□ 論 文 □

確率 및 經濟要素를 考慮한 道路線形 設計

Probabilistic & Economic Factors in Highway Geometric Desgin : A Climbing Lane Example

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要 約

本 稿는 道路線形 設計에 確率 및 經濟要素를 반영시키는 방법과 그에 따른 효과를 파악하는데 목적을 두고 있다. 接近方法은 道路使用者 費用과 道路施工 및 維持管理 費用 등을 포함하는 總費用을 豫想費用函數로 나타내어 이 費用函數를 최소화함으로써 最適 道路線形 設計方法을 찾는 것이다.

이와 더불어 本 稿는 이 개념에 의한 最適 設計方法을 등반차선(Climbing lane)의 설계에 적용하였다. 즉, 오르막길의 傾斜 및 거리에 따른 등반차선의 最適 位置를 찾기 위해 우선 오르막길에서의 승용차와 트럭의 速度分布를 찾아내고 豫想費用函數를 等式化하여 그 最適解를 구함으로써 등반차선의 최적 위치를 구하였다.

현재 등반차선의 設計過程(AASHTO, 1984)은 트럭 走行能力에 따른 가정과 트럭과 승용차의 最大 速度 差異를 임의로 선택하여 등반차선의 최적 위치를 찾고 있다. 따라서 本 稿에서는 기존 등반차선 設計方法과 새로운 설계방법을 상호 비교하면서 새로운 방법의 妥當性과 그에 따른 設計利點들을 證明하였다.

Ⅰ. INTRODUCTION

This paper addresses the practice of using rigid standards in geometric highway design. There exist examples of successful highways where standards have not been adhered to, and on the other hand, there exist examples of highways designed with rigid standards that have not operated smoothly or safely. These observations have led to the conclusion that the designer must understand the underlying rationale, and must consider all the operational consequences of the design standards.

The geometric design standards widely used in highway design are deterministic in character and thus ignore important probabilistic factors of highway operations. Most design values of highway elements are determined by only two factors : the highway type and the design speed. The experience gained from the application of highway design standards reveals that the omission of probabilistic and economic factors may, in some situations, lead to significant inefficiencies.

The design values for highway elements are determined from the input parameters using physical and driver behaviour relationships. The value of an input parameter is usually chosen as the 85th percentile value of its distribution (see for example, AASHO, 1984 and Kerman, 1980). Using a given percentile, a single design value is then derived, instead of a distribution of possible design values. However, it is possible, for example, that by using the 85th percentile value of the speed and the lateral acceleration distributions, the resulting design value of an horizontal radius may represent only the 70 the percentile value of all desired radii. Moreover, there are design problems that cannot be solved by using unique percentile values(Craus and Lirneh, 1978 and Craus et al., 1980).

The second drawback of existing design standards is that they are not sufficiently sensitive to important operational and economic factors such as : volume, accident rate and monetary costs. Recently, a stronger emphasis in design has been placed on the economic factors. Highway construction agencies are relying increasingly on expedient and cost effective standards which are essentially below current AASHTO(1984) standards.

This paper develops an approach to geometric design of highways that takes into account probabilistic and cost factors. It is suggested that the concept of "design standard" be replaced with a broader concept of "design process." In this process, the designer uses all the available information about values of input parameters and employs explicit design criteria. We employ a "cost function" that takes different values for alternative designs, and select that optimal design value that brings to a minimum the objective function. Section 2 of this paper describes the various stages of the design process, and the remainder of the paper develops the application of this approach to the design of a climbing lane on a two-lane two-way rural highway.

Ⅱ. THE PROPOSED APPROACH TO GEOMETRIC DESIGN OF HIGHWAY ELEMENTS

The proposed design process for the selection of an optimal design value for a highway element is schematically shwon in Fig. 1 and discussed below.

1. Defining Input Values Distributions

The design of highway elements requires many input parameters. For example, in the design of a vertical curve, one needs to know parameters such as : speed, driver perception-reaction time, friction factor, height of a driver's eye height of possible obstacles. These parameters are not constant since they vary among drivers, vehicle types, weather conditions, etc. Therefore, the designer is faced with distribution of parameter values. The most studied distribution is that of the running speed of vehicles. It is often approximated by the normal distribution. Distributions of other input parameters have also been investigated and documented, e.g., sight distance by Glennon(1988), height of a driver's eye by Farber(1982), and vehicle performance by Walton and Lee(1977).

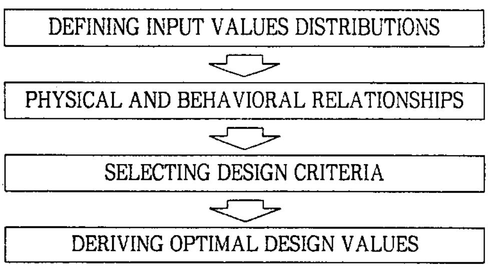

Fig. 1 A Design Process to Derive an Optimal Geometric Design Values for a Highway Elements

These distributions are different from place to place and from time to time, because they are dependent on conditions such as : weather, elevation above sea level, traffic composition and driver's performance. Thus, for the design of a specific highway, the distribution of input parameters can be estimated either from field measurements or by making reasonable assumptions about the distributions, and using previously collected data.

2 . Physical and Behavioural Relationships

Vehicle performance is governed by basic physical and behavioural relationships. For example, in the design of a horizontal curve, the centrifugal forces acting on the vehicle must be considered. An example of a behavioural relationship can be found in the design of a vertical curve where perception-reaction time of the driver is used to estimate his stopping distance.

These basic relationships define the functional dependence of an intermediate variable on the input parameters. For example, to determine the radius R of a vertical curve, one uses the following input parameters : V—speed, f—friction factor, t—driver perception-reaction time, h_1—driver height of eye, and h_2—obstacle height. These parameters are used in the following two relationships : define the stopping distance S as

$$S = V \cdot t + \frac{V^2}{2 \cdot g \cdot f} \quad \cdots\cdots (1)$$

where g is gravity acceleration ; and from the geometry of the curve define the sight distance s by

$$s = (2 \cdot R)^{1/2}(h_1^{1/2} + h_2^{1/2}) \quad \cdots\cdots (2)$$

We can derive from these relationships a radius denoted by r such that the sight distance is equal to the stopping distance. Thus, substitute

$$s = S \quad \cdots\cdots (3)$$

in (2) to obtain

$$r = \frac{S^2}{2(h_1^{1/2} + h_2^{1/2})^2} \quad \cdots\cdots (4)$$

In this example, we have defined three intermediate variables : s, S and r. In the following subsection we will show how we use these variables to determine a design value for R.

3 . Applying the Design Criterion

In order to select a design value, one needs to adopt a design criterion that can be based on safety considerations as well as on operation and construction costs. Existing design standards tend to be based exclusively on safety considerations. The design values are computed from the physical relationships which satisfy some minimum safety conditions.

The proposed approach is to define an explicit objective function which attains its optimal level by the selection of the "optimal" design value. This objective function includes several cost components, taking into account road users, construction and maintenance costs. The road users cost function may include vehicle operating costs, value of travel time and the costs of road accidents. This cost function is dependent on the traffic volume. The construction and maintenance cost function determined by the geometric design value and by highway construction and maintenance cost parameters includes pavement cost, earth movement costs, etc., expressed in terms of equivalent annual cost. The objective is thus to minimize total cost associated with a highway element.

Suppose, for example, that the design value X is being sought for a highway element. Denote the input parameters and intermediate variables as x, the probability density function of x as f(x), the road user cost function as C (X, x), and the construction cost function as I (X). We write the following objective function :

$$\operatorname*{Min}_{X}\left[\int_{x} C(X, x)\cdot f(x)\,dx + I(X)\right] \quad \cdots\cdots\cdots (5)$$

and the optimal value of X as X_{opt}. Note that the objective function can be subjected to safety and environmental constraints. For example, one may wish to limit the maximum value of a vertical curve radius. The same objective function can be applied also in a deterministic approach with constant values for the input parameters.

As a specific example, consider again the case of selecting the radius of a vertical curve denoted by R. Suppose that fixed values of the input parameters are used to calculate a value for r which is the currently used design standard. Define the following road user's cost function :

$$C(R, r) = \begin{cases} \theta_1 (r-R)^2 & \text{for } R<r \\ 0 & \text{for } R \geq r \end{cases} \quad \cdots\cdots\cdots\cdots (6)$$

where θ_1 is a constant parameter. This road user's cost function is a quadratic function of the difference between r and the radius R for curve radii smaller than the value of the radius necessary to have the stopping distance equal to the sight distance. It seems reasonable to assume that the costs of accidents and delays due to speed changes are increasing more than linearly with increasing difference between stopping and sight distance. The construction cost function which mainly explains differences in earth movement costs associated with different values of R can be approximated by the form :

$$I(R) = \theta_2 R^2 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (7)$$

where θ_2 is a constant. For for $R \geq r$, it is evident that $R_{opt} = r$.

In the range of R⟨r, the objective function is :

$$\mathrm{Min}_{R}[\theta_1(r-R)^2 + \theta_2 R^2] \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots (8)$$

The solution R_{opt} is given by :

$$R_{opt} = \frac{r}{1 + \dfrac{\theta_2}{\theta_1}} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (9)$$

that is, the optimum design value is related to the intermediate value r by the ratio of the parameter of road user cost to the parameter of the construction cost. It can be seen that unless $\theta_2=0$, R_{opt} is always less than r. Thus, when r is accepted as the design standard, it means that the cost of earth movement it taken to be negligible relative to the costs associated with road accidents and delays.

It should be mentioned that several previous studies have dealt with the monetary costs of the design standards (e.g., Kerman, 1979). However, the monetary costs were evaluated for arbitrarily selected design values, and no attempt was made to search for a general optimal design.

The remainder of this paper developes in detail the application of the proposed approach to the design of a climbing lane.

Ⅲ. A CLIMBING LANE EXAMPLE

1. The Problem

A climbing lane runs alongside the regular lanes on upgrades. It is constructed for operational and safety reasons for the use of slow moving vehicles, especially trucks. The design problem is to determine the length of a climbing lane, (see Fig. 2). Consider an uphill highway section with a given grade, 1%, and a total length of L. Let x be the distance of to a point on the upgrade from the starting point of the grade, 0. The design value X_{cr}, is the distance from point 0 to the beginning of the climbing lane.

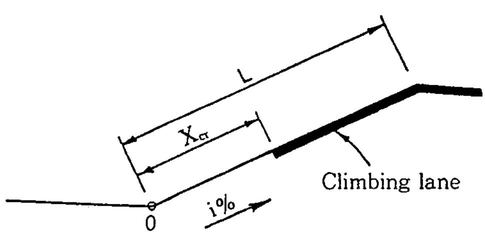

Fig. 2 The design problem of a climbing lane

2. The AASHO(1984) Design Procedure

The current design practice is to find a value of X_{cr} such that the maximum difference between the average speeds of light vehicles and trucks will not exceed a predetermined critical value $\triangle V_{cr}$, e.g., 16km/h (AASHO, 1984 and Craus et al., 1981). The design procedure proposed by AASHO is based on the following assumptions :

- The average entrance speed of light vehicles to the upgrade is equal to the average running speed. The light vehicle average speed is not affected by the upgrade, i.e., the level running speed is maintained.
- The average entrance speed of trucks to the upgrade is equal to the average running speed of light vehicles. It is decreasing with x according to performance curves which were developed during the 1970's for a truck with a weight to power ratio of 400 1b/h.p.

The AASHO procedure can be summarized by using the following notations :

V_R is the level running speed of light vehicles, V_{op} is the entrance speed of a passenger car to the upgrade, V_{ot} is the entrance speed of trucks, V_{xp} is the speed of a car at point x on the upgrade, and V_{xt} is the speed of a track at point x. It is assumed that

$$V_{xp}=V_{op}=V_R$$
$$V_{ot}=V_R \qquad \text{(10)}$$
$$V_{xt}<V_{ot} \quad \text{for } x>o \quad \text{and} \quad i>o$$

This approach is subject to the following shortcomings :

- It considers only an average or a percentile value instead of the full distribution of running speed.s
- It assumes that the upgrade has no effect on the light vehicle speed.
- Since vehicle performance is a function of its weight, engine size, vintage, and driver's ability, there is, in fact, a distribution of vehicle deceleration rates for a given upgrade.

Serveral authors have proposed the use of stochastic simulation models to improve on the AASHO procedure(See, for example, 1980, Homburger, 1987). The approach developed by Homburger(1987) considers a specific upgrade with given traffic conditions and a budget constraint, and searches for the location of the climbing lane on the upgrade that is the most cost effective. However, in order to perform this evaluation, he developed a very detailed microscopic simulation model.

3. The Proposed Approach

In this paper, we also search for the cost effective design but employ a simplified, analytical macro-scopic model at the same level of detail as the AASHO procedure.

In the following paragraph, we describe the various stages of the proposed approach.

4. Defining Input Parameters Distributions

The inputs needed for the climbing lane design problem are :

- The entrance speeds of cars and trucks to the upgrade.
- The deceleration rates of cars and trucks on the upgrades.

It is assumed that the entrance speeds of cars and trucks are normally distributed as follows :

$$V_{op} \sim N(\mu_{vop}, \sigma^2_{vop})$$
$$V_{ot} \sim N(\mu_{vot}, \sigma^2_{vot}) \quad \cdots\cdots (11)$$

where μ_{vop} and μ_{vot} and σ^2_{vop} and σ^2_{vot} are the means and variances of the entrance speeds for cars and trucks, respectively. In previous studies it was shown that the speed profile of trucks along the upgrade can be approximated by a linear function as follows :

$$V_{xt} = V_{ot} + C_t \cdot x \quad \cdots\cdots (12)$$

where C_t is the rate of truck speed change per unit distance for a given upgrade of 1%. Assume that C_t is approximately normally distributed as follows :

$$C_t \sim N(\mu_{ct}, \sigma^2_{ct}) \quad \cdots\cdots (13)$$

where $\mu_{ct}$and σ^2_{ct} are the mean and the variance of C_t, respectively.

Assumptions (11), (12), and (13) imply that the speed of trucks at any point x along the upgrade is also normally distributed, as follows :

$$V_{xt} \sim N(\mu_{vot} + x \cdot \mu_{ct}, \sigma^2_{vot} + x^2 \cdot \sigma^2_{ct}) \quad \cdots (14)$$

Similar considerations applied to the performance of cars on upgrades lead to the following distribution of car speeds at any point x along the upgrade,

$$V_{xp} \sim N(\mu_{vop} + x \cdot \mu_{cp}, \sigma_{vop} + x^2 \cdot \sigma^2_{cp}) \quad \cdots (15)$$

where μ_{cp} and σ_{cp} are the mean and variance of C_p, the rate of speed change for cars on a given upgrade of i%

5. Deriving the Distributions of Intermediate Variables

The key intermediate variable for this design problem is the profile of speed differences between cars and trucks along the upgrade. Define the speed difference at distance x from point 0 as

$$\triangle V_x = V_{xp} - V_{xt} \quad \cdots\cdots (16)$$

From equations (14) and (15) it follows that it is equal to the difference between two normally distributed variables. Thus, $\triangle V_x$ it is also normally distributed with the following mean and variance :

$$\mu_{\triangle vx} = \mu_{vop} - \mu_{vot} + x \cdot (\mu_{cp} - \mu_{ct}) = a + b \cdot x$$

$$\sigma^2_{\triangle vx} = \sigma^2_{vop} + \sigma^2_{vot} + x^2(\sigma^2_{cp} + \sigma^2_{ct}) = d + g \cdot x^2 \quad \cdots\cdots (17)$$

6. Applying the Percentile Design Criterion

Find the point along the upgrade, denoted as $x(\triangle V_{cr})$, where the critical speed difference is not exceeded for a prespecified percentile denoted p. This percentile criterion is being used when strong safety considerations exist, and explicit monetary cost function cannot be developed. To apply this percentile criterion, recall from (17) that :

$$P[\triangle V_x \leq \triangle V_{cr}] = \phi\left(\frac{\triangle V_{cr} - a - bx}{\sqrt{d + gx^2}}\right) \quad \cdots\cdots (18)$$

and find the solution $x(\triangle V_{cr})$ such that

$$P(\triangle V_x \leq \triangle V_{cr}) = p \quad \cdots\cdots (19)$$

The positive solution of the quardratic equation obtained from (18) and (19) is

$$x(\triangle V_{cr}) = \frac{2b(\triangle V_{cr} - a) + \{4b^2(\triangle V_{cr} - a)^2 - 4(b^2 - g\phi^{-1}(p))[(\triangle V_{cr} - a)^2 - d\phi^{-1}(p)]\}^{1/2}}{2(b^2 - g\phi^{-1}(p))} \quad \cdots\cdots (20)$$

A deterministic criterion equivalent to the AASHO procedure is obtained as a special case of the above percentile criterion by substituting in (20)

$$d = g = 0 \quad \cdots\cdots (21)$$

to get

$$x(\triangle V_{cr})_{det} = \frac{\triangle V_{cr} - a}{b} \quad \cdots\cdots (22)$$

In the AASHO procedure it is further assumed that $a = 0$ and $b = \mu_{ct}$.

The designer must assign values to two parameters ; $\triangle V_{cr}$ and p. The critical speed difference between the trucks and cars is usually in the range of 16km/hr to 24km/hr and the critical percentile of the speed difference distribution is usually in the range of 80～95 percent. The choice of these values is fairly arbitrary. It is usually based on previous experience and values that are reported in the literature to produce "good" results. In the following section, we shall develop a rational approach that does not require to assign such arbitrary values to obtain the design criterion.

7. Applying the Expected Cost Design Criterion

This approach determines directly the value of X_{cr}. The objective function includes the following two cost components which are functions of X_{cr} :

1) The construction cost :

Define the equivalent annual construction cost per unit length of the climbing lane as β ; the construction cost of the climbing lane starting at point X_{cr} on the upgrade is given by :

$$I(X_{cr}) = \beta(L - X_{cr}) \quad \cdots\cdots (22)$$

2) The road user's cost :

A speed difference of $\triangle V$ between cars and trucks results in additional costs to the road users, due to the increase in accident rates, increase in time delays, and increase in fuel consumption due to speed changes. It was found that the road user cost is increasing more than lineraly with the increase of $\triangle V$, Assume that these road user costs per unit distance on the upgrade can be expressed as

$$C(\triangle V)=\alpha \cdot \triangle V+\gamma \cdot \triangle V^2 \quad \cdots\cdots (23)$$

where α and γ are the cost function parameters. (The empirical values of these parameters will be considered in the following section).

The speed difference, and as a consequence the additional road user costs, exist only up to the starting point of the climbing lane, and the expected total road user cost is calculated as follows :

$$C(X_{cr})=\int_0^{X_{cr}} \int_{-\infty}^{\infty} C(\triangle V) \cdot f(\triangle V \mid x) \cdot d\triangle V \cdot dx \quad \cdots\cdots (24)$$

where $C(X_{cr})$ is the expected total road user cost, and $f(\triangle V \mid x)$ is the probability density function of the speed difference at point x along the upgrade. The sum of (22) and (24) define the expected total cost associated with the climbing lane. The optimal design value is obtained from

$$\underset{X_{cr}}{\mathrm{Min}}\Big[\int_0^{X_{cr}}\int_{-\infty}^{\infty} C(\triangle V) \cdot f(\triangle V \mid x) \cdot d\triangle V dx + (L-X_{cr}) \cdot \beta\Big] \quad \cdots\cdots (25)$$

where $C(\triangle V)$ is given by (23) and $f(\triangle V \mid x)$ is a normal density with parameters given by (17). Using the definitions of 1st and 2nd moments, we get for the expected user costs per unit distance the following expression :

$$\int_{-\infty}^{\infty}(\alpha \cdot \triangle V+\gamma \cdot \triangle V^2) \cdot f(\triangle V \mid X) \cdot d\triangle V = \alpha\mu_{\triangle vx}+\gamma(\mu^2_{\triangle vx}+\sigma^2_{\triangle vx}) \quad \cdots\cdots (26)$$

Substitute (26) and (27) in the objective function (25) to obtain

$$\underset{X_{cr}}{\mathrm{Min}}\Big[\int_0^{X_{cr}}[\alpha(a+bx)+\gamma(a^2+2abx+b^2x^2+d+gx^2)]dx+(L-X_{cr}) \cdot \beta\Big] \quad \cdots\cdots (27)$$

The optimal value of X_{cr} is derived by setting the derivative of the objective function in (27) with respect to X_{cr} equal to zero, as follows :

$$\gamma(b^2+g) \cdot X^2_{cr}+(\alpha \cdot b+2a \cdot b \cdot \gamma)X_{cr} + [\alpha a+\gamma(a^2+d)-\beta]=0 \quad \cdots\cdots (28)$$

The positive solution to the quadratic equation (28) (for γ〉0), is given by :

$$X^{opt}_{cr}=\frac{-\alpha b-2ab \cdot \gamma+\sqrt{(\alpha b+2ab\gamma)^2 - 4\gamma(b^2+g) \cdot [\alpha a+\gamma(a^2+d)-\beta]}}{2\gamma(b^2+g)} \quad \cdots\cdots (29)$$

The general solution for the optimal value of X_{cr} in a case of quadratic road user cost function, given in (29), has the following properties :

○ There might be a theoretical case where eq. (29) does not have a real solution, when

$$(\alpha b+2ab\gamma)^2 \langle 4\gamma(b^2+g)\ [\alpha a+\gamma(a^2+d)-\beta] \quad \cdots\cdots (30)$$

A necessary condition for (30) is :

$$\alpha a+\gamma(a^2+d)-\beta \rangle 0 \quad \cdots\cdots (31)$$

However, note that the first two terms on the left side of eq. (31) are in fact equal to C(X

$_{cr}=0$) which is the expected total user cost if the climbing lane were to start at the beginning of the upgrade. This is due to the fact that speed differences between cars and trucks exist at leveled highways as well. Thus, the meaning of the necessary condition (31), is that even at the beginning of the upgrade, the total road user costs are greater than the construction cost per unit distance. Therefore, if (31) holds, it is economical to add an extra lane, even on the highway section preceding the upgrade.

The economic design criteria can also be applied independently of the distributional assumptions by using a deterministic approach. In the above example, the deterministic approach can be represented by setting

$$d=g=0 \quad \cdots\cdots (32)$$

A simpler deterministic version which corresponds to the assumptions made in the AASHO procedure is obtained by assuming that the entrance speeds of trucks and cars are equal, i. e., a=0, to get the following result :

$$X_{cr}^{opt}=\frac{\alpha b+\sqrt{\alpha^2b^2+4\gamma b^2\cdot\beta}}{2\gamma b^2} \quad \cdots\cdots (33)$$

The ratio between the general solution of X_{cr}^{opt} in (29) and the special case of (33) is not straightforward, and it depends on the values of the various parameters.

In the case of a linear road user cost function, i. e., $\gamma=0$, the solution of the first order condition in (28) is given by

$$X_{cr}^{opt}=\frac{\beta-\alpha a}{\alpha b}=\frac{\beta/\alpha-a}{b} \quad \cdots\cdots (34)$$

This solution implies that the design value of X_{cr} for a linear cost function is independent of the variance of $\triangle V$. Note that this solution is identical to the deterministic solution given by equation (21) by letting $\triangle V_{cr}=\beta/\alpha$

The approach presented above has the following advantages :

- It is sensitive to the construction costs of the climbing lane given by the parameter β.
- It is sensitive to the traffic volume and local conditions such as value of time and accident costs that are represented by the parameters α and γ.
- It is sensitive to local conditions of driver performance and traffic compositions represented by the parameters a, b, d and g.

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