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Optimum Structural Design of Stiffened Cylinders Based on Reliability Analysis

by

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신뢰성 해석에 기초한 보강된 실린더 부재의 최적구조설계

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Abstract

This study is concerned with the optimum design of stiffened cylindrical members frequently found in floating offshore platforms with constraints on reliability. Minimised is the expected total cost which is composed of the structural cost and the expected failure cost. Some design requirements drawn from variotus design codes are also considered as constraints. Reliability of critical component in a structure only is considered in this paper and the system failure is discarded since the probability of system failure is in general much smaller than the probability of component failure and it is very difficult to evaluate the cost due to system failure. Ultimate strength only is considered and not the fatigue strength.

Several parametric studies are illustrated and the optimum solutions for different strength models which are now in use for the design of stiffened cylinders are derived to show the optimum designs against different strength models for the same type of structural component. The present results lead to the important conclusions relating to the posibility of more cost saving in the design of such structure through the reliability-based optimisation process.

요 약

본 논문에서는 신뢰도(reliability)에 제한조건을 두어, 부유식 해양구조물의 주요 부재인 보강된 실린서의 최적설계를 다루었다. 기대되는 총비용을 목적함수로 하여 최소화하였다. 그 총비용은 구조적 비용과 파괴로 인해 예상되는 비용(expected failure cost)으로 구성된다. 여러 설계규정에서 요구하는 설계 요구사항을 역시 제한조건으로서 고려하였다. 본 논문에서는 안전성 측면에서 중요한 구조부재만의 신뢰도를 고려하였고, 시스템의 파괴확율은 일반적으로 부재의 파괴확율보다 상당히 작고 또한 시스템 파괴에 의한 비용을 추정하는 것은 매우 어려우므로 시스템의 파괴는 고려하지 않았다. 또한 파괴모드로써 최종파괴만을 다루었고 피로파괴는 고려하지 않았다.

몇가지 최적설계의 예를 본 논문의 결과로서 보여 주었으며 또한 동일한 구조부재의 설계공식에

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따른 최적설계를 비교하기 위해 보강된 실린더의 설계시 현재 사용하는 다른 형태의 설계공식을 갖고 최적설계 결과를 유도하였다. 본 논문에서는 그 결과들로부터 신뢰성해석에 기초한 최적화 과정을 통해 다른 구조물의 설계시 보다 많은 비용의 절감을 꾀할 수 있는 가능성으로부터 그 중요성이 강조되었다.

1. Introduction

The major goal of structural design is to obtain the balanced design between safety and economy. During the last two decades reliability-based optimum design has developed in conjuction with the classical deterministic optimisation formulations. It has been proposed for some times that a more rational criterion for structural design is that structural safety be represented by reliability (or alternatively by the failure probability) and hence, the reliability constraints should be included in the optimisation procedure to get the balanced design between safety and cost [1~4]. In the light of this point the optimisation problem may be based on reliability analysis to ensure the safety of the design.

Deterministic optimisation in structural design has enjoyed its popularity for many years. In some cases, however, with regard to the structural safety the optimum design may lie on the unsafe side. For example, Das and Frieze[5] found that the minimum weight design (deterministic optimum design) were not the most safe. They compared the safety level of stringer-stiffened cylinders designed based on reliability analysis with the minimum weight designs. The results are presented in Table 1 in terms of failure probability.

Table 1 Comparison of failure probability between reliability-based design and minimum weight design of stringer-stiffened cylinders subjected to compressive force

COV of axial compressive force	10%	20%
reliability-based design (non-optimum)	0.440×10^{-2}	0.401×10^{-1}
minimum weight design	0.329×10^{-1}	0.119

The general idea of optimisation is to minimise (or maximise) the objective function subjected to reliability (or failure probability) constraints. The possible objective functions are structural weight, total cost, failure probability and utility function. Structural weight and cost are usually chosen as for the deterministic optimisation and in cost, failure cost may be included. References 3 and 6 well summarise the formulation of reliability-based optimisation problems.

In this study optimum design of stringer-stiffened cylinder has been derived and the Rosenbrock algorithm is employed. An expected total cost is to be minimised with constraint on the allowable reliability index. The cost model is composed of the structural cost and the expected cost due to failure.

2. Formulation of Problem

Let $\{X\}=\{x_1, x_2, \cdots, x_n\}$ be design variable vector where n is the number of design variables considered in the optimisation procedure. Then, the optimisation problem is:

Find $\{X\}$ such that minimises the total cost given by

$$C_T=C_s+P_f*C_f \tag{1}$$

subjected to constraint on reliability index

$$\beta\geq\beta^0 \tag{2}$$

where C_s is the structural cost, P_f the failure probability and C_f the expected failure cost. β^0 is the allowable reliability index.

All terms in the above equations are function of design variables and the above problem is non-linear. The structural cost, C_s is assumed to be proportional to the structural weight, and all costs are to be converted in terms of structural weight. The expected failure cost, C_f may be given by[2]:

$$C_f=\lambda T_D\gamma\rho(0.15+0.5A) \tag{3}$$

where ρ : material density ($=7.25\text{ton/m}^3$)

γ : factor proportional to water depth of the component ($=1$ to 7)

A : cross-sectional area

λ : average annual frequency of storm occurence

T_D : design life

This cost model represents the maintenance cost after a component failure.

3. Structural Model

The stringer-stiffened cylinder shown in Fig. 1 is chosen for the present study, which is from TLP-B.[7] Geometric and material properties are listed in Table 2. Load effects are in Table 3, for which bias factors are in Table 4. COV of 4% are assumed for geometric properties and elastic modulus and COV of 8% for yield stress. For dynamic, quasi-static and static load effects 10, 20 and 10% are given, respectively as in Reference [7]. The interaction equation under axial compression and radial pressure given by Eq. (4) is used as the strength model.[8]

$$\left(\frac{\sigma_\theta}{\sigma_{\theta u}}\right)^2+\left(\frac{\sigma_x}{\sigma_{xu}}\right)=1 \qquad (4)$$

where σ_θ is the hoop stress and obtained from $\sigma_\theta=PR/t$, P is radial pressure and σ_x is the axial stress resulting from pure axial force, bending moments and radial pressure. The subscript 'u' means the ultimate value. The mean bias and modelling uncertainty (COV) of the strength modelling parameter for the above strength model are 0.99 and 13%. With regard to the distribution types of design variables, geometric properties are normal and others are log-normal.

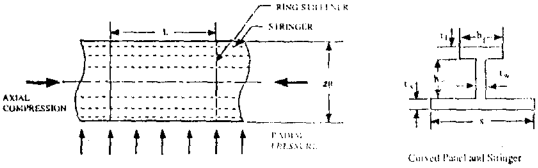

Fig. 1 Stringer-Stiffened Cylinder Model

Table 2 Data of stringer-stiffened cylinder in Fig. 1

radius of cylinder	$R=8.88$ m
thickness of cylinder	$t_s=0.025$ m
number of stringers	$S_N=60$
stringer scantling	$=0.3\times0.015\times0.19\times0.019$ m(T)
ring frame scantling	$=0.525\times0.025\times0.25\times0.03$ m(T)
ring frame spacing	$L=2.2$ m
elastic modulus	$E=200000$ MN/m^2
yield stress	$\sigma_Y=391$ MN/m^2

Table 3 Load effects including bias factors

load effect	static	dynamic	quasi-static
axial force(MN)	91.31	21.69	2.01
bending moment(MN-m)	293.31	251.01	51.9
radial pressure(MPa)	0.2125	0.0478	0.0039

note: hydrodynamic load is evaluated by using Morison approach and load effect due to wave drift force is not considered.

Table 4 Bias factors of load effects

static	load effect : 1.0
dynamic	load effect : 1.2 for axial load, 1.0 for others
quasi-static	load effect : 1.0

4. Results of Optimisation

The allowable reliability index of $\beta^0=3.72$ is chosen, which has been proposed by the TLP Rule Case Committee[9] for the design of stiffened cylinders. Following design requirements for stringer portion are drawn from various design codes:[9~11]

(1) $1/t_1<1.4\sqrt{e}$ (5)

where $e=E/\sigma_Y$

(2) $\sigma_t/\sigma_Y>2.5$ (6)

wher $\sigma_t/\sigma_Y=(e/I_0)(J/2.6+(\pi h/a)^2I_2)$

h=distance between shear centre of stringer and toe, divided by h_w $=1+t_2/2$

J=St. Venant tortional constant $=h_w^4((t_1^3+bt_2^3)/3)$

I_0=polar moment of inertia of stringer cross-section about toe $=h_w^4(t_1(t_1^2+4)/12+bt_2(b^2+t_2^2)/12+bt_2(1+t_22)^2)$

I_2=moment of inertia of stringer cross-section about its axis

$$=h_w^4(t_1^3+b^3t_2)/12$$

$$(3)\ a<8 \tag{7}$$

In the above equations L is the unsupported length, $t_1=t_w/h_w$, $t_2=t_f/h_w$, $b=b_f/h_w$ and $a=L/h_w$.

Optimum design of the stringer-stiffened cylinder shown in Fig. 1 has been drawn for $\beta°=3.72$ varying the number of design variables considered in the optimisation procedure. The results are presented in Table 5, in which the cost ratio is defined as:

$$\text{cost ratio}=\frac{\text{weight of optimum design}}{\text{weight of original design}} \tag{8}$$

As can be found in Table 5, optimum design have about 13 to 21% less cost than the original design. That is, introducing the reliability analysis into the optimisation procedure can provide us with much cost saving and hence weight saving. To compare the optimum designs against defferent strength models, the improved strength model given by Eq. (9) is used.

$$\left[\frac{R_x}{\phi_x}\right]^2+R_xR_\theta\left[\frac{2\sqrt{(1-\phi_x{}^2)(1-\phi_\theta{}^2)}}{\phi_\lambda\phi_\theta}-1\right]+\left[\frac{R_\theta}{\phi_\theta}\right]^2=1 \tag{9}$$

For details of this model refor to References 12 or 7. The results are shown in Table 6 when $n=6$. Using Eq.(9) gives lower cost than Eq.(3) and this implies that the improved strength model can provide a more economic benefit.

Table 5 Optimum design of stringer-stiffened cylinder (unit : m)

variable \ n	2	4	6
t	0.0178	0.0178	0.0213
s	0.719	0.708	0.697
(S_N)	(78)	(78)	(80)
h_w	—	0.275	0.278
t_w	—	0.014	0.0127
b_f	—	—	0.100
t_f	—	—	0.009
cost ratio	0.864	0.845	0.788

n : number of design variables considered in the optimisation procedure
s : curved panel width ($=2\pi R/S_N$)
S_N : number of stringers
"—" : same figure as in Table 2

Table 6 Optimum design against different strength model

variable \ strength model	Eq. (4)	Eq. (9)
t	0.0213	0.0161
$s(S_N)$	0.697(80)	0.700)(80)
scanting of stringer	0.278×0.0127× 0.100×0.009	0.288×0.015× 0.113×0.0112
cost ratio	0.788	0.667

Table 7 Least weight design of stringer-ctiffened sylinders for $\beta°=3.72$(after reference 12)

N	$A_T\ m^2$	$t\ mm$	R/t	s/t	A_s/st
50	2.02	31.2	284	35.7	.164
60	1.85	27.0	329	34.4	.227
70	1.70	23.4	380	34.1	.306
80	1.58	20.2	439	34.5	.404
90	1.44	16.7	530	37.0	.549
100	1.42	15.3	581	36.5	.668
110	1.47	15.2	584	33.4	.739
120	1.53	15.1	586	30.7	.808

Regarding the number of stringers, S_N, its optimum value has been found to be 80 rather than 100 appeared in Reference 12(see Table 7). As far as the present results are concerned, it seems that as more design variables are considered in the optimisation procedure, more cost saving can be achieved.

5. Conclusion

This paper has been concerned with optimum design of stringer-stiffened cylinder with constraints on the allowable reliability and design requirements specified in various design codes. From the present numerical results followings can be drawn:

(1) when the optimisation procedure based on reliability analysis is adopted, more than 10 to 13% cost saving may be possible for the allowable reliability index of 3.72 compared with the original design.

(2) the use of the improved strength model can guarantee more cost saving while retaining the same level of safety.

Because of the nature of the present model of the objective function〔see Eqs. (1) and (3)〕, there is a close correlation between weight saving and cost saving.

In this study the structural cost is assumed to be proportional to the structural weight. It should be, however, more completely assessed with considering the realistic construction cost. Nevertheless the reliability based optimisation procedure seems to provide the design with the design having safety as well as economy compared with the a deterministic optimisation.

Further study in this area is required before applying the concept to practical designs and it may be valuable to pursue such study to achieve a more economic benefit in the design stage.

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