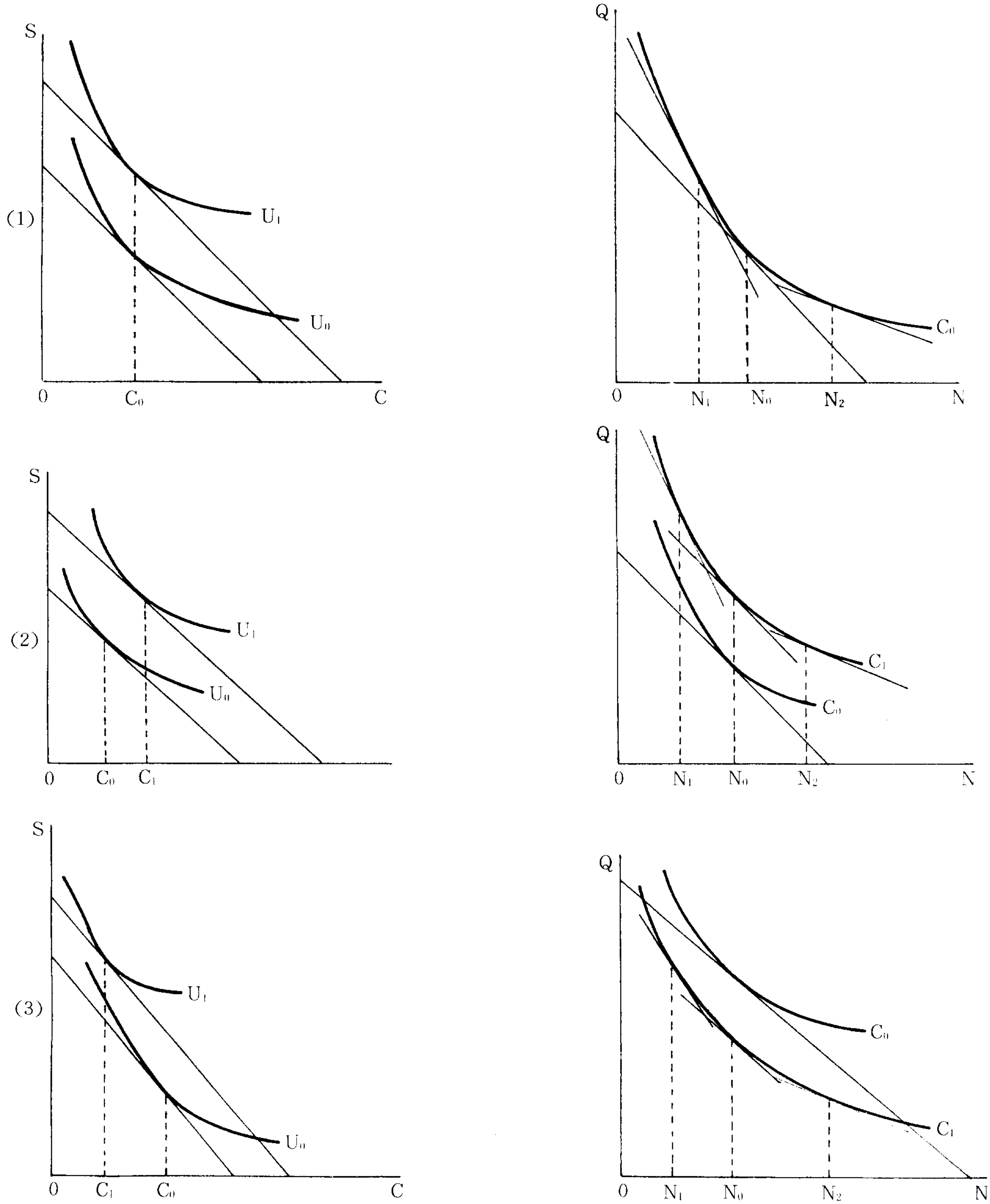
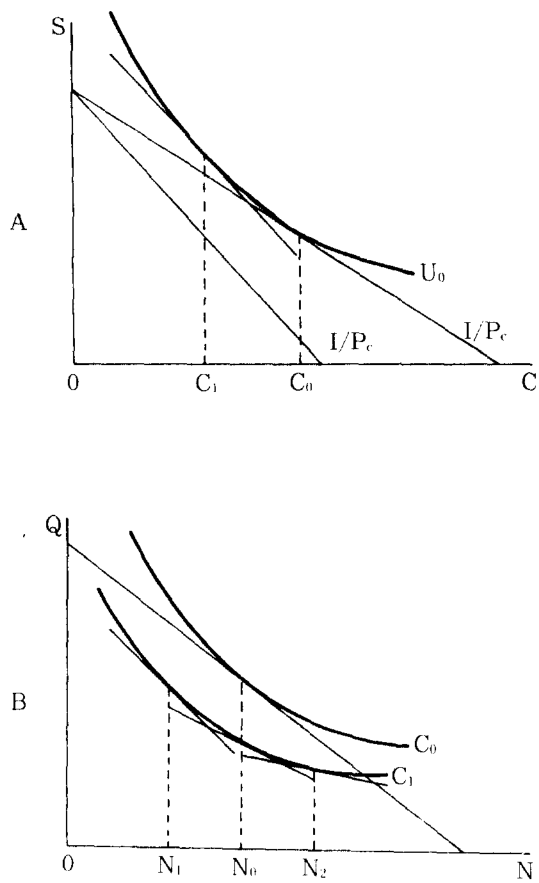


Figure 2.



V. Concluding Smmary

In this very short paper, I did a little bit of extension of Willis' Economic Theory of Fertility Behavior to the case, in which the quality of child is not linearly homogeneous of parents time and goods input. I can derive almost the same properties of demand for child quality, child quantity, and non-child home product only with a little more complicated algebraic form.

In the graphic analysis of wealth and compensated substitution effect, I can show various cases of the effect of the change in wealth and the shadow price of total child-service upon the quantity of the demand for children by adding the assumption that the relative shadow price, P_n/P_q, for childquantity can vary independently.

This analysis is limited to the demand side of the Willis' model. Hence it would be the next step to see what effect of $C=C(N, Q)$ and independently varying relative shadow price of child-quantity would be in the supply side of the model.

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≪國文要約≫

非同次生產函數의 경우 子女需要에 對한 所得 및 代替效果

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윌리스의 生產力－所得水準 法定模型에서 子女에 대한 需要가 子女의 數와 子女養育의 質에 대하여 非同次函數 形態를 취할 때 子女需要에 미치는 潛在價格의 變化에 따른 所得效果 및 代替效果를 代數的으로 誘導하고, 이 意義를 幾何學的으로 확인해 보는 데 本 글의 意義가 있다.

代數的으로 子女需要(C)를 단순히 子女의 數(N)와 子女養育(Q)의 곱으로 C＝NQ와 같이 나타낸 윌리스의 模型에서 이를 C＝f(N, Q)와 같이 일반적인 函數形態를 취할 때에 消費者 理論의 基本的인 效用極大化를 통하여 子女需要를 導出하였다.

幾何學的으로는 代數的으로 導出된 子女需要에 대한 價格效果들을, 子女의 數와 子女養育의 質에 대한 相對的 潛在價格을 고려하여 子女需要의 變化, 子女數 및 子女養育의 質 사이에 관계를 圖示하였다.