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Radial Thickness of Ice Jam in Channel Bends

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ABSTRACT/The characteristics of radial thickness of ice jam at the center part of channel bends were analyzed briefly in this paper. Jam thickness in channel bends increases both toward the inner bank, and downstream. For this study, slope at the jam's underside was assumed to be linear with similarity of radial slope of bed in alluvial bends. Radial slope at the jam's underside in floating ice elements was estimated using the force equilibrium theory in the radial direction. The eqution which can be estimated the radial slope of ice jam was suggested using Falcon and Kennedy's bed layer theory. Experimental data, which were measured at the center part of cross-section in a single 180-degree bend, were compared to the calculated values using the suggested equtions. The result shows that the calcultated values were smaller than the measured ones. It is considered that the estimated value of shear stress in the radial direction may be smaller than the actual and two-layer model may be not suitable for alluvial bend flow.

1. Introduction

In northern countries, ice represents an environmental hazard that can endanger human lives and influence human activities such as industrial production, transport of goods, housing and winter sports. Under the natural conditon, ice jams frequently develop in river bends, where curved channel alignment and attendant strong transverse gradients of flow depth and velocity, together with the influence of centrifugal forces acting on flows of water body and ice. They may inhibit ice conveyance and initiate jamming in river bends. Ice jams confront river engineers with a variety of problems, including flooding caused by blockage of channels, damage to structures,

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interference with navigation, and obstruction of diversion intakes, to mention but a few.

There is scant information on ice-jam development and topography in river bends. Most studies of jams reported have dealt with straight channel [Uzuner(1976), Tatinclaux(1977), Ashton(1986)] . Case studies on troublesome ice jam in river bends were reported [Burgi(1971), Beltaos(1981), Garbrecht(1982), Chen(1984)] . The lack of information on jam development and topography in channel bends is understandable and the fact that channel curvature may complicate the mechanics of ice jam development and topography. It also complicates the mechanics of flow, alluvial-sediment transportaion and channel bed topography. Urroz(1988) documented the variations of ice jam topography in bends. He found that jams were generally thicker along the inner bank of the bend than along its outer bank. This variation is akin to the radial variation of bed elevation in alluvial bends, and is attributed to the effect of a double helical flow field. Yoon and Ettema(1988) reported a brief summary of observation from a laboratory investigation aimed at determining ice jam topography and development in a sinuous channel, and in a single-bend channel.

A bend jam created a flow field that contains a reflected image of the flow field associated with open water flow in alluvial bend. The flow field in an ice-covered bend is distinguished by the radial components of flow which are directly inwards both at the bed and along the jam's underside. The inward components flow along the jam's underside cause broken ice to be transported inwards and, thereby, results in greater thickness of jam along the inner bank of bends.

The purpose of this paper is to report the equation which could estimate the radial slope of jam thickness in alluvial bends, and to compare the radial slope of jam's underside with that of water surface, with and without jam, and that of bed layer. It is assumed that the radial slope of jam thickness is nearly linear as the radial bed slope in alluvail bends. Force equilibrium theory in an element of floating ice jam was enable to determinate the radial slope of jam thickness. Another equation was derived from bed-layer theory(Falcon and Kennedy, 1983). The estimated valus of this equations were compared to the experimental data. This research was tired to extend the Urroz's study.

2. Radial Force Equilibrium within Ice Jam

Fig. 1 shows the froces acting through an element of ice jam. The normal and shear stresses are indicated using symbols σ and τ , respectively. Weight increment, dw, represents the weight of the ice rubble in the cover plus that of the water contained in jam pores. Side views of the ice jam element in the radial direction is shown in Fig. 2. The angle β in Fig. 2 was assumed to be small.

An equilibrium balance of forces acting a floating jam element in alluvial bend was

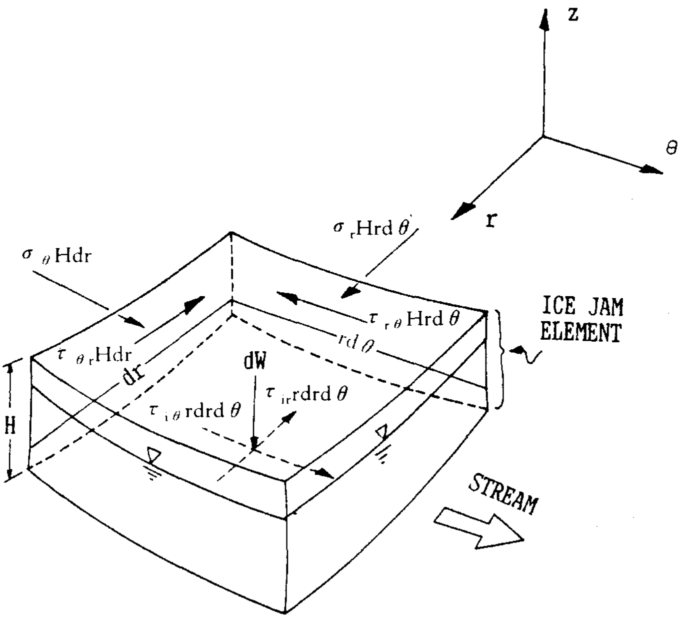

Fig. 1 Forces acting through an element of ice jam in bends.

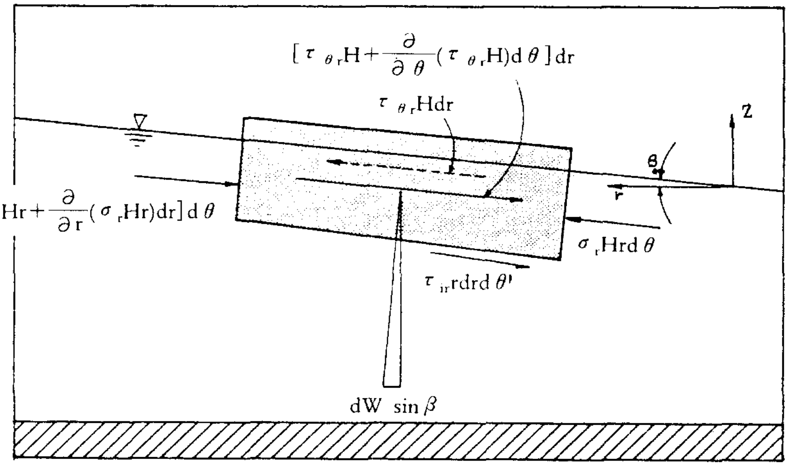

Fig. 2 Radial equilibrium of forces through an element of bend jam.

obtained by equating to zero the sum of force in radial direction.

Radial equilibrium of forces yields.

$$-\frac{\partial}{\partial H}(\sigma_r Hr)+\frac{\partial}{\partial \theta}(\tau_{\theta r}H)-\rho_i gHr\ \sin\beta_0-\tau_{ir}r=0 \qquad (1)$$

Eq. (2) is modified from Eq. (1) with the assumptions of fully developed flow conditions and $\sin\beta = S_r$(Urroz, 1988).

$$\frac{d}{dr}(rH^2)+\frac{\rho_i g S_r}{K_r \gamma_e}H-\frac{C_{\tau i}\rho_w D_i V_i^2}{2K_r \gamma_e}=0 \qquad (2)$$

where σ_r : the normal stress in the radial direction

$\tau_{\theta r}$: the shear stress in a floating layer of broken ice.

H : ice jam thickness

ρ_i : bulk density of ice jam.

g : gravity acceleration

S_r : radial slope of surface water.

ρ_w : water density

τ_{ir} : the radial shear stress at the ice covered underside.

r : radius of curvature

$C_{\tau i}$: $\dfrac{m_i+1}{m_i+2}-K_w\left(\dfrac{V}{V_i}\right)^2$

K_w : $1/\left[\dfrac{m_b}{m_b+1}\left(\dfrac{D_d}{D}\right)+\dfrac{m_i}{m_i+1}\left(\dfrac{D_b}{D}\right)\right]$

V : average velocity in cross section

V_i : average velocity in ice layer

m_b : power–law exponent in bed layer

m_i : power–law exponent in ice layer

D : total depth

D_i : depth of ice layer

D_b : depth of bed layer

K_r : $\dfrac{1+\cos\phi}{1-\cos\phi}$

ϕ : angle of internal friction of the accumulation

γ_e : $\dfrac{1}{2}(1-p)\left(1-\dfrac{\rho_i}{\rho_w}\right)\rho_i\, g$

P : jam porosity

Radial normal stress, σ_r and thickness, H can be calculated from the experimental date and the equation of $\sigma_r = K_r \cdot \gamma_e \cdot H$(Uzuner and Kennedy, 1976). Radial distribution of H is expressed in Eq. (2). Because the equation is first order, one boundary condition is required. The most useful boundary condition is $H = H_c$ at $r = R_c$. The solution for H still requires more knowledges on such coefficients as ϕ .

3. Formulation of Radial Slope at the Jam's Underside.

Fig. 3 shows the distribution of streamwise velocity and secondary velocity as jam developing. The strict deference between openwater flow and ice–coverd flow in alluvial bends is the secondary flow distribution. The distribution of secondary flow in ice–coverd bends is similar to the close–conduit flow in a curved duct. In general, there is one cell of secondary flow in openwater bend flow, while there are two cells of secondary flow with developing ice jam.

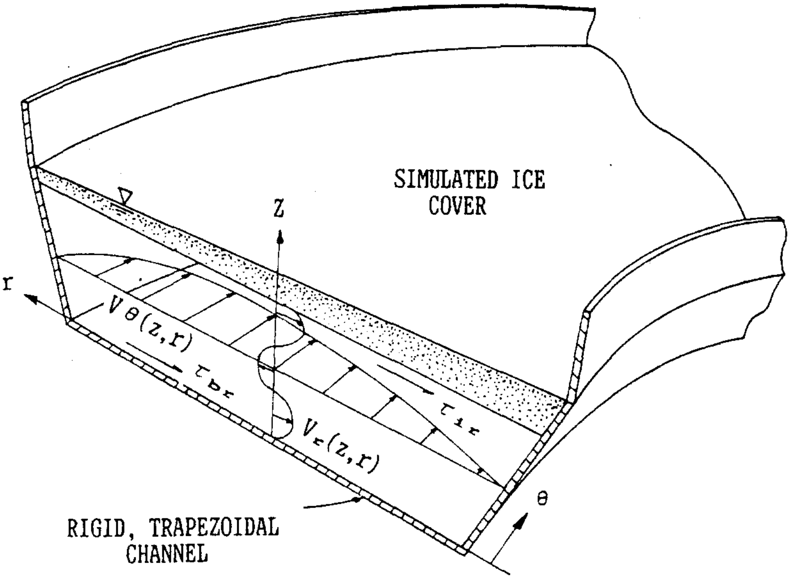

Fig. 3 Hydarulic characteristics in developing ice jam

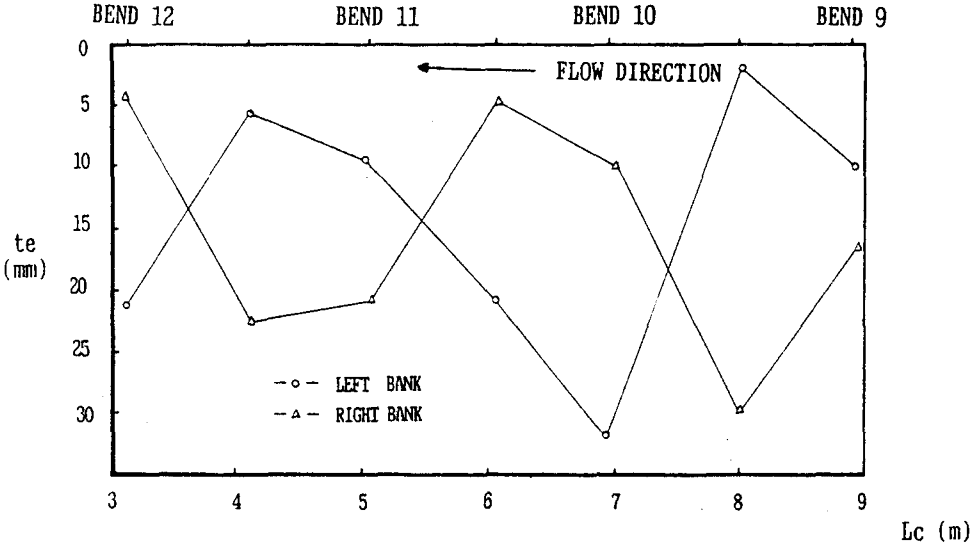

Fig. 4 Streamwise jam thickness in a sinuous channel.

The radial slope at the jam's underside is remarkably similar, but inverted, to typical radial slope of bed in alluvial bends. For the purpose of practies, the radial slope of bed in alluvial bends was assumed to be linear(Odagaard, 1984). If the shape and size of ice particles were uniform over the bends and the radial slope of surface water with ice jam was small, it could be assumed that the radial slope at the jam's underside is also linear. In order to investigate the constant radial slope at the jam's underside, experimental data of jam thickness in a sinuous channel were plotted in Fig. 4 and 5(Yoon and Ettema, 1988).

The Streamwise distribution of ice jam thickness in a sinuous channel is shown in Fig. 4. L_c is the centerline length in meter from the screen barrier, which is located at the end of laboratory flume. t_e is the thickness of ice jam. A sinuous channel has thirteen 90-degree bends. The flow direction(from bend 1 to bend 13) is also shown Fig. 4. The distribution of jam thickness at each side of bank in a sinuous channel is nearly alternative. From Fig. 4, it is obvious to say that jam thickness along inner bank is thicker than along that of outer bank. Inner bank has small radius of curvature, while outer bank has large one.

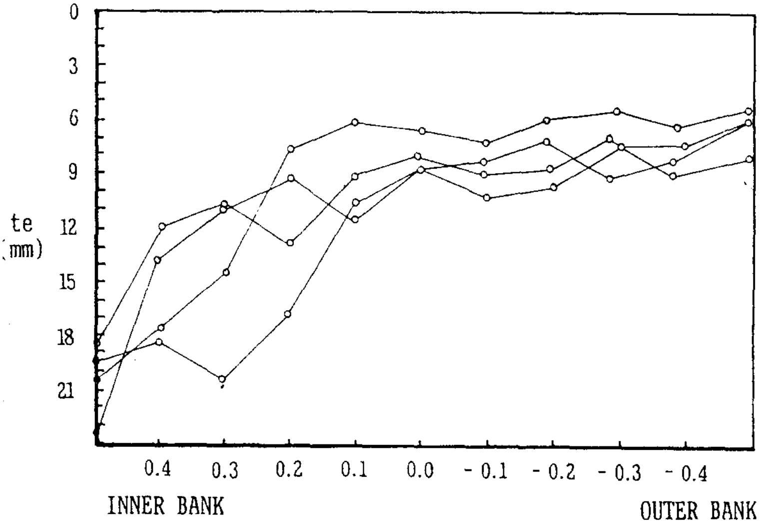

Fig. 5 Raidal jam thickness in a sinuous channel [bend 12 ~ bend 13]

Fig. 5 shows the radial jam thickness from section of bend 12 to bend 13 in a sinuous channel. Ten points at each cross-section were selected to measure the jam thickness. From Fig. 4 and 5, it was decided that the assumption of constant radial slope at the jam's underside is approximately adequate to estimate the radial slope at the jam's underside.

Equilibrium of the bed layer of alluvium in the radial direction was suggested by Falcon and Kennedy(1983) in order to predict the radial slope of bed in alluvial bends. It is expressed in Eq. (3).

$$\tau_{br}=H_b\cdot(1-P_b)\cdot(\rho_s-\rho_w)\cdot g\ \sin\ \beta_b \tag{3}$$

where ρ_s : density of bed particles
β_b : radial slope angle of the bed
H_b : bed layer thickness
P_b : porosity of bed particles

Because the balance of weight and flow shear forces in ice layer is necessary in equilibrium zone, Eq.(3) could be used to estimate the radial slope at the jam's underside. If the subscript b, represented the bed layer, is replaced the subscript i, represented ice layer, Eq.(3) leads

$$\tau_{ir}=H_i\cdot(1-P_i)\cdot(\rho_i-\rho_w)\cdot g\ \sin\ \beta_i \tag{4}$$

Radial shear stress acting on the jam's underside, τ_{ir}, could be calculated from the Rozovskii equation(1961), which contained the fully–developed and steady flow condition in channel bends. It was suggested in Eq.(5)(Urroz, 1988).

$$\tau_{ir}=-0.5\cdot C\tau_i\rho_w g D_i\frac{V_i^2}{gr} \tag{5}$$

Radial slope at the jam's underside, S_{ir}, is estimated with Eq.(4) and (5) and the assumption of $\sin\beta\ i = S_{ir}$, and it is expressed in Eq.(6).

$$S_{ir}=0.5\frac{C\tau_i\rho_w D_i}{H_i(1-\rho_i)(\rho_w-\rho_i)}\cdot\frac{V_i^2}{gr} \tag{6}$$

The term V_i^2/gr indicated the radial of surface water in bend. This new equation, suggested here, is similar form to Rozovskii's(1961) equation.

Another approximate method which can be estimated the radial slope at the jam's underside is that Eq.(2) is directly integrated using the assumption of linear radial slope, such as $h = ar + b$. Substituting this relationship to Eq.(2), Eq.(2) can be solved approximately. If jam thickness data of equilibrium zone were avaialbe, the radial slope of ice jam at center point of cross–sections in alluvial bend could be calculated. With above mentioned condition, the result of integrating the Eq.(2) leads

$$S_{ic}=\frac{H_c^2-H_cR_ck-k'}{2H_cR_c} \tag{7}$$

where H_c : ice thickness at the center point of cross–section
R_c : radius of curvature at the center point
k : $\rho_i g\ S_r/K_r\ \gamma_e$
k' : $C\tau_i\rho_w D_i V_i^2/2K_r\ \gamma_e$

S_{ic} : radial slope at the jam's underside

4. Results and Discussion

Eq.(6) and (7) are only available at the center part of shallow and mild alluvial bends. Experimental data for a sinuous channel are not adopted to compare the calculated results because it is sharp and small aspect–radio (B/D)bend, and is rigid bed channel. It is hard to get the data because each section has different parameters such as D_i, D_b, M_i, M_b, and so on. Experimental data adopted in this paper were measured at the 150–degree section in a single 180–degree bend which the rations of R_c/B and B/D were 5.4 and 13, respectovely [Yoon and Ettema(1988), Urroz(1988)] . Equilibrium zone was found at the section which was located near the downstream barrier in a single 180–degree bend. Table 1 shows the flow parameters for section 150–degree in sand bed bend.

Table 1. Flow parameters

Variable	Unit	Measured Value
Q	m^3 / sec	0.091
B	m	2.44
R_c	m	13.11
D	m	0.196
D_i	m	0.094
D_b	m	0.102
ϕ	degree	30.00
P_i	N / D	0.5
P_b	N / D	0.4
ρ_i	kg / m^3	910.0
ρ_s / ρ_w	N / D	2.65
d_s	cm	0.3
m_b	N / D	10.41
m_i	N / D	4.69
V	m / sec	0.207
K_w	N / D	1.15
H_c	mm	80
g	m / sec	9.8

* N / D:No Dimension

Radial slopes of surface water in alluvial bends, with and without ice jam, were suggested in Eq.(8) and (9), respectively.

$$S_r = \frac{V^2}{gR_c} \tag{8}$$

$$S_{cr}=k_w \cdot (\frac{V^2}{gR_c}) \qquad (9)$$

Radial slope of bed with ice jam in alluvial bend, S_{ib}, was suggested as in Eq.(10) [Urroz. 1988] . But he did not compare the calculated values with experimental data.

$$S_{ib}=\frac{\alpha}{2}\frac{D_b}{r}F_{Db}\frac{(8\theta_b)^{1/2}}{1-P_b} \qquad (10)$$

where $\alpha : m_b[\frac{m_b+1}{m_b+2}-K_w(\frac{V_b}{V})^2]$

F_{Db}: densimetric particel Froude number $[V/\{(\rho_s/\rho_w-1)gd_s\}^{1/2}]$

θ_b: Sheid parameter $[U_{*c}^2/\{(\rho_s/\rho_w-1)gd_s\}^{1/2}]$

Since numerical experiments performed by Lau(1982) indicated that for practical purpose, the condition $V = V_i = V_b$ appeared to be reasonable, the condition was adopted in this paper.

Radial slopes of suface water, with and without jam, were calculated by Eq.(8) and (9) with the data in table 1. Radial slopes of ice jam's underside were calculated by Eq.(6) and (7), and that of bed was calculated by Eq.(10). Table 2 shows the calculated radial slopes by using Eq. (6) ~ (10) with the data from the section 150–degree in a 180–degree sand bed bend, and the measured values in same section.

Table 2. Radial slopes in sand bed bend

Radial slope		Calculated Value	Measured Value
Surface water without jam	(Eq. (8))	3.3×10^{-4}	N / A
surface water with jam	(Eq. (9))	3.4×10^{-4}	N / A
jam's underside	(Eq. (6))	1.2×10^{-3}	6.1×10^{-3}
jam's underside	(Eq. (7))	2.5×10^{-3}	6.1×10^{-3}
bed with jam	(Eq. (10))	2.1×10^{-2}	3.3×10^{-2}

* N / A : Not Available

Even if the calculated radial slope of surface water, with and without jam, in bend could not be campared with the measured values, it was known the fact that the radial slope of surface water increased with ice jam developing. The magnitude of the calculated radial slope of surface water, with and without jam, may be same order and very small. In fact, they were ignored, in practical purposes, in mild and small aspect–ratio bends. Both of the calculated value of radial slope at the jam's underside using Eq.(6) and (7) were smaller than the measured values and the calculated value by Eq.(6) was smaller than that of Eq.(7). It might be the reason that the estimated value of shear strees in radial direction was smaller than actual, and some uncertain coefficients affected on the calculated results. And Eq(7) was drived from the forces balance of jam's weight, internal stress and flow shear forces, while Eq.(6) was determined by the balance of weight and flow shear force, only. However, those of the calculated by Eq.(6) and (7) show same order of the measured ones. Radial slope at the ice jam's underside was larger than that of surface water, with and without. The calculated value of radial slope of bed with Eq.(10) shows also smaller value comparing to the measured one. The reasons of differences between the calculated values with Eq.(10) and the measured ones may be similar to those reasons which were mentioned previously. But they were also same order between the calculated value with Eq.(10) and the measured one. It is interesting to note that radial slope of bed with jam in alluvial bend is the largest among them, and next that of jam thickness, and next surface water with jam, and that of surface water without jam is the smallest among them.

It is difficult to estimate correctly the radial slope with ice jam because the mechanism of flow field in alluvial bend is very complicate and until now, there are some uncertain coefficients such as ϕ . If every coefficient was known exactly, numerical solution for Eq.(2) may extent the study of radial distribution at the jam's underside.

5. Conclusion

The characteristics of radial slope of ice jam were analyzed briefly. The equation which could be estimated the radial slope at the jam's underside was suggested in Eq.(6) and (7), introducing forces equilibrium in radial direction. For this study, the calculated radial slopes of two different conditions were compared to the measured ones at the center part of 150–degree cross section in a single 180–degree alluvial bend. Even though the former is smaller than the latter, both of them are in same order. The complicate mechaincs of bend flow with ice jam inhibit to estimate ice jam topography more closely. Two–layer model which was used to divide the total depth of water into ice layer and bed layer may be not adapt to be applied the calculation of the radial slope at the jam's underside. Shear stress in radial direction is one of the most important coefficient to estimate the radial slope at the jam's underside. The rank of magnitude of radial slopes in alluvial bend were supposed:(1) that of bed with jam, (2) that of ice jam's underside, (3) that of surface water with jam, (4) that of surface water without jam.

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