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〈論 文〉

Analysis of Mechanical Face Seals for Design Purpose

Part II : Thermoelastic, Wearing and Vibrational Effects

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설계목적을 위한 기계평면시일의 해석

제2보 : 열탄성, 마모 및 진동의 영향에 관하여

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요약－기계평면시일의 접촉 운동면에서 유체가 비압축성이고, 점성의 영향을 받는 경우에 대한 체적 누설 유동량과 마찰 토오크를 멱급수의 방법을 이용하여 추정하였다. 본 연구에서 고려되고 있는 설계인자로 시일의 경사도, 접촉 운동면에서의 사인파형, 코우닝, 열탄성 변화량, 마모량, 시일의 스프링 강성도에 따른 축방향의 변화량을 종합적으로 고려하여 해석하였다. 계산된 결과에 의하면 특히 회전속도가 증가되면 열탄성 변화량에 의한 시일의 누설 유동량과 마찰 토오크는 커다란 영향을 받고 있는 것으로 나타나고 있다.

Introduction

A variety of geometric factors affecting mechanical face seal performance have been discussed in earlier papers [1-14].

The frictional heating of two bodies in sliding contacts causes thermoelastic deformation on the edge of a body which may concentrate the heating and greatly increase the contact pressure and surface temperature. Burton et al. [15-18] showed that high temperature thermally expanded mounds are integral to sliding contacts and seal operation. These mounds move along the seal face, partially support loads, and prevent the sealing gap from closing.

In general, an analysis based on the simple geometry of face seals does not adequately predict the mechanical seal performance. More complex analyses are believed to yield better prediction.

In the first part of this study [19], the general characteristics of face seals for the combined geometric effects were analytically investigated, and it was found that the misalignment term plays an essential role compared with the surface waviness and seal face coning in the volumetric flow rate.

In the present study, the combined geometry shown in Part I [19] and operating conditions such as shaft speed, surface wear, and axial movement due to spring force and fluid film pressure are simultaneously included for an analytical analysis of the leakage flow rate and frictional torque in face seals. These considerations may clarify the thermoelastic instability caused by frictional heating.

Theoretical Analysis

The equations such as temperature distribution across the scaling gap, velocity, and pressure expression in infinite series are treated as in the previous paper [19].

In the present analysis, the overall sealing gap 2h includes the combined geometry and operating conditions;

$$2h(r,\theta,t)=2\bar{h}+h_m+\hat{h}+h_c+\delta_{th}+\delta_w+\delta_d \quad (1)$$

where the first four terms representing geometric components are described in reference [19]. Here $2\bar{h}$ is the mean film thickness, h_m represents the sealing gap due to the misalignment, $\hat{h}$ is the film thickness due to surface waviness, h_c is the sealing gap variation due to coning, δ_{th} is a thermoelastic displacement due to a non-uniform viscous heating, δ_w denotes displacements caused by material removal through wear, and δ_d desecribes axial movements due to pressure forces arising from misalignment, surface waviness, and spring reactions.

Thermoelastic Displacements. Under the assumptions of a Newtonian fluid and Couette flow, the viscous heat generated across a thin film may be written for any type of fluid as

$$q=\int_{z_l}^{z_u}\eta\left[\frac{\partial v_\theta}{\partial z}\right]^2 dz \quad (2)$$

where v_θ is the fluid velocity in the tangential direction and η the viscosity of the fluid.

Substituting a temperature dependent viscosity given by Eq. (12) and tangential velocity given by Eq. (14) in refernece [19] into Eq. (2) gives the heat flow to first order in $\tilde{h}$;

$$q=\frac{\eta_r G_0 U^2}{2\bar{h}}\left[1-\frac{\tilde{h}}{2\bar{h}}\right] \quad (3)$$

where

$$G_0=\frac{\alpha_2 e^{\alpha^* T_r(1-\Lambda_l)}}{e^{\alpha_2-1}},\quad \tilde{h}=h_m+\hat{h}+2\bar{h}\cdot H_c$$

Since the first term which represents the uniform heating of the parallel surfaces does not change the shape of surface curvature, the uniform heating term is substracted from the overall heating. Then non-uniform heating $\tilde{q}$ is given by

$$\tilde{q}=-\eta_r G_0 U^2\frac{\tilde{h}}{4\bar{h}^2} \quad (4)$$

The thermoelastic displacement δ_{th} produced on the edge of a body with non-uniform heating has been shown by Burton et al. [17] to be

$$\frac{d^2\delta_{th}}{d\theta^2}=\frac{\alpha r^2\tilde{q}}{K} \quad (5)$$

Here α is the coefficient of thermal expansion and K the thermal conductivity of the body. Integrating Eq. (5) twice with respect to θ yields the thermal deformation of the surfaces

$$\delta_{th}=-\frac{\eta_r G_0 r^2 U^2}{2\bar{h}}(\delta_{thm}+\delta_{thu}+\delta_{thl}+\delta_{thc}) \quad (6)$$

where

$$\delta_{thm}=\frac{\alpha_l\varepsilon}{K_l}\left[R+\frac{\bar{R}}{1-\bar{R}}\right](1-\cos\theta)$$

$$\delta_{thu}=\frac{\alpha_u\xi_u}{K_u n_u^2}[\sin(n_u\theta-\Omega_u t)-n_u\theta]$$

$$\delta_{thl}=\frac{\alpha_l\xi_l}{K_l n_l^2}[\sin(n_l\theta-\Omega_l t)-n_l\theta]$$

$$\delta_{thc}=\left[\frac{\alpha_u}{K_u}+\frac{\alpha_l}{K_l}\right]\frac{H_c\theta^2}{2}+\delta_{cu}+\delta_{cl}$$

The first term in Eq. (6) represents the thermoelastic distortion on the boundary of the tilted lower surface. The second and third terms denote the thermoelastic deformation caused by surface waviness on the edge of the upper and lower mating surfaces. The last term is the thermoelastic deformation caused by coning shape of mating surfaces. Here δ_{cl} and δ_{cu} are the sealing gap variations due to the thermoelastic coning effects at the outer radius of the lower and upper surfaces.

Wear displacements. A wear displacement δ_w will be produced due to contact pressure p_a. A simple theory that involves pressure and sliding distance was developed by Burwell and Strang [20]; it is given by

$$\delta_w = -w\int_0^t p_a U dt \tag{7}$$

where w is an experimentally determined wear coefficient and Udt is the sliding distance during an infinitesimal time dt.

The contact pressure is associated with a heat flow between contacting surfaces. It is assumed that wear occurs only where there exists asperity contact; it is futher assumed that heat flow is produced solely by frictional dissipation; that is

$$q = -K\frac{\partial T}{\partial z} = \mu_a p_a U \tag{8}$$

The parameter μ_a is the coefficient of friction at asperity spots. The heat flow on the edge of the body will be given by differentiating with respect to z Eqs. (49) and (50) given in reference [18]. With the help of Eq. (8) the local contact pressure at the upper and lower surfaces may be written as

$$p_a = \frac{1}{\mu_a U}[|q_u|\sin(n_u\theta - \hat{\phi}_u) + |q_l|\sin(n_l\theta - \hat{\phi}_l)] \tag{9}$$

Since the positive pressure in Eq. (9) produces an extrusion of the edge of a contacting surface, it will be exlcuded from the contact pressure expression. This defines a new asperity pressure P_a that is periodic in θ ; P_a can be expressed using the Fourier series

$$p_a = \frac{|q_u|}{\mu_a U}\left\{-\frac{1}{\pi}+\frac{1}{2}\sin[n_u\theta - \hat{\phi}_u] + \right.$$

$$\left. +\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos[2n(n_u\theta - \hat{\phi}_u)]}{4n^2-1}\right\}$$

$$+\frac{|q_l|}{\mu_a U}\left\{-\frac{1}{\pi}+\frac{1}{2}\sin(n_l\theta - \hat{\phi}_l)\right.$$

$$\left. +\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos[2n(n_l\theta - \hat{\phi}_l)]}{4n^2-1}\right\} \tag{10}$$

where

$$|q_u| = K_u|T_u|[a_u^2+b_u^2]^{1/2}$$

$$\phi_u = \tan^{-1}[\frac{a_u}{b_u}]$$

$$a_u = \left\{-\frac{\chi_u}{2}+\frac{\chi_u}{2}[\chi_u^2+(\frac{c_u+U}{\alpha_u})^2]^{1/2}\right\}^{1/2}$$

$$b_u = \left\{\frac{\chi_u}{2}+\frac{\chi_u}{2}[\chi_u^2+(\frac{c_u+U}{\alpha_u})^2]^{1/2}\right\}^{1/2}$$

$$|q_l| = K_l|T_l|[a_l^2+b_l^2]^{1/2}$$

$$\phi_l = \tan^{-1}[-\frac{a_l}{b_l}]$$

$$a_l = \left\{-\frac{\chi_l}{2}+\frac{\chi_l}{2}[\chi_l^2+(\frac{c_l}{\alpha_l})^2]^{1/2}\right\}^{1/2}$$

$$b_l = \left\{\frac{\chi_l}{2}+\frac{\chi_l}{2}[\chi_l^2+(\frac{c_l}{\alpha_l})^2]^{1/2}\right\}^{1/2}$$

where c_l and c_u are the wave velocities at the lower and upper bodies given by the reference [18]. The wear displacement due to the contact pressure P_a at the upper and lower surfaces is given by integrating Eq. (7)

$$\delta_w = \delta_{wu} + \delta_{wl} \tag{11}$$

where

$$\delta_{wu} = \frac{w|q_u|}{\mu_a}\left\{\frac{t}{\pi} - \frac{\sin[\frac{\Omega_u t}{2}]}{\Omega_u}\sin[n_u\theta - \frac{\Omega_u t}{2} - \phi_u]\right.$$

$$\left. -\frac{2}{\pi\Omega_u}\sum_{n=1}^{\infty}\frac{\sin[n\Omega_u t]\cos[2n(n_u\theta - \frac{\Omega_u t}{2} - \phi_u)]}{n[4n^2-1]}\right\}$$

$$\delta_{wl} = \frac{w|q_l|}{\mu_a}\left\{\frac{t}{\pi} - \frac{\sin[\frac{\Omega_l t}{2}]}{\Omega_l}\sin[n_l\theta - \frac{\Omega_l t}{2} + \phi_l]\right.$$

$$\left. -\frac{2}{\pi\Omega_l}\sum_{n=1}^{\infty}\frac{\sin[n\Omega_l t]\cos[2n(n_l\theta - \frac{\Omega_l t}{2} + \phi_l)]}{n[4n^2-1]}\right\}$$

Axial Displacements. A rigid body motion between mating rings may be developed due to variations of pressure in the sealing gap. This motion changes the sealing gap in the axial direction.

The total pressure in a mixed friction regime is composed of fluid pressures and asperity contact pressures. If the seal surfaces approach each other closely enough, metal-to-metal contact at asperity spots can help support the load. Otherwise, fluid film pressures provide support. The force due to the film pressure is the main component which acts axially on the sealing interface.

To estimate the fraction of fluid film pressure for the region where asperities on the rotor interfere with asperities on the sealing ring, a model developed by Christensen [21] may by used. It will be assumed that roughness is superimposed over the nominal surface of the seals as shown in Fig. 1. Then the expected value E_f for fluid sealing due to the film pressure in the fluid-metallic contact region is given by

$$E_f = \begin{cases} \frac{1}{32}[16+35\bar{H}-35\bar{H}^3+21\bar{H}^5-5\bar{H}^7] & \text{for } \bar{H}\leq 1 \\ 1 & \text{for } \bar{H}>1 \end{cases} \tag{12}$$

$$E_a = 1 - E_f \tag{13}$$

where E_a is the expected value for supporting the asperity contact pressures. The fluid film pressure P_f can be determined by Eqs. (3) and (8) with the subscript a replaced by f. The result is

$$P_f = \frac{\eta_r G_0 U}{2\bar{h}\mu_f}[1-\frac{\tilde{h}}{2\bar{h}}] \tag{14}$$

where μ_f is the coefficient of fluid friction. The overall separating force F caused by the combined fluid and asperity pressures in the mixed regime may be calculated as

$$F = n\int_0^{2\pi/n}\int_{r_i}^{r_o}[E_f P_f + E_a P_a]rdrd\theta$$

$$= \frac{\pi\eta_r G_0 r_o^3[1-\bar{R}^3]\omega}{3\bar{h}\mu_f}E_f - \frac{2r_o[1-\bar{R}]}{\mu_a\omega}\cdot$$

$$[|q_u|+|q_l|]E_a \tag{15}$$

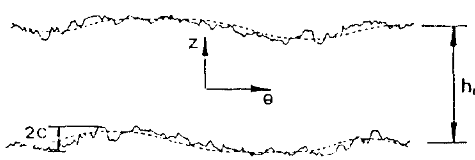

Fig. 1. Roughness distribution on the wavy seals.

The first term represents due to the uneven heating. The second term describes the axial movement caused by wear on the metallic contact zone. Eq. (15) indicates that when the speed is increased, the axial force caused by the uniform heating will dominate the overall axial motion compared with that of the asperity contact. Lebeck [11] has similarly concluded that a high percentage of fluid film pressure can be obtained even for heavily loaded, low viscosity seals if the magnitude of the as-running surface roughness is sufficiently small. The axial displacement due to the overall separating force may be carried by the spring force system as shown in Fig. 2; That is

$$\delta_a = F/k_{eq} \tag{16}$$

where the stiffness k_{eq} is an equivalent spring constant. The macroscopic sealing concepts of equations (15) and (16) are used to produce crude a priori estimates of displacements due to misalignment, waviness, and surface roughness.

Pressure Distribution. The pressure distribution between the sealing gap is given by solving a simplified Reynolds equation (17) in reference [19] using the power series; $P=\sum_{j=0}^{\infty} P_j R^j$ and the boundary conditions; $P_o = P_{in}$ at $R=0$ $(r=r_i)$ and $\sum_{j=0}^{\infty} P_j = P_{ex}$ at $R=1$ $(r=r_o)$. The dimensionless parameters for the recursive equation (21) in the previous paper [19] are given by

$$\hat{H}_0 = 1+a_2\cos\theta - a_3[a_4(1-\cos\theta)+\delta_{cu}+\delta_{cl}$$

$$+\delta_{thu}+\delta_{thl}]+\frac{1}{2\bar{h}}[\hat{h}+\delta_w+\delta_a]$$

$$\hat{H}_1 = \varepsilon\cos\theta + C_1 - a_3[3a_0(1-\cos\theta)+\frac{2}{\bar{a}_2}(\delta_{thu}$$

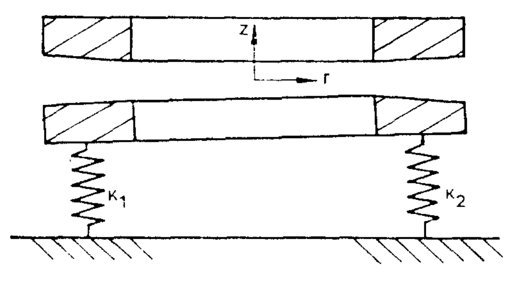

Fig. 2. Seal ring with back-up spring in parallel.

$$+\delta_{thl}))]+\frac{1}{2}a_1C_1\theta^2+\frac{2}{\bar{a}_2}(\delta_{cu}+\delta_{cl})]$$

$$\hat{H}_2=C_2-\frac{a_3}{\bar{a}_2^2}[3a_0\bar{a}_2(1-\cos\theta)+\delta_{thu}+\delta_{thl}$$

$$+\frac{a_1\theta^2}{2}(2\bar{a}_2C_1+\bar{a}_2^2C_2)+\delta_{cu}+\delta_{cl}]$$

$$\hat{H}_3=C_3-\frac{a_3}{\bar{a}_2^2}[a_0(1-\cos\theta)+\frac{a_1\theta^2}{2}(C_1+2\bar{a}_2C_2+\bar{a}_2^2C_3)]$$

$$\hat{H}_i=C_i-\frac{a_1a_3\theta^2}{2\bar{a}_2^2}(C_{i-2}+2\bar{a}_2C_{i-1}+\bar{a}_2^2C_i)\ \text{for } i\geq 4$$

$$A_0=-\frac{\Gamma_2}{\Gamma_1}[(a_2+a_0\bar{a}_2a_3)\sin\theta-\hat{H}'$$

$$+a_3(\delta'_{thu}+\delta'_{thl})+\frac{\delta'_w}{2\bar{h}}]+\frac{\hat{H}'_u}{\Gamma_1}$$

$$A_1=-\frac{\Gamma_2}{\Gamma_1}\{(\varepsilon+3a_0a_3)\sin\theta-a_3[\frac{2}{\bar{a}_2}(\delta'_{thu}+\delta'_{thl})$$

$$+a_1C_1\theta]\}$$

$$A_2=-\frac{\Gamma_2a_3}{\Gamma_1\bar{a}_2^2}[3a_0\bar{a}_2\sin\theta+a_1\bar{a}_2(2C_1+\bar{a}_2C_2)$$

$$+(\delta'_{thu}+\delta'_{thl})]$$

$$A_3=-\frac{\Gamma_2a_3}{\Gamma_1\bar{a}_2^2}[a_0\sin\theta+a_1\theta(C_1+2\bar{a}_2C_2+\bar{a}_2^2C_3)]$$

$$A_i=-\frac{\Gamma_2a_1a_3}{\Gamma_1\bar{a}_2^2}(C_{i-2}+2\bar{a}_2C_{i-1}+\bar{a}_2^2C_i)\theta\quad\text{for } i\geq 4$$

where the primes denote differentiation with respect to θ .

$$a_0=\frac{\alpha_l\varepsilon}{K_l},\quad a_1=\frac{\alpha_u}{K_u}+\frac{\alpha_l}{K_l},\quad \bar{a}_2=\frac{\bar{R}}{1-\bar{R}}$$

$$a_2=\frac{\varepsilon\bar{R}}{1-\bar{R}},\quad a_3=\eta_rG_0(\frac{r_iU}{2\bar{h}})^2,\quad a_4=\frac{\varepsilon\bar{R}\alpha_l}{K_l(1-\bar{R})}$$

Volumetric Flow Rate. The radial flow rate of an incompressible fluid through the sealing gap is defined as

$$Q=n\int_0^{2\pi/n}\int_{z_l}^{z_u} rv_r dzd\theta \tag{17}$$

This equation can be nondimensionalized using the radial velocity in Eq. (13), dimensionless parameters (16), $r=r_i+(r_o-r_i)R$, and pressure distribution (19) given in refernece [19]. Then the volumetric flow rate in dimensionless form is given by

$$\frac{\dot{Q}}{2\bar{h}\omega r_0\bar{w}}=\bar{Q}=n\tau\int_0^{2\pi/n}(B_0+B_1R+\cdots)d\theta \tag{18}$$

where

$$\tau=\frac{e^{\alpha^*T_r(\Lambda_l-1)}}{\alpha_2(1-e^{-\alpha_2})}\{1+\frac{1-e^{\alpha_2}}{[\alpha^*T_r\Lambda_l(\beta_\Lambda-1)]^2}\cdot$$

$$[1+\alpha_2+(1-\alpha_2)e^{\alpha_2}]\}$$

$$B_0=\bar{R}\hat{H}_0^3P_1,\quad B_1=[1-\bar{R}]\hat{H}_0^3P_1$$

$$+[3\hat{H}_1P_1+2\hat{H}_0P_2]\bar{R}\hat{H}_0^2,\ \cdots$$

The dimensionless flow rate may be simplified with $R=0$ because it cannot vary with R. Thus the flow rate equation (18) is given by

$$\bar{Q}=\tau\bar{R}\ P_1\sum_{i=1}^{13} I_i \tag{19}$$

The integration constant I_i is given in Appendix. $K_{11}-K_{13}$, $K_{22}-K_{25}$, and K_{34} are given in reference [19].

Torque. The equation for frictional torque is

$$T_t=\int_0^{2\pi}\int_{r_i}^{r_o}\eta[\frac{\partial v_\theta}{\partial z}]r^2drd\theta \tag{20}$$

By substituting the temperature dependent viscosity (12) and velocity gradient given by Eq. (14) in reference [19] into Eq. (20), the frictional torque can be obtained as

$$T_t=\frac{\pi\eta_r\alpha^*T_r\Lambda_l\bar{w}Ur_0^2(1-\beta_\Lambda)}{\bar{h}(1-e^{\alpha_2})}\sum_{i=1}^{\infty}\frac{1}{i}$$

$$[\bar{R}^2T_{i-1}+2(1-\bar{R})\bar{R}T_{i-2}+(1-\bar{R})^2T_{i-3}] \tag{21}$$

where

$$T_{-2}=0$$

$$T_{-1}=0$$

$$T_0 = 1 + a_3 B^* - \frac{1}{2\bar{h}} [\frac{t}{\pi}(|\delta_{wu}| + |\delta_{wl}|) + \delta_d]$$

$$T_1 = -C_1 + a_3 [\frac{2B^*}{\bar{a}_2} + a_0 + \frac{2\pi^2 a_1 C_1}{3}]$$

$$T_2 = -C_2 + a_3 [\frac{B^*}{\bar{a}_2} + \frac{2}{\bar{a}_2} + (a_0 + \frac{2\pi^2 a_1 C_1}{3}) + \frac{2\pi^2 a_1 C_1}{3}]$$

$$T_3 = -C_3 + a_3 [\frac{1}{\bar{a}_2^2}(a_0 + \frac{2\pi^2 a_1 C_1}{3}) + \frac{2\pi^2 a_1}{3}(\frac{2C_2}{\bar{a}_2} + C_3)]$$

$$T_i = -C_i + \frac{2\pi^2 a_1 C_3}{3}[\frac{C_{i-2}}{\bar{a}_2^2} + \frac{2C_{i-1}}{\bar{a}_2} + C_i] \quad \text{for } i \geq 4$$

and

$$B^* = a_4 - \pi [\frac{\alpha_u \xi_u}{K_u n_u} + \frac{\alpha_l \xi_l}{K_l n_l}] + \delta_{cu} + \delta_{cl}$$

Results and Discussion

The algorithm described in the previous study [19] was implemented and used to explore the seal performance using the data used for the calculation of reference [19].

Figs. 3 and 4 show the variation of volumetric leakage flow rate for various radius ratios from $\bar{R}$ = 0.82 to 0.98. The maximum value of a tilt parameter ϵ is calculated for $(1-\bar{R})/2$ and the wear coefficient w is 0.2×10^{-12} (m^2/N). The reduction of the radius ratios lowers considerally the volumetric leakage rate. Fig. 3 shows the effect of misalignment on the seal leakage flow rate when the geometric components and the thermoelastic deformation caused by the viscous friction are considered. The leakage rate is considerably small in comparison to the result calculated for the geometric components only (Fig. 5 of reference [19]). Every figure appears to have a minimum leakage rate for the whole range of tilt parameter; this is because the thermoelastic deformation reduces the sealing gap for the allowable range of operating conditions. In Fig. 4, the leakage rate due to the geometric components and the operation conditions is plotted against the tilt parameter. It is seen that when the geometric components and the operating conditions are simultaneously considered, the leakage rate in Fig. 4 is high compared with the result shown in Fig. 3. This indicates that the combined effects of wear and axial displacements due to the dynamic movement of the sealing gap may slightly increase the sealing gap. It is interesting to note that in the tilt parameter region where the minimum value of each cruve exists, the leakage rate in the low radius ratio is high compared with the high radius ratio; this is because the combined effects of the geometry and operating conditions play a key role

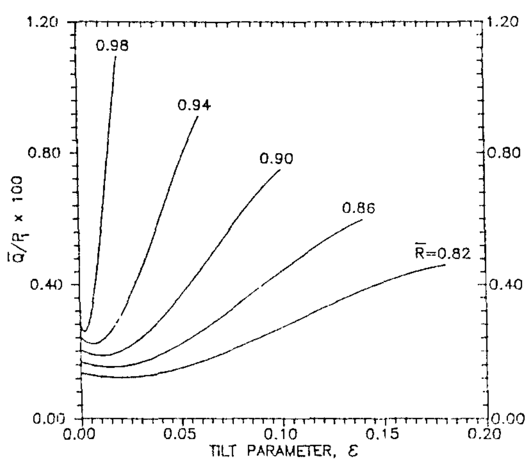

Fig. 3. Effect of tilt parameter on dimensionless leakage flow rate for $2h = 2\bar{h} + h_m + \hat{h} + h_c + \delta_{th}$.

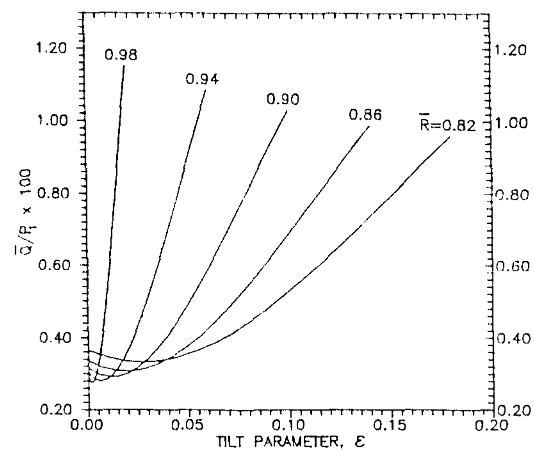

Fig. 4. Effect of tilt parameter on dimensionless leakage flow rate for the overall sealing gap equation (1), n = 4000 rpm, and $2\bar{h}$ = 40 (μm).

even though the influence of the individual component is not high.

Typical results of the frictional torque are obtained for $\overline{R}=0.82\sim0.98$, $\epsilon=(1-\overline{R})/2$ and $n=4000$ rpm. In Figs. 5 and 6, frictional torque is plotted against the mean sealing gap. These figures show the combined effects due to the geometric and operation components may play an important role on torque. Figs. 5 and 6 show that the thermal effects on torque are particularly increase for small sealing gap. Figs. 7 and 8 show the effect of shaft speeds for $\overline{R}=0.82\sim0.98$, $2\overline{h}=40$ (μm), and $\epsilon=(1-\overline{R})/2$ on friction torque. If the geometric terms are only considered, the torque is linearly increasing for shaft speeds as shown in Fig. 7 of reference [19]. But in Figs. 7 and 8, when the radius ratio is decreased and shaft speed increased the friction torque is considerably increased by the thermoelastic effect. When the overall sealing gap components (1) are considered for the result of Fig. 8 the torque

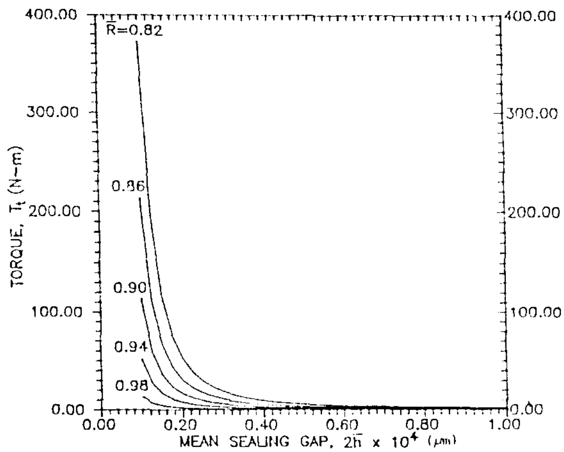

Fig. 5. Torque as a function of mean sealing gap for $2h=2\overline{h}+h_m+\hat{h}+h_c+\delta_{th}$, $n=4000$ rpm, and $\epsilon=(1-\overline{R})/2$.

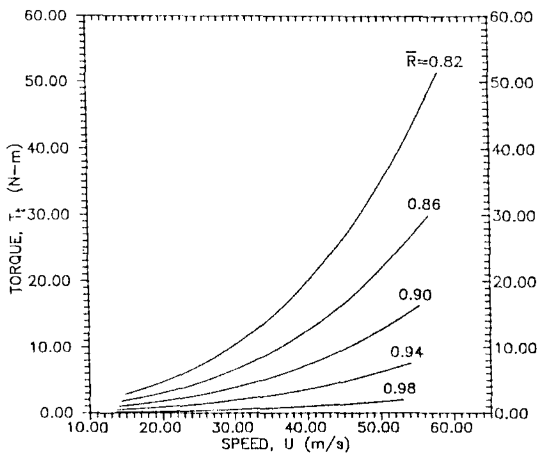

Fig. 7. Torque as a function of shaft speed for $2h=2\overline{h}+h_m+\hat{h}+h_c+\delta_{th}$ and $2\overline{h}=40$ (μm).

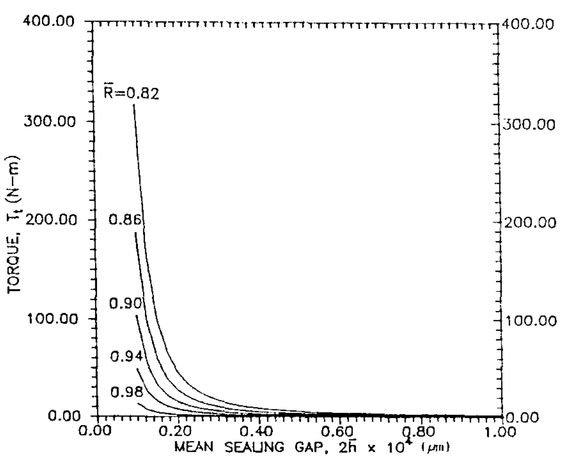

Fig. 6. Torque as a function of mean sealing gap for the overall sealing gap equation (1), $n=4000$ rpm, and $\epsilon=(1-\overline{R})/2$.

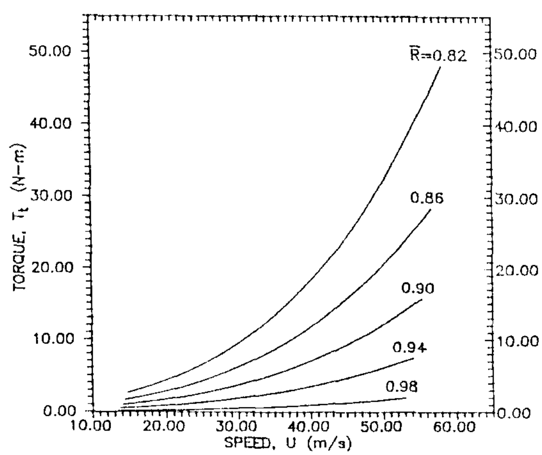

Fig. 8. Torque as a function of shaft speed for the overall sealing gap equation (1) and $2\overline{h}=40$ (μm).

is low compared with the result of Fig. 7. This is because the combined effects of wear and axial displacements may increase the sealing gap.

Conclusions

Using a power series technique, a Reynolds equation having a combination of geometry and operation components was analytically solved for an incompressible liquid with temperature dependent viscosity. This analysis was connected to the volumetric leakage rate and frictional torque of mechanical faces seals.

The results seem to indicate that the thermoelastic displacement term plays an important role on the leakage flow rate and frictional torque. But wear and axial displacements due to the dynamic movement in the axial direction appear to increase the sealing gap. As the speed increases and the sealing gap decreases, the results indicate the importance of the thermoelastic effects on the leakage flow rate and torque. In order to estimate the leakage rate and torque it is recommendable to combine in a single analysis many of the overall sealing gap components including operational conditions as described by the equation (1).

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Nomenclature

c : one-half of the roughness height
C_0, C_1 : coefficients determined by coning effects
H_c : dimensionless initial sealing gap due to coning
h_n : nominal film thickness
$\bar{h}$: half of the mean sealing gap
$\bar{H}$: h_n/c, dimensionless film thickness
n : number of wave
$\bar{R}$: r_i/r_o
t : time
$|T_l|$, $|T_u|$: temperatures of the surfaces at the stator and rotor
$\bar{w}$: $r_o - r_i$
α^* : temperature-viscosity coefficient
α_2 : $\alpha^* T_r \Lambda_l (\beta_l - 1)$
β_l : T_u/T_l
γ : angle of tilt
ϵ : $\gamma (r_o - r_i)/(2\bar{h})$, tilt parameter
κ : $2\pi/\lambda$, wavenumber
λ : wavelength
Λ_l : T_l/T_r
ξ_l : $|\hat{h}_l|/(2\bar{h})$
ξ_u : $|\hat{h}_u|/(2\bar{h})$
ϕ_l : $\Omega_l t - \phi_l$
ϕ_u : $\Omega_u t + \phi_u$
ω : angular velocity
Ω_l : κc_l
Ω_u : $\kappa_u (c_u + U)$

Subscripts

c : coning
Cl : coning at the lower surface
cu : coning at the upper surface
i : inner radius
l : lower surface
o : outer radius
r : reference conditions
u : upper surface

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Appendix : The Appendix of this paper will be continued on the next issue.