Korean Journal of Hydrosciences Vol 1. pp. 73 ~ 97 1990.

Numerical Models of Wave–Induced Currents

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ABSTRACT/A literature review is made on the numerical models of wave–induced currents. The major processes of the flow system are wave breaking, bottom friction of combined wave–current flow and mixing processes primarily caused by wave breaking as well as the flow fields of waves and currents themselves. The survey is given to each item with great emphasis on numerical implication as well as physical mechanism. As noted is the importance in recent investigations, a brief treatment is also given on the currents driven by random or spectral waves.

1. Introduction

Once waves are generated many of them eventually reach the coastal region or man–made structures, and then break and disappear. But before they totally disappear through any dissipation mechanism, some of their energy are transferred to generate unidirectional currents. As the unidirectional currents are far more efficient than the oscillatory waves alone for the transport of beach materials particularly when the currents are combined with the waves, the flow mechanism of wave–induced currents draws great attention to the coastal engineer and it has been recognized that the proper description of the wave–induced currents is vital to the prediction of coastline change or topographical deformation in the coastal region.

The generation of wave–induced currents, particularly rip currents, was a big concern in the military landing operation during the last world war. The awareness has made the engineers investigate the problem more seriously and hence our history of the study in fact started with the military purpose. A lot of field measurements were taken along with some theoretical works.

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However, the most important step has been made by Longuet–Higgins, who has developed the concept of radiation stress(Longuet–Higgins and Stewart, 1960) and shown that the currents are produced by the spatial gradients of radiation stresses(Longuet–Higgins, 1970). He indicated that the bottom friction is the major force for balancing the driving forces driven by the radiation stresses and the mixing process is mostly responsible for the smooth profile of longshore currents. See Fig. 1. He also developed an analytical solution for a uniform, plane beach, after linearizing the expressions of the influencing factors. Since the epochal studies of Longuet–Higgins, engineers have tried to develop mathematical models, certainly numerical models for general application, for the description of wave–induced currents along with more theoretical, field and/or laboratory works on each item independently or on the whole process together.

The evolution of its numerical study has also grown significantly with furthering our understanding of the physical process in detail, perceived primarily from laboratory experiments. In this review, various numerical models are presented which are considered to have made important progress from predecessors with regard to numerical technique, mathematical formulation or numerical implication of physical concepts. Because many research groups are involved world –widely in this study and the number of papers published exceed the limit of one's personal capacity while he does his own modelling work, some of the important works may be missing in the survey but it was tried to include all non–negligible physical processes involved in the mechanism.

Extensive reviews of surf zone currents, which is nearly the same subject as the present one apart from the numerical side, were made by Basco(1983) and later by Battjes(1988) and Arcilla(1989). Various aspects of the physical mechanism involved in the subject were explained relatively in detail and some of the numerical aspects and mathematical formulations were also discussed in the reviews. But the areas of the numerical implication are found to be insufficient in the works particularly when one really wishes to develop his own numerical model or to refine any existing model. Therefore, the present work lays much emphasis on the numerical implication of various physical processes found in the theory and laboratory or field measurement.

As described in the schematic diagram of the flow system(Fig. 1) the numerical model system of the wave–induced currents may have to contain two primary units:

- waves
- currents

and various secondary units:

- radiation stress
- wave breaking
- mixing processes
- bottom friction

The main programme may consist of the two primary units and the secondary units might be

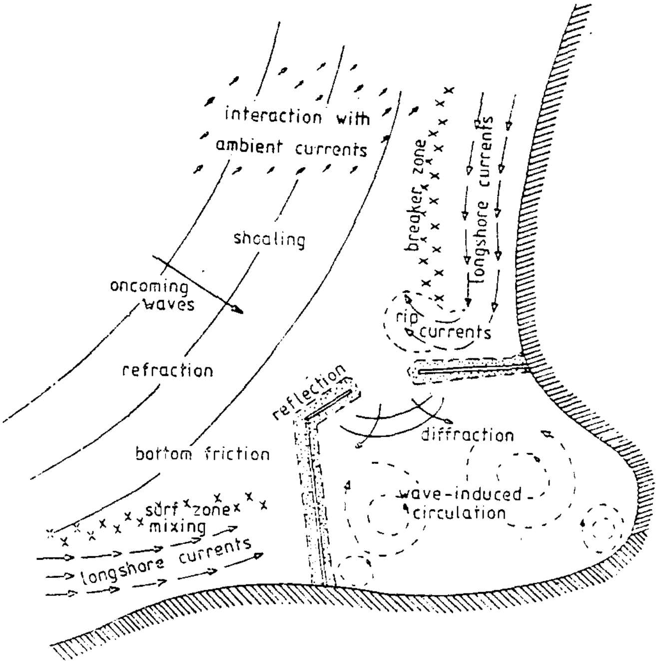


Fig. 1 Skematic diagram of hydraulic environments in coastal region.

included in several subroutines respectively or simply expressed using appropriate parameters or criteria. The survey follows on each item specifically, but synthetic views are also given when they are considered to be necessary.

2. Wave Models

As we treat the currents induced by waves, a proper wave model is essential for the accurate estimation of current field. The wave–induced currents being driven by spatial gradients of radiation stresses which are normally significant where waves break or diffract behind structures, the model should be capable of handling wave diffraction as well as refraction and shoaling. The wave breaking mechanism may not be sufficiently descriptable by pure mathematics, but its description much relys on the empiricism for engineering usage. It will be treated in Section 5. After inshore waves generate currents, they will be further transformed by currents driven by themselves. Therefore, consideration should be given to full interaction between waves and currents in the numerical modelling even if ambient currents do not exist in the investigation area.

A literature survey on shallow water wave models has been made by the author in recent years(Yoo, et al., 1988). The summary is given in Tabel 1. The models are classified according to the stage of model development and the type of governing equations, which may determine the

Table. 1 Numerical models of wave climate under refraction, diffraction, reflection and current-interaction

Model Types		Basic Equations	Solution Techniques	Comment	References
1st Stage	Boussinesq (I)	$\frac{\partial \zeta}{\partial t}+\frac{\partial R_i}{\partial x_i}=0$ $\frac{\partial R_i}{\partial t}+\frac{\partial}{\partial x_j}(\frac{\sigma}{d}R_iR_j)+gd\frac{\partial \zeta}{\partial x_i}$ $=\frac{h^2d}{3}\cdot\frac{\partial^3}{\partial x_i\partial x_j\partial t}(\frac{R_j}{d})$	ADI	Valid only in shallow water, good for nonlinear transfer and reflection.	(1), (22) (39)
	Elliptic (II)	$(\frac{\partial}{\partial t}+U_i\frac{\partial}{\partial x_i})(\frac{\partial \zeta}{\partial t}+U_i\frac{\partial \zeta}{\partial x_i})$ $+\frac{\partial U_i}{\partial x_i}(\frac{\partial \zeta}{\partial t}+U_i\frac{\partial \zeta}{\partial x_i})-\frac{\partial}{\partial x_i}nC^2\frac{\partial \zeta}{\partial x_i}$ $+(1-n)\sigma_0\zeta=0$	Implicit	Valid in all depths good for reflection.	(25)
	Hyperbolic (III)	$\frac{\partial \zeta}{\partial t}+(1+\frac{K_iU_i}{\sigma_0})\frac{1}{n}\frac{\partial}{\partial x_i}(nR_i)=0$ $\frac{\partial R_i}{\partial t}+(1+\frac{K_iU_i}{\sigma_0})C^2\frac{\partial \zeta}{\partial x_i}=0$	ADI or Explicit	Valid in all depths, good for reflection.	(16), (36) (56), (57)
2nd Stage	Parabolic (IV)	$i\{2\sigma_0\frac{\partial A}{\partial t}+2\sigma_0U_i\frac{\partial A}{\partial x_i}+\frac{\partial}{\partial x_i}(U_ia)$ $+A\frac{\partial}{\partial x}(\sigma_0nC)+2\sigma_0nC\frac{\partial A}{\partial x_i}\}-\frac{D^2A}{Dt^2}$ $-\frac{\partial U_i}{\partial x_i}\frac{DA}{Dt}+\frac{\partial}{\partial x_i}(nC^2\frac{\partial A}{\partial x_i})=0$	ADI (one sweep)	External diffraction is not accounted, weak reflection.	(58)
	Elliptic (V)	$\frac{\partial}{\partial x_i}(U_i+nC_i)A^2=0$ $K^2=k^2+\frac{1}{a}\frac{\partial^2 a}{\partial x_i^2}$	One-direction expicit	no reflection	(17)
	Hyperbolic (VI)	$\frac{\partial A}{\partial t}+i\sigma_0A+[1+\frac{K_iU_i}{\sigma_0}]\frac{1}{n}\frac{\partial}{\partial x_i}(nP_i)=0$ $\frac{\partial P_i}{\partial t}+i\sigma_0P_i+[1+\frac{K_iU_i}{\sigma_0}]C^2\frac{\partial A}{\partial x_i}=0$	ADI or Explicit	Good for any type diffraction and reflection.	
3rd Stage	**Hyperbolic** (VII)	$\frac{\partial a}{\partial t}+\frac{1}{2a}\frac{\partial}{\partial x_i}(U_i+nC_i)a^2+\frac{S_{ij}}{\rho ga}\frac{\partial U_j}{\partial x_i}=0$ $\frac{\partial K_i}{\partial t}+(U_j+nC_j)\frac{\partial K_i}{\partial x_j}+K_j\frac{\partial U_j}{\partial x_j}$ $+\frac{\sigma_0(2n-1)}{2h}\frac{\partial h}{\partial x_i}+\frac{nC}{2ka}(\frac{1}{a}\frac{\partial a}{\partial x_i}\frac{\partial^2 a}{\partial x_i^2}$ $-\frac{\partial^3 a}{\partial x_i\partial x_j^2})=0$	Upstream Explicit	Good for internal and external diff. no reflection.	(51)

Symbols

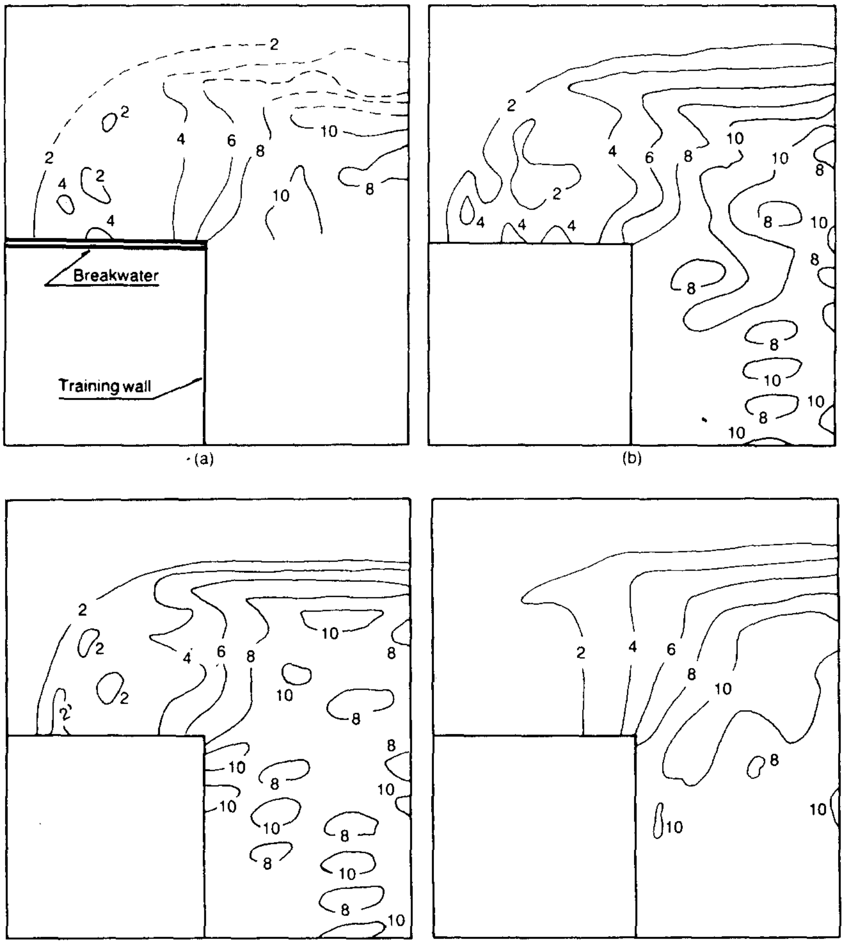
a=wave amplitude; $A=a/\sigma_0$; $C=\sigma_0/K$, celerity; $d=\eta+h$; g=gravity acceleration; h=period-average water depth; $i=\sqrt{-1}$ or i, j=1, 2; k=separation factor; K=wave number; n=group velocity/celerity; P=a/K; $R=\eta/K$; S_{ij}=radiation stresses; U=current velocity; $\zeta=\eta/\sigma_0$; η=surface elevation; σ=apparent frequency; σ_0= Doppler-shifted frequency.

range of application and efficiency of each model. They are categorized into three groups, namely 1st stage inter–period models, 2nd stage evolution models and 3rd stage period–average models. The basic equations of the 1st stage models are obtained by depth–integrating the primitive mass and momentum(Navier–Stokes) equations, and those of the 3rd stage models by further period–averaging the depth–integrated energy equation. Kinematic conservation equation is supplemented to the energy conservation equation for the completion of the 3rd stage models. For the development of the 2nd stage evolution models, harmonic forms of the primary variables are introduced into the basic equations of the 1st stage models and the time–independent amplitude terms are extracted to form the basic equations of the 2nd stage models. Most of the wave models were used for the study of wave–induced currents, but some of the early forms might have only historical values and such models are now rarely employed. Nonetheless, it is worth mentioning them, because they were the foot steps for the development of the latest comprehensive models and the problems found in their usage drove the refinements on such directions.

Due to historical reasons, the first numerical study of wave–induced currents employed one of the 3rd stage wave models, i.e. ray tracking approach. The ray tracking approach is not necessarily simpler than the others, but it was the only possible solution technique in those days with expecting reasonable computation time. Noda(1974) was amongst the first modellers to attempt a numerical description of the wave–induced currents. In order to estimate properly the gradients of radiation stresses, the ray tracking approach was modified so that the wave conditions were specified at predetermined grid points. Full interaction between waves and currents was tried to be taken into account, but the model seriously suffered from numerical instability. The degree of interaction was arbitrarily decreased in order to obtain solutions.

Successive refinements of Noda's numerical model have been made by Dalrymple and his co–workers. Birkemeier and Dalrymple(1975) successfully completed the full interaction of waves and currents using the time–varying form of the wave energy equation. They found that their success was due to the slow manner in which the longshore currents were built up in the model. However, the model was unable to handle waves in caustics or diffractive waves.

To account for the effects of diffraction, the kinematic conservation equation or wave number vector equation has been modified in recent year(Yoo and O'Connor, 1986, 1988) using the empirical dispersion relation of Hedges(1976) and Battjes eiconal relation, which tells that the wave number is not anymore the same as the separation factor given by the frequency dispersion relation when amplitude curvature(second order derivatives of amplitude) exists. See the equations(Ⅶ) shown in Table 1, noting the last terms related to the amplitude curvature. The amplitude curvature is solely responsible for the diffraction effect and the inclusion of the amplitude curvature ensures the kinematic conservation when diffraction occurs. The model was successfully used for the description of wave–induced currents behind a semi–detached breakwater where wave diffraction is obviously the major process of the flow system(Yoo and O'Connor, 1986). No instability problem has occurred, and the agreement between the computational results



(a) (b)

(c) (d)

Fig. 2 Wave height distributions in metres: (a) laboratory data ; (b) hyperbolic inter-period model with current interaction ; (c) hyperbolic inter-period model without current interaction ; (d) hyperbolic period-average model with current interaction.

and laboratory results is good although some disagreement is found near coast where reflection effect is considered to be significant, see Figure 2.

The kinematic conservation equation was also successfully used by Yamaguchi(1986) for his description of wave–induced currents, but the diffraction term was not included in his model. Instead nonlinear effects were accounted for based on a cnoidal wave theory. He found that the wave distribution was detectably improved but the current distribution was hardly altered by the inclusion of nonlinearity.

Because wave reflection is not largely negligible in the region of wave induced currents, particularly where man–made coastal structures are located, consideration on the wave reflection is generally recommendable. Proper description of wave reflection can be achieved only when the full interaction between the incoming waves and the reflected waves are taken into account. The 1st stage models are most suitable for the consideration of wave reflection. The hyperbolic 1st stage linear model was used by Watanabe and Maruyama(1984) and Copeland(1985a) for the description of wave–induced currents but without considering the interaction effect of current on wave transformation. After wave condition was used for the generation of currents, the wave condition was assumed to be frozen and no more interaction was considered. Big discrepancies were found in both wave and current fields when Copeland applied his model to the laboratory situation of semi–detached breakwater.

The refinement of the hyperbolic linear model was made, for the inclusion of current–interaction effect on wave transformation, almost at the same time by three research groups(Dong, 1987 ; Ohnaka, et al., 1988 ; Yoo, et al., 1989 or Yoo, 1989a). See the equation Ⅲ developed by Yoo, et al.(1989) in Table 1. All of them applied their models to the flow system of wave–induced

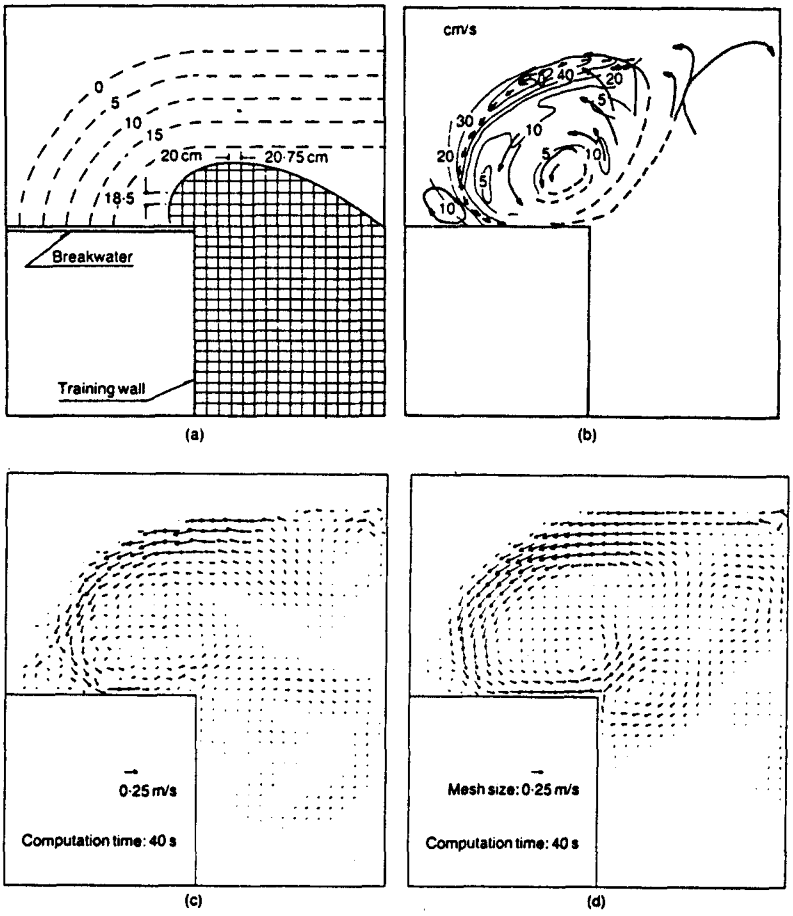


Fig. 3 Finite difference mesh systems for Gourlay's laboratory tank (1) and wave induced currents. (b) laboratory data ; (c) computational result associated with hyperbolic inter-period wave model ; (d) computational result associated with hyperbolic-average wave model

currents with good success. When the refined model was applied to the same situation of semi–detached breakwater(Gourlay, 1974), much improvement was made in both flow fields, see Fig. 2 for the wave field and Fig. 3 for the current field(Yoo, et al., 1989).

In general real seas have in fact random and spectral characteristics, and the flow mechanism driven by random waves is different from that driven by monochromatic waves with regards to wave–induced currents primarily in two aspects. One is the view that the breaker zone becomes broad while the breaker of the monochromatic waves has in principle a sharp edge line. Therefore, the mixing processes of the currents are strongly influenced by the random breaking motion. It will be discussed in Section 6. Another is the fact that waves of higher or lower frequency are usually generated by non–linear interaction. It was noted that the wave energy growth is significant particularly in the high frequency band(Dingemans, et al., 1986).

Spectral waves can be modelled using one of the 3rd stage models with superposition technique(Yamaguchi, 1988) or global parameterization technique(Dingemans, et al., 1986). The inclusion of nonlinear generation mechanism does not seem to be impossible using one of the 3rd stage models particularly in the non–breaking and relatively deep area. However, in shallow water region where the nonlinear interaction effect is more pronounced, the 3rd stage model may find some difficulty in handling the mechanism.

The nonlinear 1st stage model might be most suitable for the description of wave–induced currents especially when the nonlinear interaction effect is significant. Although the model of 2–D version has been developed by Abbott, et al.(1978), the description of wave–induced currents was deferred probably due to the probelm of handling the breaking waves in the surf zone. Hauguel and Pechon(1986) employed the Serre–type nonlinear equations to describe the wave–induced currents on groyned uniform beach. They simulated the breaking waves with including mixing terms. A constant value was chosen for the dispersion coefficient from the breaker line to the coast, and no dispersion was imposed outside the breaker line. Although the computational results do not yet seem to be satisfactory and the nonlinear interaction mechanism was not considered in their modelling since only monochromatic waves were tested, it promises us to have really accurate solution technique in the near future for the description of nearshore current induced by random waves. The nonlinear 1st stage model has also been modified for the transformation of waves on ambient currents in recent years by Prüser and Zielke(1989).

Another nonlinear 1st stage model, the so called "unified model", may have to be mentioned, which was developed by Witting(1984), because it has various interesting features. The basic equations have no third order time derivatives and hence they can be solved using an explicit technique, while the equations of the other models may have to be solved only using an imlicit technique. Solving the wave equations in an explicit way is presumably better for considering the full interaction with current and for handling breaking waves than in an implicit way. Furthermore the model is applicable to fairly deep water condition up to Kd = 8, while the previous nonlinear models would produce significant error when Kd excceds the value of unity. The model is still at the early stage of development and needs much refinement to account for 2-dimensional case in plan and the wave-current interaction. When such refinements are achieved, our modelling capability will approach nearer to the perfection .

The 2nd stage, evolution, models are also attractive for the description of the wave field in the flow system, since they are almost independent of the wave length in the choice of grid size(Kirby, 1984, ; Yoon and Liu, 1986 ; Ebersole, 1986 ; Madsen and Larsen, 1987). Particularly the hyperbolic evolution model developed by Madsen and Larsen draws great attention to us, because it matches the hyperbolic inter-period model in handling reflected waves while saving some computer storage and the computation time if rough resolution were possible. Port designers or coastal planners may also eagerly wait for its application to this flow system. If a dynamic nesting procedure were implemented in the model grid system which is possible for the wave length-independent-resolution model, the model will have a great value for engineering application.

3. Current Models

We have a great variety in the choice of wave model, at least among 7 different types of basic forms of equations as shown in Table 1. The variety will further increase if numerical techniques are concerned, i.e. whether we choose a method between "finite element" and "finite difference", or a method between "explicit" and "implicit". On the other hand, fortunately enough the choice of current model is relatively restricted to one form of equations, i.e.

$$\frac{\partial \eta}{\partial t}+\frac{\partial}{\partial x_i}(dU_i)=0 \tag{1}$$

$$\frac{\partial U_i}{\partial t}+U_j\frac{\partial U_i}{\partial x_j}+\frac{1}{\rho d}\frac{\partial S_{ij}}{\partial x_j}+g\frac{\partial \eta}{\partial x_i}+\frac{1}{d}f_u \mid U \mid U_i=\frac{\partial}{\partial x_j}\nu_t\frac{\partial U_i}{\partial x_j} \tag{2}$$

where η is the mean surface elevation, U_i is the depth-mean current velocity tensor, d is the water depth, ρ is the water density, g is the acceleration due to gravity, f_U is the friction coefficient associated with U_i, ν_t is the coefficient of mixing processes, and finally S_{ij} is the radiation stress tensor driven by wave motion.

The radiation stress is not the only one that is responsible for the generation of wave-induced currents. As recently emphasized by several researchers the so-called undertow mechanism is largely caused by the mass flux due to wave motion and large vortices formed after wave breaking(Svendsen, 1984). Though, however, the sea-ward return flow is clearly detected in the case of wave propagation normal to the beach, it is uncertain whether the role of mass flux is as important as that of radiation stress in a general 2-dimensional situation. For this reason our discussion is made primarily on the currents induced by radiation stresses, strictly speaking their spatial gradients. The subject will be more clearly defined when more experiences are gathered using 3–D or Quasi–3–D models, for example Stive and de Vriend(1987).

Noda may also be named as the first man who solved the equations (1) and (2) using a numerical technique. Although there has been a long history of numerical model development for the general forms of the equations, the inclusion of the third term of equation (2) is regarded to be made first by him. The terms of advection and diffusion being excluded, he might not have found any numerical difficulties apart from the problem of full wave–current interaction. The problem seems to be primarily caused by the defect of his wave model as discussed in the previous section.

The most difficult feature he met in the modelling seems to be the lateral boundary condition which is unique for the modelling of wave–induced currents and probably still problematic to most modellers. The conditions to be reproduced on the solid boundary at the coastline and the open offshore boundary are straight–forward, i.e. no flow at both boundaries. However, since the lateral boundaries normally include surfzones where both wave and current flow fields vary rapidly, the treatment of the flow conditions on the lateral boundaries becomes far more difficult. Noda(1974) proposed the use of a periodic lateral boundary condition which assumes that the hydraulic motion at each row parallel to the beach repeats periodically with a particular spatial interval.

Allender, et al.(1978) applied the model of Birkemeier and Dalrymple(1975), which may be in principle almost the same as the Noda's model, to the field situation in the Great Lakes in Canada, and found some interesting results. By progressively extending the alongshore length of the model, while employing the periodic boundary condition, they found that the numerical results showed insensitivity to the longshore length used after a certain limit(about 200 meters in their case study). This does not necessarily mean that the periodic condition is correct, but it would indicate that it might be acceptable in certain areas, probably where sea bed contours do not change rapidly. The comparison between model results and observed data showed some discrepancy, however. For example, the longshore current profiles across the surf zone computed by the model generally displayed a much greater rate of change than those observed in the field. From this comparison, they concluded that provision for horizontal mixing of momentum would improve the model results.

Ebersole and Dalrymple(1980) included the nonlinear convective acceleration terms and lateral mixing terms, but excluded the current interaction on waves. On the other hand Da Silva Lima(1981) attempted to include all the above processes and tested his findings against his own laboratory data of groyned beach case, but was not able to achieve a proper solution. He found that after a certain number of time steps, the wave heights in the offshore area grew too high due to a strong interaction with the induced currents. In contrast to the Ebersole and Dalrymple's attempt, he decided to exclude the convective acceleration terms, while retaining full interaction between waves and currents.

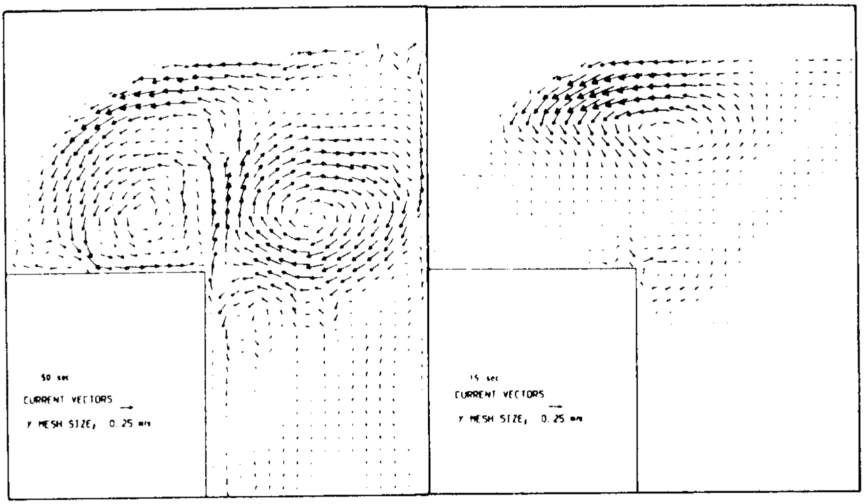
Several investigations have been made using finite element techniques. Bettess, et al.(1978) took into account convective accelerations and the set–up contribution to the mean surface elevation, while Liu and Lennon(1978) excluded them. Both models estimated the wave field using the early type of 1st stage elliptic model, and hence ignored the influence of current on wave transformation and the flooding and drying of flat areas.

Since water levels in a coastal region can change significantly due to tide, wave and wind action, it follows that any coastal model should be able to cope with situations involving the "wetting" and "drying" of computational grids. Birkemeier and Dalrymple(1975) proposed a simple test to allow a dry mesh to be flooded. When the set–up at the landwardmost wet grid became greater than the depth of the adjacent dry grid, the dry grid was set to be flooded. This was accomplished by moving a small percentage of the difference between the surface elevation of the wet block and the bottom elevation of the dry block. The technique, however, found some problem. That is, their wave model did not converge accurately if mixed wet and dry block existed in the same computational row, and hence provision was made to allow the set–up to build up until all grids in a dry row are flooded. It is not clear whether the need for the wetting of a whole row was because of the method of checking the flooding procedure or because of the inaccurate wave model. The technique also has the disadvantage that the scheme allows flooding in the cross–shore direction only and not in the longshore direction. Extension to 2–dimension is possible, but a very complex procedure may be required since it needs to be imposed in four directions and special care must be taken not to violate the mass continuity.

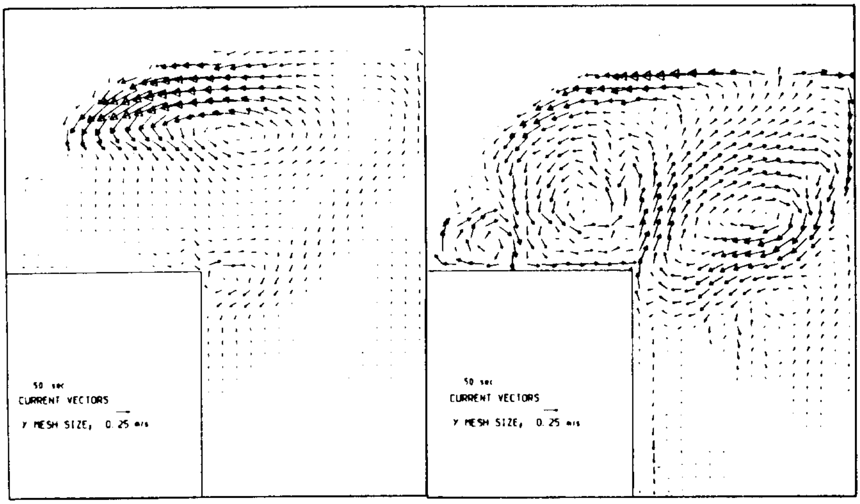
On the other hand, the scheme developed by Flather and Heaps(1975), which was used for tide modelling was implemented for modelling of wave–induced currents with minor modification(Yoo and O'Connor, 1986). The scheme checks the wet condition at the velocity point instead of the surface elevation point, which is considered to be safer for ensuring the mass continuity. Two meshes directly related to a velocity point are checked to see if they were wet or dry. When one of them is dry, further consideration is given whether the surface gradient has potential enough to flood the dry mesh.

A test has been made using the 3rd stage wave model for demonstrating the significance of the role of each item ; advection, current–interaction and flooding(Yoo and O'Connor, 1986). The computational results at 50 seconds were produced excluding each item and compared with the results from the laboratory measurement or the complete scheme, as shown in Fig. 4. Probelms were found when excluding the advection terms. The flow developed so rapidly near the coastline that the whole system eventually became unstable at a run time of about 18 seconds. Therefore, the intermediate result at 15 seconds was presented for the no–advection case.

Both schemes of no–advection and no–current–interaction are shown to be incapable of representing the small gyre at the left–hand corner behind the breakwater, while the center of the main gyre is located a considerable distance from the measured one. Totally wrong current patterns are thus given by the no–advection and no–current–interaction schemes. In contrast, the no–flooding scheme seems to have described the current pattern nearly as well as that of the complete scheme.



(1) complete (2) no-advection



(3) no-current-interaction (4) no-flooding

Fig. 4 Velocity distributions of wave-induced currents from different versions of the present model.

However, it generally over–estimates the current velocity particularly near the coast ; the maximum velocity of the main gyre being predicted at over 65 cm/s compared with about 50 cm/s from the measurement and the maximum velocity of the small gyre at over 25 cm/s compared with about 10 cm/s from the measurement. It is concluded that the energy to be used for flooding is falsefully directed to increase the current velocity when the flooding process is not undertaken.

The modelling technique of current field does not vary significantly whether we use the 1st stage wave model, 2nd stage wave model or 3rd stage wave model, except one aspect, i.e. the estimation of the radiation stresses. As the 3rd stage model produces the period–average parameters at each time step, the values of the depth–integrated, period–averaged radiation stresses are given immediately at any time when they are required. However, the 1st or 2nd stage models need an accumlation process for the estimation, since the primary variable of the models vary over the whole wave period. It may also be possible to use the depth–integrated radiation stresses at each computational time step without the averaging process beforehand. In this case the accumulation process is not required for the estimation of the period–averaged radiation stresses. In the following section it will be further discussed.

4. Radiation Stress

Due to the presence of waves a net momentum flux over a wave period is produced in the fluid. The excess momentum due to waves or the so–called radiation stress is obtained by subtracting the momentum flux due to the hydrostatic pressure from the total momentum flux. Integrating it over a verical and averaging it over a wave period, the radiation stress tensor is given by(Longuet–Higgins and Stewart, 1960):

$$S_{ij} = \langle \int_{-h}^{s} (\rho\, \tilde{u}_i \tilde{u}_j + p\,\delta_{ij}))dz - \int_{-h}^{\eta} p_0\,\delta_{ij} dz \rangle \tag{3}$$

where s is the surface elevation, h is the bottom elevation, ũ is the wave particle velocity, p is the dynamic pressure, p_0 is the hydrostatic pressure, δ_{ij} is the Kronecker delta, and the brackets ⟨ ⟩ is the period–averaging operator.

For the progressive linear waves, employing a potential formulation the expression can be further simplified so as to give(Yoo and O'Connor, 1986):

$$S_{ij} = \frac{1}{2} \{(1+G)\frac{K_i K_j}{KK} + G\,\delta_{ij}\}\, E \tag{4}$$

where K_i is the wave number tensor, E is the wave energy density and G = 2Kd/sinh 2Kd. For waves in caustics some modification was found in the form(Yoo, 1986), but its effects are recognized to be generally ignorable. The equation (4) has widely been used when the wave field is specified by employing one of the 3rd stage models which provide the wave amplitude and wave number vector or direction in an explicit way.

Watanabe and Maruyama(1984) have produced an expression of the radiation stress for its use in the modelling when employing the 1st stage wave model. But the expression is essentially the same as the equation(4) so that any features of wave reflection do not seem to be included. The expression has also been used for the refined model which accounts for the current interaction on wave transformation as presented by Ohnaka, et al.(1988). On the other hand Copeland(1985 b) has developed a rather complicated method of computing the radiation stresses. The terms of cross products were considered in his method when introducing the vertical momentum equation to compute the water pressure under wave motion, expecting the effects of reflection are well included. It is, however, thought that the later introduction of cross product terms for the estimation of pressure does not guarantee the accuracy of the method because the velocity potential used for the formulation is developed excluding the effects of product terms and hence it may possibly add only unnecessary complexity(Yoo, 1989a).

The wave particle velocity can be expressed by using the variable of the 1st stage model R_i and the depth integration of pressure by the combination of the wave amplitude and the surface elevation(Yoo, 1989a).

Then the depth–integrated form of the radiation stress tensor is given by:

$$\tilde{S}_{ij} = \rho g \{ nK^2 R_i R_j + (n - \frac{1}{2}) \sigma_0^2 \zeta^2 \sigma_{ij} \} \tag{5}$$

and its period–average form simply by averaging the time–varying variables:

$$S_{ij} = \rho g \{ nK^2 \langle R_i R_j \rangle + (n - \frac{1}{2}) \sigma_0^2 \langle \zeta^2 \rangle \delta_{ij} \} \tag{6}$$

where $n = (1 + G)/2$, and the primary variables are explained in Table 1. In the modelling either of the forms may be used for the estimation of radiation stresses but in a different fashion. When equaiton (5) is used, a continuous full interaction is able to be taken into account. But equation(6) being used instead, a discrete form of interaction may have to be carried out since the accumulation process may need no interruption over a wave period.

As noticed from the governing equations of momentum(eqs. I in Table 1), the nonlinear 1st stage wave model already contains the terms of advection and high–order surface elevation which are in fact the excess momentum fluxes of wave motion. It is, therefore, not necessary to employ any further expression to estimate the radiation stresses. Using a post–processor of time–averaging after running only the wave model, it is possible to extract the time–average parts of velocity, i.e. the wave–induced currents. For the description of surf zone currents, the major difficulty in the usage of the nonlinear model, however, lies in handling the breaking waves, possibly because an implicit technique was so far used for its solution.

Figure 5 demonstrates the influence of breakwater reflection on the distribution of wave–induced currents around a semi–detached breakwater. When the reflection is none, i.e. if the whole wave energy is absorbed by the breakwater, the energy gradient at the front face of the breakwater becomes almost zero so that the driving forces for the current generation, radiation stress gradients, become negligible. The resulting current pattern for this case is almost equivalent to the one for the case of training wall deployment as shown in Fig. 3. Stronger longshore currents are, however, found to develop with higher reflection, because higher wave energy is produced at the front face of the breakwater. As clearly demonstrated in the results, higher reflection causes not only higher wave heights but also stronger longshore currents in front of the breakwater.

5. Wave Breaking

The transformation of waves in the surf zone is eventually controlled or limited by wave breaking which results from the loss of stability of wave formation and in turn provides a major contribution to surf zone mixing processes. It has been common practice to describe the breaking process by a simple criterion which allows the wave growth only up to a certain limit.

The criterion developed by Miche(1944) has been widely used for numerical computation due to its mathematical reliability and broad coverage from deep to shallow water. In the late 1960's it was realised from experimental evidence that beach slope influenced the breaking mechanism and criterion, To include the slope effect, Miche's criterion was reworked by Battjes and Janssen(1978) as follows:

$$a_b = \frac{\pi}{7K} \tanh(qKd)_b \tag{7}$$

where q_b is a parameter which considers various factors influencing the breaking mechanism. It may be roughly estimated by $q_b = 7\gamma_b/\pi$ where γ_b is the ratio of wave amplitude to the water depth at the breaking point $(a/d)_b$. In this formulation, they matched equation (7) with the breaking criterion for solitary waves in very shallow water, i.e. when $Kd \rightarrow 0$ to find the value for q_b or γ_b. According to Madsen(1976), γ_b is given by:

$$\gamma_b = \begin{cases} 0.59 & m > 0.1 \\ 0.36 + 2.3m & m < 0.1 \end{cases} \tag{8}$$

where m is the bed slope.

Sakai and Saeki(1984) have proposed that for waves on opposing currents wave breaking may be characterized by a normalized unit discharge defined as $q^* = Ud/g^2T^3$ where T is the wave period. Their modified form of Miche's criterion is slightly different from equation(7), but it was found the effect of q^* is no more than the Doppler effect(Yoo, 1990).

On the other hand the parameter q_b was correlated to the Iribarren number $I = m/\sqrt{H/L}$, which implies the wave steepness as well as the bottom slope. A number of laboratory data sets were examined and the following relation was found(Yoo, 1986):

$$q_b = 0.8 + \tanh(1.06I) \tag{9}$$

The data used did not account for the effect of current–interaction, but the inclusion of Doppler relation may be sufficient for this effect.

Although the breaking criterion accounts for a number of factors involved in the mechanism, it may only comfortably be used for the 3rd stage wave models since various parameters are related to the period–average properties of wave motion. It is not clear how the various factors can be

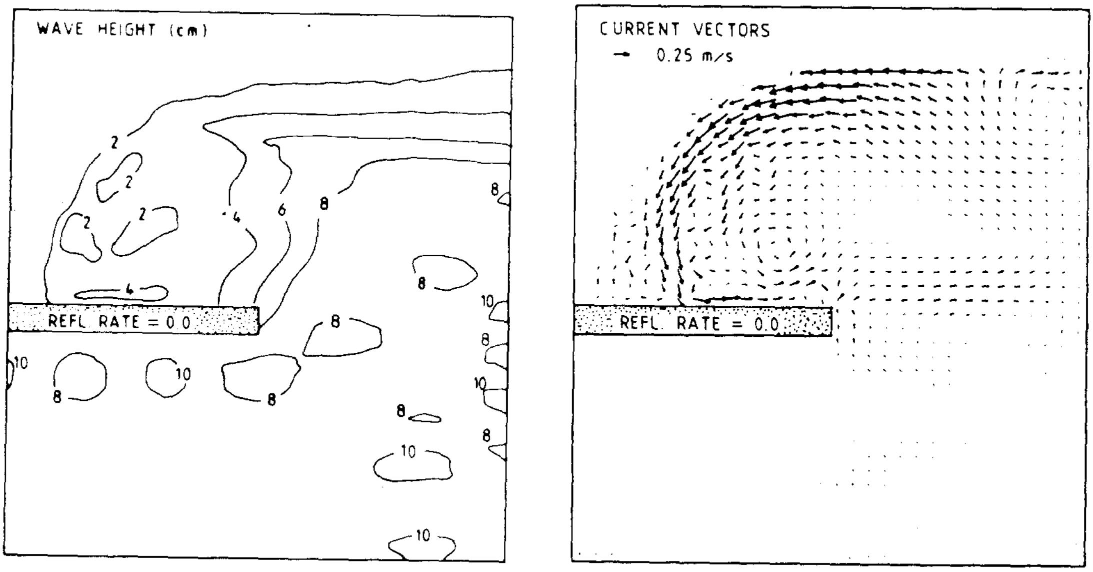


Fig. 5(1) Distributions of wave height and wave induced currents with reflection coefficient = 0.0

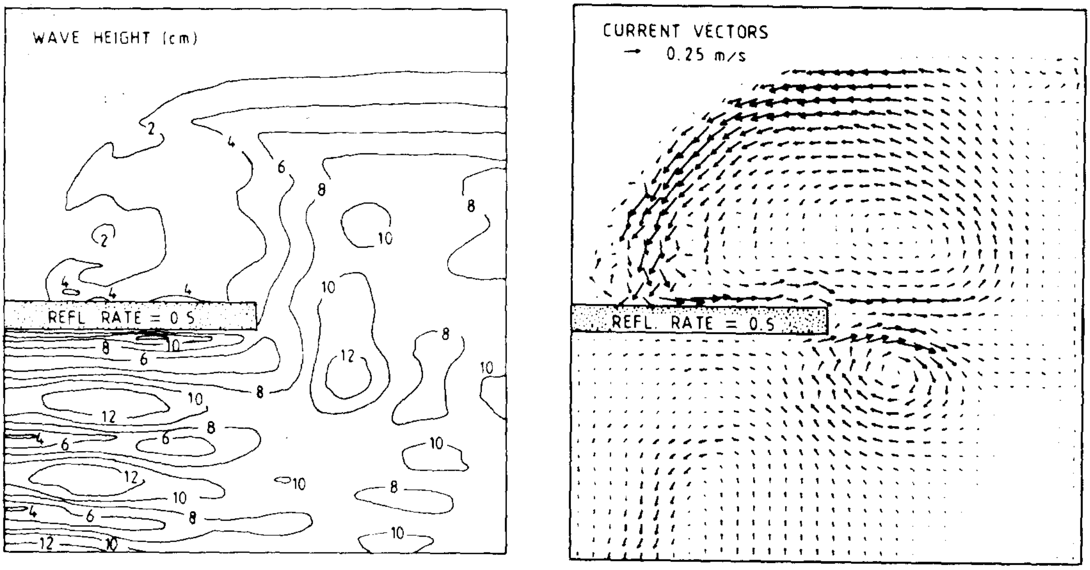


Fig. 5(2) Distributions of wave height and wave-induced currents with reflection coefficient = 0.5

accounted for when using the 1st or 2nd stage wave models. The present technique used for the 1st stage linear wave model simply considers the growth of the surface elevation with a criterion related to the water depth, but a running check of wave breaking should be made over the whole wave period(Copeland, 1985a). That is, the surface elevation is checked to see if it exceed downward limit as well as upward limit given by:

$$\zeta_b = \min(\alpha d, \max(-\alpha d, \zeta)) \qquad (10)$$

where the optimum value of the constant α was found to be about 0.5(Yoo, 1989a). For the nonlinear wave model the use of the downward limit may not be necessary since the model produces a sharp surface profile on the crest but a flat surface on the trough.

A criterion related to the surface gradient may also possibly be used to determine the breaking condition. Based on the Stokes wave theory, Michell(1893) found that the upper limit of wave steepness H/L is equal to $1/7$ where H is the wave height and L is the wave length. Higher value might be used for the running check of surface gradient over the whole wave period. It is considered that the checking of surface gradient is better than that of surface elevation because the former covers the deep water as well as the shallow water while the latter is usuable only for the shallow water because it is related to the water depth.

The breaking criterion may not be only responsible for the restriction of wave energy growth, but the dissipation caused by wave breaking itself was also found to have some influences on the wave transformation in the surf zone. Various approaches have been proposed, and a bore theory is widely recognized to be a reasonable one for the estimation of breaking energy dissipation rate(Battjes and Janssen, 1978 ; Dally, et al, 1984). A reasonable estimation of wave height was also obtained simply using the breaking criterion for the monochromatic waves(Yamaguchi, 1986 ; Yoo and O'Connor, 1986). But a significant discrepancy was found between the computation and measurement in the root–mean–square wave heights when Yamaguchi(1988) modeled irregular waves on a barred sloping beach. He thought that unsatisfactory description of wave breaking dissipation might be largely responsible for the discrepancy.

The problem of wave breaking dissipation also depends on the type of wave model employed. When using time–dependent wave equation, the breaking dissipation term may not necessarily be included in the equation particularly for the description of monochromatic waves. At each time step the computed wave height is checked to see if it exceed the criterion. On the other hand when using a steady state form of wave equation or an implicit technique for its solution, the term should necessarily be included. Otherwise the wave height will grow up continuously until it finally reaches a computation limit.

6. Mixing Processes

In the problem of wave and current interaction, considerable mixing takes place in the surf zone, where wave breaking is a major cause of diffusion. The mixing processes induced by various factors are generally described by introducing the eddy viscosity concept as represented by the last term of equation(2), and various schemes have been proposed to specify the eddy viscosity but mostly using the direct relations to the turbulence velocity and length scales.

Harris(1967) conducted a pioneering study of mixing processes in the surf zone performing both field and laboratory experiments, and proposed that the surf zone average eddy viscosity might be proportional to H_b^2/T. Longuet–Higgins(1970), in his theoretical study of longshore current, suggested that it is proportional to $l_x \sqrt{gd}$, where l_x is the distance offshore from the shoreline. On the other hand Thornton(1970) and Jonsson, et al. (1974) argued that the amplitude of the wave particle motion at the sea bed be used for the length scale and the wave particle velocity at the bed for the velocity scale without an adjustable coefficient of proportionality.

Though many researchers seem to have realized that the mixing process in the surf zone should be related to the amount of wave energy dissipated by wave breaking, it was Battjes(1975) who first derived the velocity scale of turbulence from any sort of parameter directly related to the wave breaking. He found the cube root of the energy dissipation rate to be a proper measure of the velocity scale, and suggested that the characteristic size of eddies would be primarily restricted by their vertical extent rather than their horizontal extent. His relation was confirmed to be a correct approach by O'Connor and Yoo(1987), and further improvement was made using the kinetic turbulence energy transport equation.

The two–equation k- ϵ modelling has been attempted by Wind and Vreugdenhil(1986) for the description of rip currents induced by waves. In the modelling the breaking energy dissipation rate was used for the production of the turbulence kinetic energy k. It was, however, clearly observed in the work of O'Connor and Yoo (1987) that the eddy viscosity profile given by the k- ϵ model was still suspect because it grows too big even where the turbulence agitation is understood to be weak. It was thought that it is due to the defect of the ϵ equation and the production term in the equation was modified accordingly(Yoo and O'Connor, 1988). The final form of the k- ϵ equations were then represented by:

$$\frac{\partial k}{\partial t}+U_i\frac{\partial k}{\partial x_i}=\frac{\partial}{\partial x_i}\nu_k\frac{\partial k}{\partial x_i}+\frac{1}{d}\frac{D}{\rho}-\epsilon \tag{11}$$

$$\frac{\partial \varepsilon}{\partial t}+U_i\frac{\partial \varepsilon}{\partial x_i}=\frac{\partial}{\partial x_i}\nu_\epsilon\frac{\partial \varepsilon}{\partial x_i}+C_s\left(\frac{k}{a}\right)^2-C_2\frac{\epsilon^2}{k} \tag{12}$$

where k is the depth–mean turbulence kinetic energy, ϵ is the dissipation rate of k, ν_k is the eddy viscosity associated with k, ν_ϵ is the one with ϵ, D is the energy dissipation rate primarily due to wave breaking, and C_2 and C_s are constants. When k and ϵ are estimated by using equations (11) and (12), the eddy viscosty ν_t is then given by:

$$\nu_t=C_\mu\frac{k^2}{\epsilon} \tag{13}$$

where C_μ is recognized to be a universal constant, i.e. $C_\mu = 0.09$.

A significant improvement was made in the computation of wave–induced currents in a semi–detached breakwater situation. When the eddy viscosity was directly related to the velocity and length scales without considering the transport mechnism of turbulence, the eddy viscosity was of considerable value only in the surf zone but very minute outside the surf zone and hence some numerical instability was observed at the tip of the breakwater, see Fig. 4(1). When using the k- ϵ model, as shown in Fig. 3, the whole computation was stabilized and the center of the main gyre was located at the similar position to the laboratory one with the gradual variation of current velocity. It surely assures that the model's real predicability can be achieved only when the eddy viscosity is specified employing the higher level turbulence closure since it uses the universal constants and accounts for the whole turbulence mechanism involved.

When random or spectral waves propagate to the beach, large waves would break first and the smaller waves would find their breaking points further inshore. Therefore we now have a wide region of wave breaking so that currents are generated in such wide region by random waves. It results in a broadened, smoother current profile. Such mechanism has probably long been recognized by many researchers, but Thornton and Guza(1986) and Yamaguchi(1988) might be amongst the first researchers who modelled the nearshore currents induced by random or irregular waves in a field situation particularly on this respect. Thornton and Guza obtained a sufficiently smooth profile of current velocity without implementing the term of eddy viscosity (the last term of equation(2)), and found that the inclusion of eddy viscosity did not much change the current velocity profile. Yamaguchi also found a significant reduction(about 30%) in the longshore current velocity when the currents are driven by irregular waves. It may be, therefore, considered that the last term of equation(2) and the whole process of turbulence may be ignored in case of modelling the currents driven by random waves, which need to be further clarified using the turbulence transport equation.

7. Bottom Friction

Bed friction plays a dominant role in balancing the forces driven by radiation stresses of waves as well as dissipating wave energy outside the surf zone. Therefore proper estimation of bed friction is also essential for describing well shallow water waves and/or wave–induced currents. The importance of bed friction in coastal sedimentation problems, which often involve combined wave–current flow, has led to the development of several elaborate mathematical models to estimate the combined flow bed friction. The major drawback of such elaborate models for use in flow models is the requirement of large computation time, since iteration techniques are often involved for the solution. Consequently, such models have not been used in the modelling of wave–induced currents. The model developed by Bijker(1966) was, however, an exception, because it has the solution in an explicit way, being generally valid in a wide range of current or wave dominant situations, and has no difficulties in dealing with arbitrarily–angled flows.

The essence of the Bijker's model is the proposal of a hypothetical point for the combination of particle velocities to use Prandtl's mixing length theory. Successive refinements of the model have been made in recent years. Consideration was given on the near–bed wave velocity profile in order to estimate properly the wave particle velocity at the Bijker's hypothetical point(Swart, 1974). The reduction of current velocity was implemented, which is caused by enhanced eddy moton(Yoo, 1986 ; O'Connor and Yoo, 1988), and later further simplification was made for the explicit expression of various parameters(Yoo, 1989b).

As the domain of wave–induced currents is relatively small and the breaking dissipation is predominant, any influence of bottom frictional dissipation might be negligible on the wave transformation within the region. In such case no term of frictional dissipation might be required to be added to the wave equation presented in Table 1, but if it is considered to be necessary care should be taken to implement adequately the influence of full wave–current interaction(Yoo and O'Connor, 1987 ; Yoo, et al., 1989). The combined flow bed shear stress is represented by the fifth term of equation (2) which still uses the depth–mean current velocity only. The effects of wave–current interaction and boundary layer thickness should be accounted for on the friction factor f_U.

Before the use of the refined Bijker model simple expressions had usually been used with some radical assumptions in the modelling of wave–induced currents. The simple superposition of depth–mean current velocity and near–bottom wave velocity had been a general practice in this modelling with rough estimate of the friction factor. Although Longuet–Higgins(1970) cautioned the use of his simplification and recommended the Bijker model for more accurate estimation, his model seems to have been used world–widely until quite recent years possibly because it is easy to apply. The danger of the simple models was clearly and explicitly demonstrated in the recent work of Yoo and O'Connor(1987), and it was asserted that the predicability of one's model is ensured only when the elaborate closures are used for the evaluation of the combined flow bottom friction as well as the eddy viscosity.

The boundary layer mechanism in the surf zone may differ from that outside the surf zone, and one may argue that none of the friction models so far developed may be suitable for the estimation of bed shear stress in the surf zone. It is, however, considered that the difference may not be significant as the wave boundary layer is still relatively thin and hardly affected by the direct impulse of breaking waves at least outside the swash zone, where total agitation would occur down to the bottom.

Spectral waves is again a matter of difficulty with respect to bottom friction, but the way of computation may largely depend on the type of the wave model employed. If a simple discretization technique is used for the description of spectral waves, the computation of the bottom friction in each frequency or directional band may be made totally independent of the others ignoring any effect of time delay. The turbulence level of the whole spectrum might be, or should be if the bottom frictional dissipation is determined by the characteristics of the whole spectrum, represented by the wave condition of a ceratin frequency band or the root–mean–square value of the maximum excursion lengths as done by Madsen, et al(1988). In such case a representative value of wave friction coefficient may have to be estimated in each computation sweep.

Although natural beaches have mobile beds almost everywhere in the world, nobody yet, in the elaborate modelling of wave–induced currents, seem to have considered the ripple formation for the estimation of effective roughness height and thereby the friction coefficients of both wave and current for mobile beds. Watanabe and Maruyama(1984) included the topographical change in the modelling, and its effect was allowed to deform wave condition and wave–induced currents. But the bottom friction computation was fairly simple without considering any fine structure of the combined flow near the bed. As the roughness height is one of the most important factors influencing the friction coefficients in the turbulent flow, the accurate estimation of roughness height is important as much as the choice of adequate friction model.

8. Concluding Remarks

Our modelling techniques of wave–induced currents on a natural beach have steadily been improved since the solid basis of theoretical form was constructed by Longuet–Higgins(1970). Since then significant improvements were achieved in various aspects, and a refinement in one area often required some re–shaping of the other areas. As the whole flow system of waves and wave–induced currents is highly nonlinear, a numerical technique is necessarily to be employed even for a plane, uniform beach if a reasonably accurate solution were desired to be obtained. Due to this reason the review was directed primarily to the works of numerical modelling or numerical implication made for various physical concepts.

Further improvements may continuously be made in the coming years with existing capability of modelling techniques and physical understanding. The generation mechanism due to nonlinear interaction, possibly between waves and current as well as waves and waves, appears to be significant in the flow system. It may be tackled elsewhere in the very near future using one of the nonlinear 1st stage models. More concern may have to be given to the breaking criterion used for the 1st stage models. A method may possibly be devised to imply the effects of bottom slope, wave steepness and furthermore wave–current interaction in the breaking criterion for the 1st stage wave models. The treatment of random or spectral waves cast three problems to us in the modelling of wave–induced currents. The nonlinear interaction problem is one of them, and others are mixing processes caused by random breaking motion and the bottom friction of the flow interacted with spectral waves. The studies concerned are probably still at primitive stage, and hence much work may need to be done to define the mechanisms and to implement them into the modelling system.

For most modellers the prediction of topographical change is the eventual goal of their modelling for practical applications. Efforts have also been devoted for developing a model system to predict the topographical change induced by waves and wave–generated currents. Much room may, however, still exist for further research particularly in the area of the combined flow sediment transport. Since anyone of the updated elaborate models of bottom friction may not yet have been exploited for studying the sediment transport of the wave–current interacted flow, the existing formula of sediment transport may need some corrections using one of the better qualified friction models. If it is done, we may find ourselves having far better confidence for the use of the model system, and then we can predict the wave–induced currents on a natural mobile beach more realistically with reasonable accuracy accounting for full interaction between the combined flow and mobile sea beds.

Acknowledgement

Part of the study was done during the stay in the Institute of Fluid Mechanics, Hannover University, Federal Republic of Germany, supported by the Von Humboldt Foundation and Korea Science and Engineering Research Foundation. Discussions made with Professor W. Zielke and Mr. H. H. Prüser were very helpful particularly with regard to the nonlinear modelling of waves.

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