새로운 행렬 분할법을 이용한 최적 무효전력/전압제어

Optimal Reactive Power and Voltage Control Using A New Matrix Decomposition Method

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요 약

본 논문은 최적 무효전력 전압 제어 문제를 해결하기 위한 새로운 엘고리즘을 제시하였다. 계통의 손실을 최소화 시키며, 발전기의 무효전력, 모선전압, 변압기탭, 커패시터와 리액터 등에 제한을 갖도록 제약조건을 구성하였다. 계통 손실을 최소화하는 목적함수와 독립변수와 종속변수간의 감도를 구하기 위해 새로운 자코비안 분할법을 이용한 선형화 된 감도 관계식을 유도하였다. 본 논문은 두 개의 모듈, 즉 무효전력 전압제어를 위한 Q·V 모듈과 오차보정을 위한 조류계산 모듈로 구성되어 있다. 특히 수렴특성을 증대시키기 위해 가속항을 도입하였으며 이 두 모듈을 이용하여 무효전력 전압제어 문제를 효과적으로 풀 수 있었다. 여러 사례연구를 통하여 제안된 알고리즘이 계산정확도나 수렴특성면에서 우수함을 입증하였다.

Abstract- A new algorithm is suggested to solve the optimal reactive power and voltage control (optimal VAR control) problem. The model minimizes the real power losses in the system. The constraints include the reactive power limits of the generators, limits on the bus voltages and the operating limits of control variables - the transformer tap positions, generator terminal voltages and switchable reactive power sources. The method presented herein, using a newly developed Jacobian decomposition method, employs linearized sensitivity relationships of power systems to establish both the objective function for minimizing the system losses and the system performance sensitivities relating dependent and control variables. The algorithm consists of two modules, i.e. the Q-V module for reactive power-voltage control, and load flow module for computational error adjustments. In particular, the acceleration factor technique is introduced to enhance the convergence property in Q-V module. The combined use of the afore-mentioned two modules ensures more effective and efficient solutions for optimal reactive power dispatch problems. Results of the application of the method to a sample system and other worst-case systems demonstrated that the algorithm suggested herein is compared favourably with conventional ones in terms of computation accuracy and convergence characteristics.

1. Introduction

The control of reactive power and voltages represents one of the most important activities in the operation of modern power systems. The control is known as the "voltage/reactive power" or voltage/VAR" control. Generically, any changes to the system configuration or in power demands can result in higher or lower voltages in the system. This situation can be improved by the operator by allocating reactive power sources in the system, i.e., by adjusting transformer taps, changing generator voltages and switching shunt capacitors/reactors. Also, it is possible to minimize active power losses or production cost in systems by reactive power reallocation. The main objective of this control can be regarded as an attempt to achieve an overall improvement of system security, service quality and economy. System security requires adequate voltage levels and reactive reserves in order to maintain voltage stability when critical contingencies occur. The service quality and economy require appropriate voltage control at all system nodes and consumer terminals within tolerable limits, in order to insure adequate reactive power line flows which result in minimal transmission losses.[1]

In the past, many methods using sensitivity relationships and gradient search technique have reported to solve reasonably complexity of this voltage/VAR control problem. [2, 3] Dommel and Tiney[4] minimized a nonlinear objective function of production costs or losses using Kuhn-Tucker conditions. Hano[2], Mamandur[5], Elangovan[6] developed sensitivity relationships to minimize the system losses. And, other investigators presented a method of minimizing the production cost by coordinating real and reactive power allocations in the system. [7]

This paper presents a new algorithm for solving reactive power-voltage control problem in order to

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接受日字: 1989年 7月 31日 1 次修正: 1990年 1月 8日 obtain the economic operation condition of electric power system. The suggested method is based upon two modules coupled to each other. First, Q-V module optimally determines the reactive power output of generators and shunt capacitor/reactor as well as transformer tap settings with the assumption that the real power generation is held constant.

It is assumed further that loss changes during the Q-V module is covered by the slack generator. Second, the load flow module is used to make the fine adjustment of the error resulting from the Q-V module. The main features of the algorithm suggested here are summarized as follows:

- (1) The Q-V module takes over the objective function with bus voltages and transformer taps only as the independent variables. Here, the objective function is also a linearized version.[8]
- (2) Mathematical model is developed by using the sensitivity relationships between dependent and control variables for the objective function and all network performance constraints. This model is done by decomposing the jacobian matrix of NR load flow equation which is augmented to include coefficients representing the changes in real and reactive power with respect to the changes in tap settings of the transformer. [7, 9]
- (3) In order to preserve the sparsity of constraints matrix, the system voltages and the transformer tap settings are adopted as the independent variables, thus the sparsity techinque is fully utilized in performing Q-V module.
- (4) By introducing the acceleration factor, the convergence property to the optimal operation condition, without zigzagging or oscillation, is obtained.

2. Mathematical Model

2.1 Constraints

All system performance constraints to be satisfied and control variable constraints to be maintained are as follows:

The generator voltage magnitudes are constrained by the limits on the excitation and the load bus voltage magnitudes have upper and lower limit by service performance sense.

$$V \min \le V \le V \max$$
 (1)

where.

V: vector of bus voltage $(\cdot)_{max}$: upper bound of (\cdot) $(\cdot)_{min}$: lower bound of (\cdot)

The reactive power generation of all generators have their upper and lower values limited by the design specifications. Also the load buses which have reactive power compensation devices are assumend to have finite capacity.

$$Q\min \le Q \le Q\max$$
 (2)

where.

Q: vector of reactive power generation

There are also physical limits for the upper and lower values of transformer tap settings.

$$T\min \le T \le T\max$$
 (3)

where.

T: vector of transformer tap ratio

And the system is constrained by the real and reactive power supply and demand balance equation.

$$G(P,Q) = 0 (4)$$

where.

P: vector of real power generation

In general, the transmission line capacity is constrained by the physical property of the conductor such as thermal capacity.

$$|H(V,\delta)| \le H \max \tag{5}$$

where.

 δ : vector of bus voltage angle

Defining
$$\Delta V \max = V \max - V$$

 $\Delta V \min = V \min - V$
 $\Delta Q \max = Q \max - Q$
 $\Delta Q \min = Q \min - Q$
 $\Delta T \max = T \max - T$
 $\Delta T \min = T \min - T$

constraints (1)-(5) can be modified as:

$$\Delta V \min \leq \Delta V \leq \Delta V \max_{\Delta Q \min} \Delta Q \leq \Delta Q \max_{\Delta T \min} \Delta T \leq \Delta T \max_{\Delta Q \in \Delta Q \min} G(\Delta P, \Delta Q) = 0 \\
|H(\Delta V, \Delta \delta)| \leq H \max$$
(6)

2.2 Formulation of optimization problem

From equation (6), the optimal reactive power dispatch problem is mathematically formulated as follows:

Minimize
$$f(\Delta Q)$$

subject to
$$\Delta V \min \leq \Delta V \leq \Delta V \max$$

$$\Delta Q \min \leq \Delta Q \leq \Delta Q \max$$

$$\Delta T \min \leq \Delta T \leq \Delta T \max$$

$$G(\Delta P, \Delta Q) = 0$$

$$|H(\Delta V, \Delta \delta)| \leq H \max$$
(7)

where, $f(\Delta Q)$: objective function for Q-V module which is derived in detail in the later section

Here, it is noted that ΔQ is a dependent variable depending on control variables ΔV , ΔT . The most important reason why the bus voltage V is adopted as control variable in Q-V module, is to preserve the sparsity of constraints matrix. The consideration of line flow constraints is optional to avoid unnecessary restrictions that increase computation time and deteriorate the computational efficiency.

2.3 Derivation of Sensitivity Relationships

The sensitivity relationships between the control variables and the dependent variables are derived as:

Partitioning the jacobian matrix defined from the power flow calculation using the Newton Raphson method,

| ΔPs | | Jpds | Jpvs | Jpts | Δδs | |
|---------------|-----|----------------|----------------|----------------|------------|-----|
| ΔPg | | | | | Δδg | |
| |]] | Jpd | Jpv | Jpt | | - |
| ∆Pl | = | | | | Δδl | (8) |
| $\Delta Qsgc$ | | J_{qd}^{sgc} | J_{qv}^{sgc} | J_{qt}^{sgc} | ΔV | |
| $\Delta Q l'$ | | $J_{qd}^{l'}$ | $J_{qv}^{ l'}$ | $J_{qt}^{ u}$ | ΔT | |

where.

subscript/superscript s, g, c, l, l': indices for the slack generator, other generator, load bus with the reactive power compensation devices, total load bus, load bus without the reactive power compensation devices, respectively.

Using the condition that the phase angle of slack bus does not change, $\Delta \delta s = 0$, the incremental real power can be derived from equation (8) in terms of power angle as:

$$\Delta Ps = Jpds \begin{vmatrix} \Delta \delta g \\ \Delta \delta l \end{vmatrix} + Jpvs \cdot \Delta V + JpVs \cdot \Delta T$$
(9)

$$\left|\begin{array}{c} \varDelta Pg \\ \varDelta Pl \end{array}\right| = Jpd \left|\begin{array}{c} \varDelta \delta g \\ \varDelta \delta l \end{array}\right| + Jpv \cdot \varDelta V + Jpt \cdot \varDelta T \tag{10}$$

With the use of equations (9) and (10), we get

$$\Delta Ps = Jpds \cdot Jpd^{-1} \begin{vmatrix} \Delta Pg \\ \Delta Pl \end{vmatrix} + |Jpvs - Jpds \cdot \\ Jpd^{-1} \cdot Jpv|\Delta V + |Jpts - Jpds \cdot Jpd^{-1} \cdot Jpt| \cdot \\ \Delta T$$

$$Let \quad Jb \equiv Jpvs - Jpds \cdot Jpd^{-1} \cdot Jpv,$$

$$Ic \equiv Jpts - Jpds \cdot Jpd^{-1} \cdot Jpt$$

$$(11)$$

Supposing that the real power of load bus is constant value, $\Delta Pl = 0$, equation (11) is transformed into:

$$\Delta P_{S} = Ja|\Delta P_{S}| + Jb \cdot \Delta V + Jc \cdot \Delta T \tag{12}$$

where.

Ja: row vector with the first[m-1] elements of matrix product Jpds · Jpd⁻¹ m: total number of generator buses

It is noted that the relation in equation (12) representing the mutual dependency among real generation powers replaces the conventional supply and demand balance equation. Also, using another assumption that the variation of phase angle doe not have an effect on the reactive power, the incremental reactive power in equation (8) is redefined as:

$$\Delta Q l' = J_{qv}^{l'} \cdot \Delta V + J_{qt}^{l'} \cdot \Delta T \tag{13}$$

$$\Delta Q_{SGC} = J_{qv}^{sgc} \cdot \Delta V + J_{qt}^{sgc} \cdot \Delta T \tag{14}$$

Consequently, from the above equations the Q-V module is summarized as follows:

3. Q-V Module

In constructing the objective function from equation (12) for Q-V module, it is assumed that the real power generation P is not changed since the system state with the optimal real power generation schedule is the starting point for this algorithm. With the use of above discussion, the equation (12) is transformed into:

$$\Delta P_{S} = Jb \cdot \Delta V + Jc \cdot \Delta T \tag{15}$$

In this paper, equation (15) is used the objective function for Q-V module. Apparently it looks like an absurd job, however, this is attributed to the fact that an equivalent alternative to that of minimizing the system losses is to minimize the slack power generation. [5] Consequently, the objective function for Q-V module is converted into the function of variables, ΔV , ΔT only.

$$f(\Delta V, \Delta T) = [Jb \cdot \Delta V + Jc \cdot \Delta T] \tag{16}$$

In order to develop the Q-V module with objective function in equation (16), the system performance constraints are constructed by the equations (14) and (15). The results are summarized as:

Minimize $f(\Delta V, \Delta T)$ subject to

$$J_{qv}^{t'} \cdot \Delta V + J_{qt}^{t'} \cdot \Delta T = 0$$

$$\Delta Q \min \leq J_{qv}^{sgc} \cdot \Delta V + J_{qt}^{sgc} \cdot \Delta T \leq \Delta Q \max$$

$$\Delta V \min \leq \Delta V \leq \Delta V \max$$

$$\Delta T \min \leq \Delta T \leq \Delta T \max$$
(17)

4. Load Flow Module

The above-mentioned Q- optimization problem is solved by using the optimization technique (G. P) [9] with the assumption of the approximated linearized objective function and constraints as given in equation (17). Thus, its solutions are not exact optimal values. Therefore, it is necessary to use the load flow procedure for making fine adjustments on those optimal values.

5. Computational Procedure

The following steps describe how to find an optimal solution of the optimal reactive power dispatch problem discussed above.

- step 1) Perform the initial power flow calculation to determine the state of system.
- step 2) Calculate Jb, Jc matrices and construct Q-optimization problem in terms of information drawn from load flow calculation.
- step 3) Solve the Q-optimization problem.
- step 4) With the use of control variables ΔV , ΔT , obtained in step 3) and the acceleration factor introduced to enhance the convergence property, update the states as:

$$V \leftarrow V + \alpha \cdot \Delta V$$
 $Q \leftarrow Q + Jqv \cdot \alpha \cdot \Delta V + Jqt \cdot \alpha \cdot \Delta T$
 $T \leftarrow T + \alpha \cdot \Delta T$
where.

 α : acceleration factor to ensure the convergence, the recommended value is $0.55 \sim 0.75$ (by

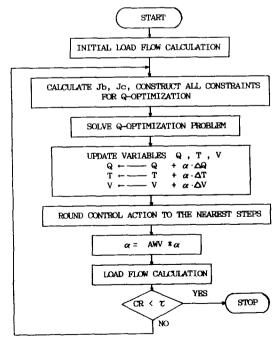


Fig. 1 Flow Chart of Algorithm

simulation)

As the procedure is iterated, the value of acceleration factor is changed by the appropriate weighting value(AWV) (about AWV=0.83 by simulation).

- step 5) For the capacitor/reactor switching and transformer tap settings, the control actions have to be rounded to the nearest step, so the control action is realizable. (This step is also optional),
- step 6) Perform the power flow calculation to adjust the error caused by linearization and obtain the improved state of system.
- step 7) Define the loss regulation as:

$$CR =$$

previous system loss previous system loss

check whether the absolute value of CR is within the predesignated limit (τ) or not. If the answer is negative, the process will be repeated from step 2), ortherwise the results will be printed out and the process will be terminated.

The above computational procedure is schematized in the flow chart in Fig. 1.

6. Sample Studies

The new algorithm developed in previous sec-

Table 1 Line data for sample-1 system

| Line number | bus number | | impe | dance | tap ratio | line charge |
|----------------|---------------|----|--------|--------|--------------|----------------|
| Humber | from | | R | X | Tatio | charge |
| 1 | 1 | 2 | 0.0015 | 0.0015 | | 0.20 |
| 2 | 2 | 3 | 0.0092 | 0.2205 | 0.900 | |
| 3 | 4 | 5 | 0.0399 | 0.1276 | | |
| 4 | 5 | 6 | 0.0399 | 0.1276 | | ĺ |
| 5 | 6 | 3 | 0.0000 | 0.5000 | | |
| 6 | 4 | 3 | 0.0000 | 0.5000 | | |
| 7 | 3 | 8 | 0.0171 | 0.0458 | | ! |
| 8 | 8 | 10 | 0.0000 | 0.6280 | 0.975 | ! |
| 9 | 8 | 9 | 0.0198 | 0.0150 | | |
| 10 | 9 | 7 | 0,0000 | 0.6280 | | ĺ |
| 11 | 10 | 7 | 0.1488 | 1.4126 | | |
| 12 | 5 | 11 | 0.0399 | 0.1276 | | |
| 13 | 5 | 12 | 0.0399 | 0.1276 | 1.050 | |

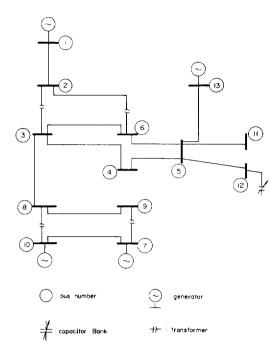


Fig. 2 One line diagram of sample-1 system

Table 2 Bus data of sample-1 system (in p.u.)

| bus | Generation | load |
|--------|-------------|-------------|
| number | P Q | P Q |
| 1 | 0.000 0.000 | 0.000 0.000 |
| 2 | 0.000 0.000 | 0.000 0.000 |
| 3 | 0.000 0.000 | 0.035 0.017 |
| 4 | 0.000 0.000 | 0.075 0.037 |
| 5 | 0.000 0.000 | 0.193 0.094 |
| 6 | 0.000 0.000 | 0.000 0.000 |
| 7 | 0.150 0.082 | 0.122 0.094 |
| 8 | 0.000 0.000 | 0.000 0.000 |
| 9 | 0.000 0.000 | 0.000 0.000 |
| 10 | 0.033 0.010 | 0.118 0.074 |
| 11 | 0.000 0.000 | 0.057 0.028 |
| 12 | 0.000 0.000 | 0.130 0.063 |

tions has been applied to the sample systems in order to demonstrate its efficiency and availability.

6.1 Sample-1 System

The sample-1 system with, 12 buses, 13 lines, 3generators, 3 tap-changing transformers and 1 shunt capacitor bank was used as the model system. Fig.2. shows one line diagram of this system.

Table 3 Operating limits (in p.u.) of generators

| bus number | Pmin Pmax | Qmin Qmax | Vmin | Vmax |
|---------------|-----------|------------|------|------|
| 1 | 0.0 1.000 | 0.00 0.500 | 0.95 | 1.01 |
| 7 | 0.0 0.150 | 0.00 0.082 | 0.95 | 1.01 |
| 10 | 0.0 0.033 | 0.00 0.016 | 0.95 | 1.01 |

Table 4 Limits of other control varibles (p.u.)

| (i) | Transformer tap ratio |
|--------|---|
| | $1.0 - 0.0125 \times NT \le Ti \le 1.0 + 0.0125 \times$ |
| | NT |
| | NT = number of steps, i = transformer |
| | bus |
| | 0.0125 = step size of transformer |
| (ii) | Load bus voltage |
| | $0.95 \le Vi \le 1.01$ i=total load bus |
| (iii) | VAR sources |
| | $0.0 \le Q12 \le 0.30$ |

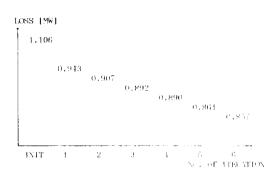


Fig. 3 Variation in losses (sample-1 system)

Table 1 summarizes the line data for the sample-1 sytem on 100 MVA base, while Table 2 gives bus data.

The operating limits of generator are summarized in Table 3.

Also the limits of other control variables needed in simulating are shown in Table 4.

The results drawn from the algorithm presented in this paper are summarized in Table 5, showing system losses at each iteration step. The initial point (initial iteration) produced 5 buses with violation of limits. This is eliminated by redistributing VAR sources. It is important to note that system losses were decreased from the initial base value of 1.106 MW to 0.857MW a decrease of 0.249 MW (29.05%). All violated variables were within limits, system losses converged in iteration

Table 5 Results for the sample-1 system

| variable | initial state | final state | | |
|----------|-----------------------|-----------------------|--|--|
| V1 | 1.0000/ 0.0000 | 1.0100/ 0.0000 | | |
| V2 | 0.9980/ -0.0050 | 1.0080/ 0.0240 | | |
| V3 | 1.0080/ -6.0760 | 1.0040/ -6.4830 | | |
| V4 | $0.9340/\!-\!13.3000$ | $0.9790/\!-\!13.3760$ | | |
| V5 | $0.9150/\!-\!14.4520$ | 0.9730/-14.6160 | | |
| V6 | $0.9400/\!-\!12.8550$ | 0.9850/-12.9710 | | |
| V7 | 0.9590/ -6.0400 | 0.9670/ -6.4960 | | |
| V8 | 1.0040/ -6.1500 | 1.0000/ -6.5610 | | |
| V9 | 1.0040/ -6.1520 | 1.0000 - 6.5450 | | |
| V10 | 0.9800/ -8.3160 | 0.9620/ -8.7730 | | |
| V11 | 0.9090/ -14.8750 | 0.9670/ -14.9900 | | |
| V12 | 0.9010/ -15.4300 | 0.9820/ -15.8800 | | |
| P1 | 0.5587 | 0.5562 | | |
| P7 | 0.1500 | 0.1500 | | |
| P10 | 0.3300 | 0.3300 | | |
| Q1 | 0.4947 | 0.2638 | | |
| Q7 | 0.0817 | 0.0819 | | |
| Q10 | 0.0160 | 0.0170 | | |
| Q12 | 0.0000 | 0.2000 | | |
| T1 | 0.9000 | 0.9500 | | |
| T2 | 0.9750 | 1.0000 | | |
| T3 | 1.0500 | 1.0250 | | |
| LOSS | 1.1060 | 0.8570 | | |

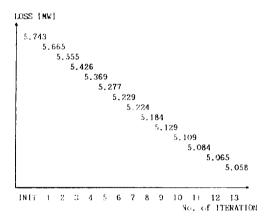


Fig. 4 Variation in losses (sample-2 system)

7 without oscillation, the good convergence property of real power losses was shown in Fig.3.

6.2 Sample-2 System

The sample-2 system used a data set based on an adaptation of a modified IEEE 30 bus system[7]. The results are given in table 6, showing that the method presented in this paper gives much better

Table 6 Results for the sample-2 system

| variable | initial state | final state |
|----------|-----------------------|-----------------|
| V19 | 0.9440/-10.4830 | 1.0040/ -9.9530 |
| V20 | 0.9470/-10.2090 | 1.0080/ -9.7370 |
| V21 | 0.9430/ -9.6830 | 1.0150/ -9.3960 |
| V22 | 0.9440/ -9.6750 | 1.0160/ -9.4100 |
| V23 | 0.9470/-10.1360 | 1.0110/ -9.7970 |
| V24 | 0.9290/-10.2240 | 1.0150/-10.2440 |
| V25 | 0.9230/-10.2580 | 1.0150/-10.1420 |
| V26 | 0.9040/-10.7700 | 0.9970/-10.5630 |
| V27 | 0.9290/ -9.9670 | 1.0240/ -9.8130 |
| V29 | 0.9070/-11.4660 | 1.0090/-11.1820 |
| V30 | $0.8940/\!-\!12.5490$ | 1.0030/-12.2140 |
| P1 | 0.9894 | 0.9826 |
| P2 | 0.8000 | 0.8000 |
| P5 | 0.5000 | 0.5000 |
| P8 | 0.2000 | 0.2000 |
| P11 | 0.2000 | 0.2000 |
| P13 | 0.2000 | 0.2000 |
| Q1 | 0.0000 | 0.0000 |
| Q2 | 0.1562 | 0.0783 |
| Q5 | 0.1654 | 0.2322 |
| Q8 | 0.1573 | 0.3295 |
| Q11 | 0.3701 | 0.0980 |
| Q12 | 0.0000 | 0.1597 |
| Q13 | 0.4081 | 0.0419 |
| Q17 | 0.0000 | 0.0920 |
| Q24 | 0.0000 | 0.1223 |
| Q27 | 0.0000 | 0.0608 |
| Q30 | 0.0000 | 0.0297 |
| T1 | 1.0750 | 1.0000 |
| T2 | 1.0625 | 0.9625 |
| T^3 | 1.0375 | 1.0125 |
| T4 | 1.0625 | 1.0125 |
| | 1.0020 | 1.0120 |

results than other methods. The difference in system losses between the initial base and the final (5.743 MW compared to 5.058 MW) clearly shows the advantages of the acceleration factor. System losses converged in iteration 13 without oscillation, the good convergence property of real power losses was shown in Fig.4.

7. Conclusions

This paper has presented a new algorithm for the optimal reactive power/voltage control problem. The distinct advantages drawn from this study are summarized as follows:

- (a) This algorithm is more accurate than the conventional methods since the sensitivity relation matrix is successively updated by the newly developed method.
- (b) The convergence property is enhanced by the acceleration factor varying with the iteration steps.
- (c) The computation time is much saved since the highly-sparse characteristics of constraints matrix can be preserved by adopting bus voltages as the control variables in Q-V module.
- (d) The operator can make the reasonable control action by using the routine which the increment of capacitor/reactor and transformer taps is rounded to the nearest step.
- (c) The reliable computation accuracy and fast convergence characteristics obtained in this methodology present the possibility for its application to the other areas.

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