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Manning's Roughness Factor in Alluvial Channels

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ABSTRACT/Manning's roughness factor to flow in sand-bed channels may be divided into the grain roughness factor and the form roughness factor. The grain roughness factor may be determined by using Keulegan's formula. By using available experimental data, it was found there is a unique relationship between the form roughness and the hydraulic radius to sediment particle size ratio for a given value of the Froude number. The form roughness and the bed form may be determined by using this unique relationship. The technique for engineering applications of the results appears to be quite simple.

1. Introduction

The hydraulic resistance is a vital factor in successful design and planning of fluvial hydraulic engineering works. But the relation of the bed form and hydraulic resistance is one of the most difficult problems in fluvial hydraulics.

For the case of steady uniform flow, the flow resistance can be predicted with a satisfactory degree of certainty. But in alluvial streams the problem is complicated by the presence of sediment in the flow. In this view, the purpose of this study is to find the method of the prediction of Manning's roughness factor and the bed form.

For this purpose, the grain roughness will be determined by using Keulegan's formula(9) for the resistance, and the form roughness coefficient and the bed form will be analized by using available experimental data.

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2. Flow Resistance in Open Channels with Rigid Boundary

2.1 Manning's Formula in Rough Channels

In 1889 Manning presented an equation to approximate the average velocity in open channels. This formula defines the resistance to flow. The most usual form is

$$V=\frac{1}{n}R^{2/3}S^{1/2} \tag{1}$$

in which V is the mean velocity of flow(m/sec), n is the Manning's roughness factor, R is the hydraulic radius and S is the energy slope. Mannign's formula can be written as

$$V=\frac{0.3193}{n}R^{1/6}\sqrt{gRS} \tag{2}$$

in which g is the acceleration of gravity (m/sec^2).

The shear velocity u is defined as

$$U_*=\sqrt{\tau_{0/\rho}} \tag{3}$$

in which τ_0 is the boundry shear stress and ρ is the density of fluid. The shear stress can be expressed as

$$\tau_\circ=\gamma RS \tag{4}$$

in which γ is the specific weight.

Substituting Eq.4 into Eq.3,

$$u_*=\sqrt{gRS} \tag{5}$$

Substituting Eq.5 into Eq.2,

$$\frac{V}{U_*}=\frac{0.3193}{n}R^{1/6} \tag{6}$$

and rearranging Eq.6,

$$\frac{V}{U_*}=0.3193\frac{k^{1/6}}{n}\left(\frac{R}{k}\right)^{1/6} \tag{7}$$

in which k is the height of roughness element.

2.2 Keulegan's Formula for the Velocity Distribution Relation

With k known, the expression for the mean flow in rough channels can be written as in the following from Keulegan's formula(9):

$$\frac{V}{U_*}=6.25+5.75\log(\frac{R}{k}) \tag{8}$$

By Keulegan, a practical range of R/k is from 15 to 500. With this range in mind, a straight line for 1/6 power of R/k is drawn in Fig. 1 as fairly good approximations to actual observations. The straight line in Fig.1 represents the velocity distribution relation(Eq.9) which is obtained by the analytical solution using the least squares method.

$$\frac{V}{U_*}=8.179(\frac{R}{k})^{\frac{1}{6}} \tag{9}$$

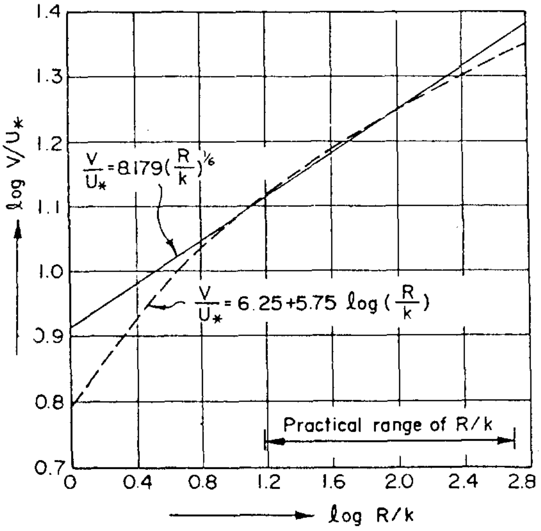

Fig. 1 The Relationship between V/u_* and R/k in Rough Channel

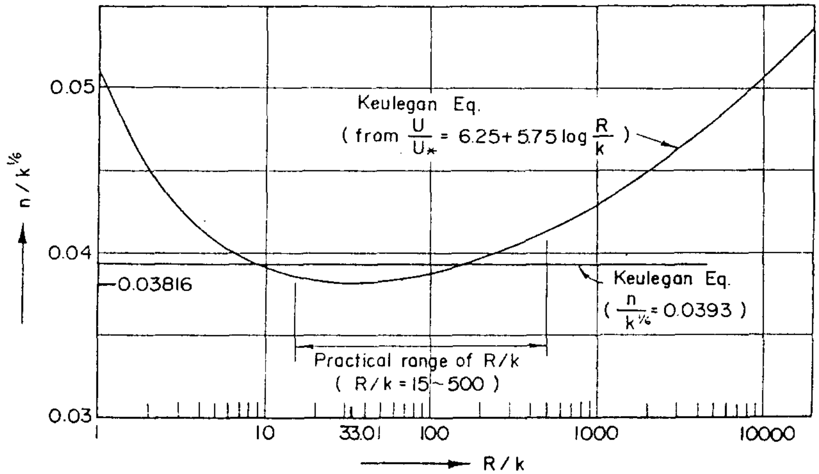

Fig. 2 The Relationship between $n/k^{1/6}$ and R/k

But let's consider 8.12 instead of 8.179 because Keulegan(9) used 8.12 in his paper.

$$\frac{V}{U_*}=8.12\left(\frac{R}{k}\right)^{\frac{1}{6}} \tag{10}$$

Substituting Eq.10 into Eq.7,

$$\frac{n}{k^{\frac{1}{6}}}=0.0393 \tag{11}$$

in which k is the equivalent roughness of Nikuradse and is expressed in meter(Fig. 2).

3. Division of Manning's Roughness Factor

Manning's roughness factor of flows in erodible–bed channels are usually larger than those of comparable flows in rigid–boundary channels. For conceptual as well as analytical convenience, the resistance to flow may be regarded as composed of two parts. A certain fraction of the energy is consumed overcoming the resistance due to surface drag and the remainder is used to overcome the form drag, Myer–Peter and Muller(1, 12) proposed that the energy slope S be divided into two components, that is

$$S = S' + S'' \tag{12}$$

in which S' corresponds to the fraction of the energy slope required to overcome surface drag and S" the fraction required to offset form drag.

On the basis of Eq.12 Bajorunas(2) assumed that the Manning's roughness factors n is composed of two parts.

$$n=n'+n'' \tag{13}$$

in which n' is due to the grain roughness and n" is due to the form roughness. From Manning's formula, the Manning's roughness factor

$$n=\frac{R^{\frac{2}{3}}S^{\frac{1}{2}}}{V} \tag{14}$$

From Eqs.12, 13 and 14, the following equations may be given.

$$n'=\frac{R^{\frac{2}{3}}(S')^{\frac{1}{2}}}{V} \tag{15}$$

$$n''=\frac{R^{\frac{2}{3}}(S'')^{\frac{1}{2}}}{V} \tag{16}$$

But assuming the flat bed Manning's roughness factor, n', may be determined by using Keulegan's formula for the velocity distribution relation, simply.

$$n' = 0.0393k^{\frac{1}{6}} \tag{17}$$

From Eq. 13, the form roughness, n", can be calculated as following.

$$n'' = n - n' \tag{18}$$

The variaiton of n" is somewhat different for erodible-bed and rigid-bed channels, and n' never attains values smaller than those corresponding to rigid, smooth boundaries. Therefore, if n' of the data utilized is larger than n, the data are neglected.

4. Dimensional Analysis of Manning's Roughness Factor

From the relationship between Manning's and Darcy-Weisbach's formulas, Manning's roughness factor is given as

$$n = \frac{R^{\frac{1}{6}} f^{\frac{1}{2}}}{\sqrt{8g}} \tag{19}$$

in which f is the Darch-Weisbach's resistance coefficient. This relation can be expressed as

$$n = F(R, f, g) \tag{20}$$

Chow(5) gave an excellent discussion on the factors affecting Manning's n. However, the relationship of Eq. 20 generally means that the dimensional analysis for the Darcy-Weisbach's resistance coefficient f by Yen(16) can be adopted. From the division of Manning's roughness factor,

$$n'' = n - n' \tag{18}$$

where

$$n = \frac{R^{\frac{2}{3}} S^{\frac{1}{2}}}{V}$$

$$n' = 0.0393k^{1/6} (k \text{ in meter}) \tag{17}$$

In general, k is different from the mean size of sediment particles, d.

$$k = Cd \tag{21}$$

in which C is a constant and varies form 1.0 to 5.1 in Rijn paper(11). But in this study, C = 1.0 will be considered for simplicity and the results by Yen(16) and Yen & Liou (17) will be adopted directly as following.

For steady flow the Manning's form roughness factor n" can be expressed as

$$n''=F_1(\rho,\ \mu,\ \rho_s,\ d,\ \sigma,\ g,\ s,\ V,\ D,\ \phi) \tag{22}$$

in which μ is dynamic viscosity of the fluid ; ρ_s and σ are density and standard deviation of the size of the sediment particles, respectively ; D is hydraulic mean depth of the flow and ϕ represents a nondimensional cross-sectional shape factor.

Through dimensional analysis, Eq.22 can be written nondimensionally as

$$n''= F_2(\mathbb{F},\ \mathbb{R},\ \frac{R}{d},\ \sigma,\ \frac{\rho_s}{\rho},\ s,\ \phi) \tag{23}$$

in which the Froude number of the flow, $\mathbb{F}$, represents the effect of gravity on the flow and the Reynolds number, $\mathbb{R}$, represents the effect of viscosity of the fluid.

Usually the magnitude of S is small and its effect on n" may be relatively less important. Therefore, the parameters ρ_s/ρ and S may be combined into one representing the relative mobility of the sediment. Using the shear velocity u_* and the terminal fall velocity of the sediment, V_t,

$$V_t=\frac{\rho g}{18\mu}(\frac{\rho_s}{\rho}-1)d^2 \tag{24}$$

$$n''=F_3(\mathbb{F},\ \mathbb{R},\ \frac{R}{d},\ \sigma,\ \frac{V_t}{U_*},\ \phi) \tag{25}$$

In most of the laboratory experiments, the channel has a rectangular cross section so that the cross-sectional shape factor can be represented by B/D, where B is the width of the channel, and the sediment particles are approximately uniform size. Hence, Eq.25 may be simplified as

$$n''=F_4(\mathbb{F},\ \mathbb{R},\ \frac{R}{d},\ \frac{V_t}{U_*},\ \frac{B}{D}) \tag{26}$$

in which B is water surface width.

5. Analysis of Data

The dependence of the Manning's roughness factor on six dimensionless control paramenters as indicated in Eq.25 necessitates a large amount of reliable data to be analyzed to yield useful information on hydraulic resistance in movable-bed channels. Because of the accuracy and reliability of the measurements, only those data obtained from controlled laboratory flume experiments are used in the present study.

The summary of the labortory experimental data used in the present study is listed in Table 1.

Table. 1 Summary of Data Utilized (17)

Source of Data	Flume Size			Grain size		No. of Runs	Discharge	
	Width	Depth	Length	Medium Size	Gradation		min.	max.
Colorado State University	8	2	150	0.19	1.30	40	0.96	22.33
				0.27	1.56	20	5.11	21.84
				0.28	1.67	37	4.36	22.02
				0.45	1.60	45	1.84	21.62
				0.47	1.54	54	6.92	21.42
				0.93	1.54	43	4.49	22.69
Barton and Lin	4			0.18		31	0.90	9.10
Brooks	0.875	0.833	40	0.16	1.11	12	0.20	0.54
Vanoni and Brooks	2.79	1	60	0.137	1.38	16	0.51	3.84
Kennedy	0.875	0.833	40	0.549	1.14	16	0.20	0.78
				0.233	1.47	14	0.22	0.78
Kennedy	2.79	1	60	0.233	1.47	13	0.68	3.33
Kennedy	2.79	1	60	0.142	1.38	9	1.05	1.40
Vanoni and Hwang	3.61		130	0.206	1.46	6	2.26	3.82
Stein	4			0.4	1.50	51	1.80	17.00

(continued)

Source of Data	Slope		depth		Mean Velocity		Reference
	min.	max.	min.	max.	min.	max.	
Colorado State University	0.0055	0.950	0.42	1.09	0.78	4.58	(6)
	0.0077	1.022	0.45	1.13	0.79	4.93	
	0.007	1.007	0.30	1.07	0.82	4.93	
	0.015	1.01	0.19	1.00	0.65	5.54	
	0.042	0.960	0.30	1.33	1.13	5.32	
	0.013	1.28	0.38	1.11	1.00	6.07	
Barton and Lin	0.044	0.210	0.300	1.38	0.71	3.60	(3)
Brooks	0.18	0.35	0.155	0.30	0.93	2.15	(4, 14)
Vanoni and Brooks	0.039	0.280	0.203	0.553	0.77	2.53	(4, 14)
Kennedy	0.55	2.72	0.074	0.346	1.65	4.65	(8)
	0.26	1.60	0.147	0.346	1.57	3.42	
Kennedy	0.32	2.29	0.145	0.356	1.35	3.45	(8)
Kennedy	0.056	2.25	0.228	0.550	0.91	2.21	(7)
Vanoni and Hwang	0.0642		0.578	0.603	1.05	1.83	(15)
Stein	0.1013	1.079	0.30	0.81	4.50	5.52	(13)

Dtails on experimental equipment and conditions from which these data were obtained can be found in the respective original publications.

In this study, the following equations were used for the computation of the magnitude of the parameters in Eq.26.

$$R=\frac{BD}{B+2D} \tag{27}$$

$$V=\frac{Q}{BD} \tag{28}$$

$$\mathbb{R}=\frac{\rho VR}{\mu} \tag{29}$$

in which Q is a flow discharge and B is equal to the channel width because the rectangular cross sections are used.

For the Forude number, two equations were considered as following.

$$\mathbb{F}_D=\frac{V}{\sqrt{gD}} \tag{30}$$

$$\mathbb{F}_R=\frac{V}{\sqrt{gR}} \tag{31}$$

Particularly, Eq.31 was considered because B/D may not be sufficiently large values for no side-wall effects although the values are generally larger than 5.

For steady flow in straight channels of constant width and slope with the bed covered by approximately uniform size sand, the Manning's form roughness factor n" as well as the bed from is uniquely determined. In Eq.26, the width-depth ratio, B/D, represents the effect of geometry of cross-section. When B/D is small, the side-wall effect is particulary important. For the minimization of this side-wall effect, the Froude number using hydraulic radius, $\mathbb{F}_R$, is considered and if the Froude number using mean depth, $\mathbb{F}_D$, is considered, B/D will be limited to larger values than 5.

Now, the effect due to change of values of B/D on n" may be relatively small and Eq.26 can be simplified as

$$n''=F_5\left(\mathbb{F},\ \mathbb{R},\ \frac{R}{d},\ \frac{V_t}{U_*}\right) \tag{32}$$

Using this Eq.32, the bed form and n" may be determined. For the graphical solutions of the bed form and n", the groups of three parameters may be selected. Because the ranges of n", $\mathbb{F}$, $\mathbb{R}$, R/d and V_t/u_* are large, the log-log papers were used for plotting.

5.1 Manning's Form Roughness Factor, n"

As indicated in Eq.32, the Manning's form roughness factor n" varies with the ratio of the hydraulic radius to particle size, R/D. For the form roughness, it is expected that R/D is a very significant factor. Also previous researchers(1, 10, 16, 17) considered R/D or R/d as important factor.

However, the following relations are considered for the decision of n" in this study.

$$n'' = F_6\left(\mathbb{F}, \frac{R}{d}\right) \qquad (33)$$

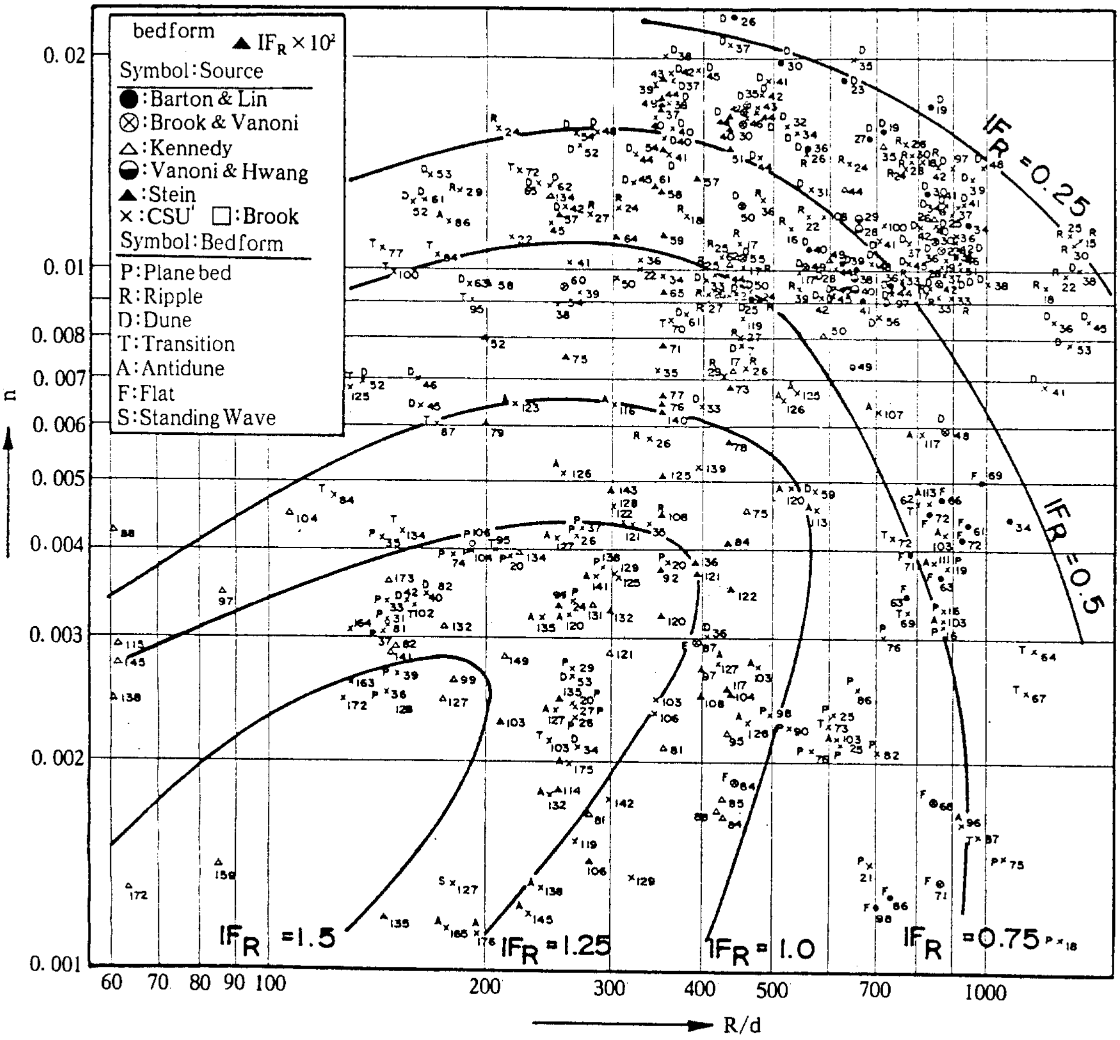

Fig. 3 Graphical Predictor for n" as a Function of R/d and $\mathbb{F}_R$

$$n'' = F_7(\mathbb{R}, \frac{R}{d}) \tag{34}$$

$$n'' = F_8(\frac{V_t}{U_*}, \frac{R}{d}) \tag{35}$$

$$n'' = F_9(\mathbb{F}, \mathbb{R}) \tag{36}$$

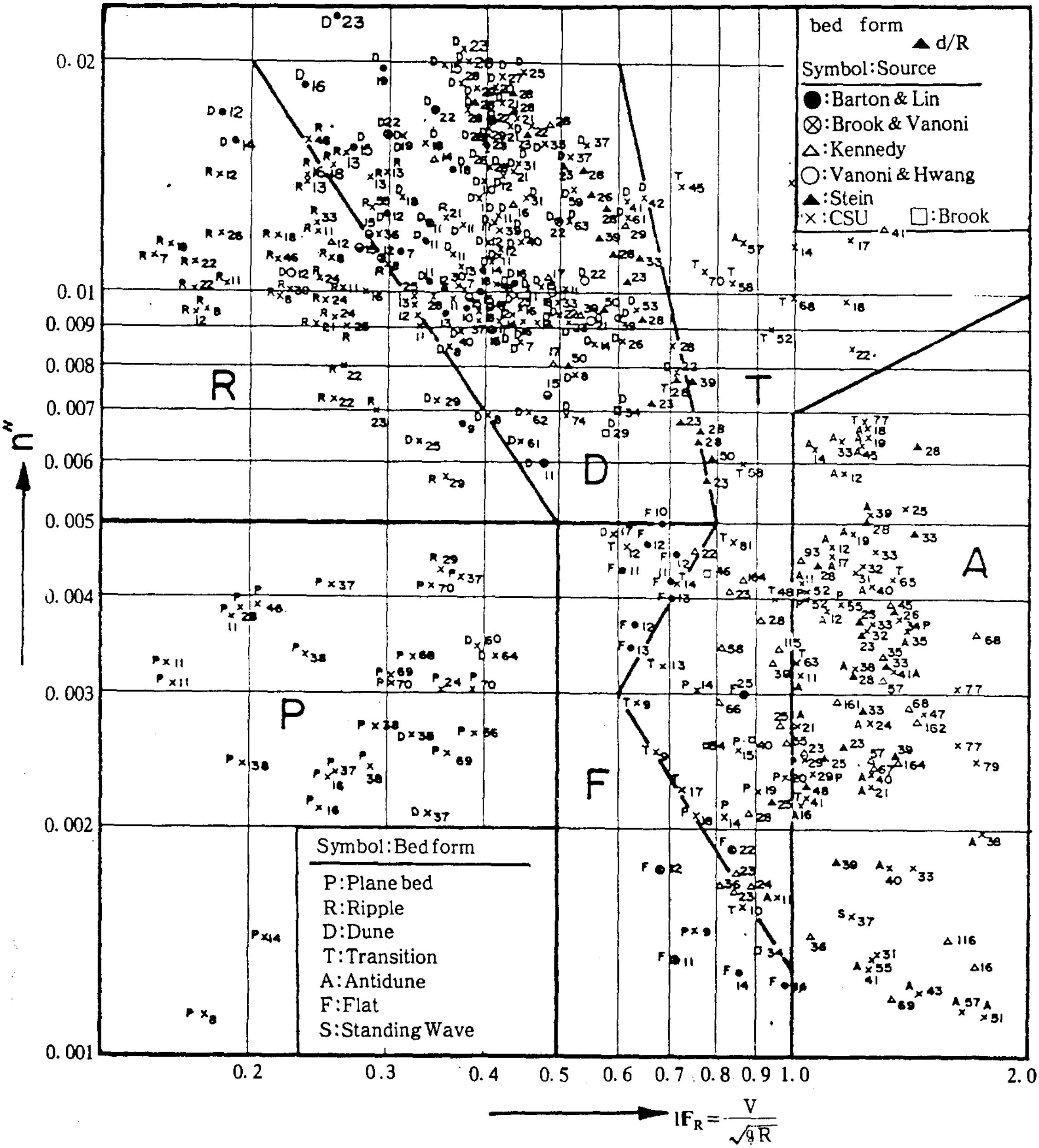

Fig. 4 Graphical Predictor for the Bed Form as a Funtion of n" and $\mathbb{F}_R$

In this study, the relationships between n" and the factors in Eq.34 through Eq.37(also $\mathbb{F}_R$ and $\mathbb{F}_D$ for $\mathbb{F}$) are tested and plotted. In these graphical tests, the best selection is that n" may be predicted by using R/d and $\mathbb{F}_R$ as shown in Fig.3.

5.2 Bed Form

Yen and Liou(17) pointed out the bed form is a function of the same parameters affecting Manning's roughness coefficient n", i. e.,

$$\text{Bed form} = F_{10}(\mathbb{F}, \mathbb{R}, \frac{R}{d}, \frac{V_t}{U}) \tag{37}$$

In this study, the relationship between n" and each parameter of right hand side in Eq.37 for a given bed form was considered. The best selection in these graphical tests is that the bed form may be predicted by using n" and $\mathbb{F}_R$ as shown in Fig.4. This also means that the region of the Manning's form roughness factor may be roughly expected by the bed form and the Froude number.

6. Conclusion

The Manning's form roughness factor and the bed form have been analyzed by using available experimental data. Through the graphical tests, the information is obtained that the Manning's form roughness factor and the bed form may be predicted by using several important parameters.

The Manning's form roughness factor, n", can be predicted on the graph using R/d and the Froude number, $\mathbb{F}_R$. Also, the bed form can be predicted if the Manning's form roughness factor and Froude number are given.

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