TIDAL DENSITIES OF GLOBULAR CLUSTERS AND THE GALACTIC MASS DISTRIBUTION

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ABSTRACT

The tidal radii of globular clusters reflect the tidal field of the Galaxy. The mass distribution of the Galaxy thus may be obtained if the tidal fields of clusters are well known. Although large amounts of uncertainties in the determination of tidal radii have been obstacles in utilizing this method, analysis of tidal density could give independent check for the Galactic mass distribution. Recent theoretical modeling of dynamical evolution including steady Galactic tidal field shows that the observationally determined tidal radii could be systematically larger by about a factor of 1.5 compared to the theoretical values. From the analysis of entire sample of 148 globular clusters and 7 dwarf spheroidal systems compiled by Webbink (1985), we find that such reduction from observed values would make the tidal density (the mean density within the tidal radius) distribution consistent with the flat rotation curve of our Galaxy out to large distances if the velocity distribution of clusters and dwarf spheroidals with respect to the Galactic center is isotropic.

I. INTRODUCTION

The globular clusters have been used to probe the large scale Galactic mass distribution since they can be traced to large distances. There are essentially two ways to explore the Galactic mass distribution using globular clusters: kinematics (e. g., Frenk and White 1980; Thomas 1988) and tidal density (e.g., Peterson 1974; Innanen, Harris and Webbink 1983).

The tidal density is defined as the mean mass density within tidal radius and it is proportional to the mean Galactic density within the (orbital) Galactocentric radius if the cluster's orbit is circular. Therefore it is possible to determine the Galactic mass distribution if tidal density distribution of globular clusters is known as a function of Galactocentric distance. However there are some complications: (1) the clusters do not move in circular orbits; (2) it is very difficult to determine the tidal density observationally due to uncertainties in both cluster's mass and the tidal radius. Because of these difficulties, this method is by no means superior to others but it is still important as an independent determination.

The conventional definition of tidal radius is the place where the density becomes zero. It is very difficult to find such a place from the observations because the surface brightness becomes very low near the tidal boundary. Actual determination of tidal radii has been based on extrapolation of surface brightness distribution of inner parts, which resembles that of the lowered istothermal models (King models) very closely. Obviously large amounts of errors could be introduced through this extrapolation process.

Innanen, Harris and Webbink (1983; referred to IHW hereafter) have made an extensive study regarding the tidal radius and its implications. They have analyzed the tidal densities of 66 well observed systems (globular clusters and dwarf spheroidals) and found that the Galactic mass distribution is consistent with that obtained from flat rotation curve (i.e., $M_G(R) \propto R$). They also noted that the inferred tidal field appeared to be systmatically weaker than that computed from the flat rotation velocity of 220 km/sec. One obvious reason is the systematic understimate of mass-to-light ratio of globular clusters, thereby reducing the tidal density by the same factor. However, careful modeling of mass distribution based on both photometric and spectroscopic observations does not seem to indicate any strong deviation from conventional values of M/L_V — 1.7 (in solar units). The other reason could be due to the systematic errors in tidal density determination since the tidal density is proportional to inverse of third power on the derived tidal radius.

The purpose of the present study is to re-examine the theoretical implications of tidal radii on the mass distribution of the Galaxy, and to see what can be learned from the analysis of tidal density distribution. Our major extension from the very extensive study by IHW on this subject is to use larger sample of clusters. We also introduce a new notion that the observed tidal radii may be systematically wrong by a constant factor based on theoretical modeling of dynamically evolving clusters in a steady Galactic tidal field by Lee and Ostriker (1986).

This paper is organized as follow: We review the expected distribution of tidal radii in in an idealized Galactic potestial §II. The results of data analysis for a large sample of globular clusters and dwarf spheroidals are presented in §III and a summary is given in the last section.

II. TIDAL DENSITY FOR NON–CIRCULAR ORBITS

We will follow the formalism by IHW for the definition of tidal density. The shape of the Galaxy is assumed to be spherically symmetric, which may be a good assumption at large distances (several kpc) from the Galactic center. We will further assume that the potential is that of a singular isothermal sphere (SIS) which represents many galaxies with flat rotation curves very well. IHW assumed that the tidal radius is determined by the tidal field at the perigalactic passage. This assumption has been subsequently supported by the numerical study by Allen and Richstone (1988).*

Under these assumptions the distance to the first Lagrangian point for a cluster moving in a non-circular orbit, whose perigalactic distance is R_p, can be calculated from

$$x_e = [1 - 2 \ln(R_p/A)]^{-1/3} \left[\frac{M_c}{2M_G(R_p)} \right]^{1/3} R_p, \tag{1}$$

where M_c is the cluster mass, $M_G(R_p)$ is the Galactic mass within R_p and A is the radius of a hypothetical circular orbit whose orbital energy is the same as the actual orbit.

The shape of Roche surface (the place at which the tidal force balances with the gravity) is not spherical. The distance to the first Lagrange point is farthest on this surface. Since the isopotential surface is spherical in the inner parts, the tidal radius determined by the extrapolation would obviously be the shortest distance (e. g., Spitzer 1988). In the case where the variation of the Galactic mass over a cluster scale is negligible, the tidal radius is two-third of the distance to the first Lagrangian point. Therefore the mean density within the tidal radius r_t (i. e., tidal density) becomes

$$\rho_{tid} = \frac{M_c}{4/3 \pi r_t^3} = \frac{81}{32 \pi} \frac{M_c}{x_e^3}$$

$$= \frac{81}{32 \pi} [1 - \ln(Rp/A)] \frac{M_G}{R_G^3}$$

$$= \frac{81}{16 \pi} [1 - \ln(R_p/A)] \frac{V_{cir}^2}{GR_p}, \tag{2}$$

where we have used the identity $M_G(R_p) = R_p V_{cir}^2/G$ for an SIS with the rotation velocity being V_{cir}.

If the orbits of clusters are circular, equation (2) simply becomes

$$\rho_{tid} = \frac{81}{16 \pi} \frac{V_{cir}^2}{GR_p^2} \tag{3}$$

Thus the Galactocentric distance versus tidal density should become a straight line in a log-log plot. The presence of observational errors will introduce the scatter in this plot.

If the orbits are non-circular, $R_p < A$ and $R_p < R_G$ so that ρ_{tid} is greater than $81V_{cir}^2/16\pi GR^2_G$. Therefore, the data points in the plot of tidal density against the present Galactocentric distance should appear above the theoretical line drawn in accordance with the right-hand-side of equation (3).

In equation (2) both R_p and A are unknown. For an SIS potential, it can be shown that

$$\langle \ln (R_p/R(t)) \rangle = \ln (R_p/A), \tag{4}$$

where $R(t)$ is the dependence of the robital radius with time and the brackets represent the time average. Multiplying equation (3) by R^2_G and replacing A by R_G, where R_G now represents the present Galactocentric distance, we get

$$\rho_{tid} R^2_G = \frac{81}{16 \pi} [1 - \ln (R_p/R_G)] \left[\frac{R_G}{R_p} \right]^2 \frac{V_{cir}^2}{G}. \tag{5}$$

From the equation (4) we can compute R_p/R_G for each cluster. The individual value of this quantity is not particlulary meaningful because it is continuously varying function of time if the cluster is moving on a non-circular orbit. However, we can get an *unbiased distribution* of R_p/R_G for a sample if the orbital phase of a given cluster is random so that current distribution of galactocentric distances has no particular bias. The distribution function can be computed theoretically for a given potential if the nature of orbits (or velocity ellipsoid) is known. Therefore the observed frequency distribution of R_p/R_G is a combination of the shape of Galactic potential and the velocity ellipsoid of clusters against the Galactic center. Unfortunately it is very difficult to separate the velocity distribution effect, but we should be able to put some constraints on that based on various reasons. We now turn into the analysis of observed data.

III. DATA ANALYSIS AND DISCUSSIONS

1. The Data Set

Webbink (1985) has made an extensive compilation of all data for 148 Galactic globular clusters and 7 dwarf spheroidal systems. We have used these data for tidal radii distribution. The observational error in tidal radii is estimated to be $\epsilon (\log r_t) > 0.1$.

In order to obtain the mass of individual cluster, we have assumed $M/L_V = 1.7$ in solar units for all globular clusters following Illingworth (1976) and Webbink (1985). Since the dwarf spheriodals are known to have widely varying M/L we used Kormendy (1987) and Aaronson and Olzewlski's (1986) indivdidual values of M/L for each system based on radial velocity measurements.

The tidal radius in King models (also in observational definition) is the place where the density becomes zero. However, since stars continuously escape from cluster through the tidal boundary, the density at tidal radius will not be zero. Numerical studies by Lee and Ostriker (1987, also by Lee, Fahlman and Richer 1991) indicate that the radius where the tidal force balances the internal gravitation is about a factor of 1.5 smaller than that defined by fitting to King models in the inner parts. This process will give a tidal radius similar to that obtained by extrapolation. We thus reduced the tidal radii in Webbink's compilation by the same factor in computing the tidal densities of clusters discussed below.

2. Tidal Densities versus Galactocentric Distances

As discussed earlier, the tidal density is expected to lie above the line computed from equation (3) if the clusters move in non-circular orbits. In Figure 1, we show a plot of ρ_{tid} *versus* R_G for the entire sample. Also shown as a solid line is the theoretical tidal densities for circular orbits in an SIS potential with $V_{cir} = 220$ kms^{-1}. Although this is a scattered diagram, observed tidal density distribution appears to be consistent with the assumed Galactic potential of an SIS in the sense that most of the data points lie above the solid line. If we had not reduced the tidal radii by a factor of 1.5, many data points lie below the straight line, which represents the minimum tidal

density for a given Galactocentric radius if the tidal density is really determined at the perigalactic passage.

We have performed the least-square fit to the relationship

$$\log P_{tid} = -a\log(R_G) + b,\tag{6}$$

where a and b are constants. We have excluded clusters with $R_G < R_c$ where R_c is the cut-off radius. We have varied R_c from 1 to 30 kpc in order to find any differences in the behavior of clusters with small R_G compared to those with large R_G. The results are listed in Table 1, which shows that the slope a remains to be a constant around 2 as long as $R_c \geq 3kpc$. The slope

Table 1. Least Square Fitting for $\log R_G$ versus $\log P_{tid}$.

R_c	N	a
1	152	1.70 ± 0.10
2	140	1.77 ± 0.13
3	118	2.03 ± 0.16
10	60	2.05 ± 0.40
30	23	$2.03 + 1.00$

becomes smaller if we include clusters whose R_G are smaller than 3 kpc. This may be caused by several effects: (1) the orbit of clusters with small R_G could be more circular than those with large R_G because of dynamical friction that destroys clusters with radial orbits, (2) the potential in the inner parts deviates significantly from that of an SIS, and (3) clusters having small R_G are expected to experience strong tidal shocks (e. g., Aguilar, Hut and Ostriker 1988) that could change the tidal density.

In Figure 1 the result for the least-square fit with $R_c = 3kpc$ is shown as a dotted line. The slope a is very close to 2, the value expected from the Galactic potential whose rotation curve is flat.

From Figure 1, one can easily guess that the use of the original tidal radii from Webbink's table will result in putting many data points lie well below the solid line. The reduction factor of 1.5 in tidal radii has raised all tidal densities by about a factor of 3.4, making the tidal density distribution more consistent with the known Galactic mass distribution.

The fact that systematically smaller r_t's give better distribution for R_p/R_G supports the idea that the density at tidal radius is indeed non zero. As pointed out by IHW, raising M/L will have the same desirable effect, but the required increase in M/L would be $1.5^3 \approx 3.4$ which cannot easily be reconciled with the mass estimates based on velocity dispersion measurements of globular clusters.

The mass-to-light ratios for globular clusters have been determined by various means. King's core fitting formula (e. g., Richstone and Tremaine 1986) could be used to obtain central M/L. Illingworth (1976) derived the M/L for selected clusters based on central velocity·dispersion
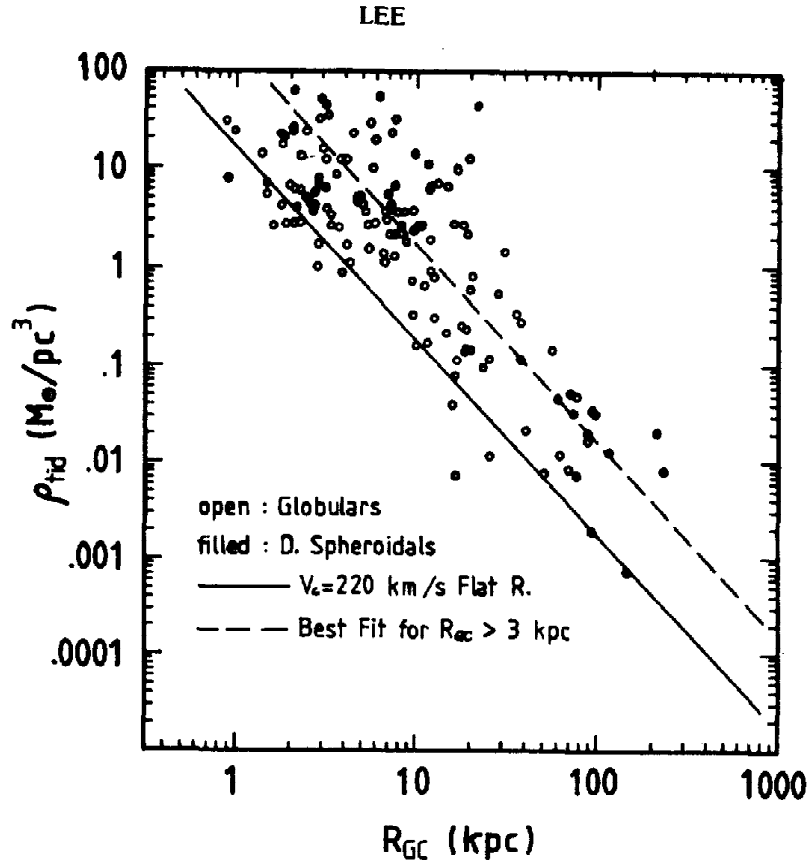
Fig. 1. Tidal density versus Galactocentric distance for the entire sample. The solid line is for a SIS potential with V_{cir} = 220 kms^{-1} and the dotted line is the least square fit to the data with R_G > 3 kpc.

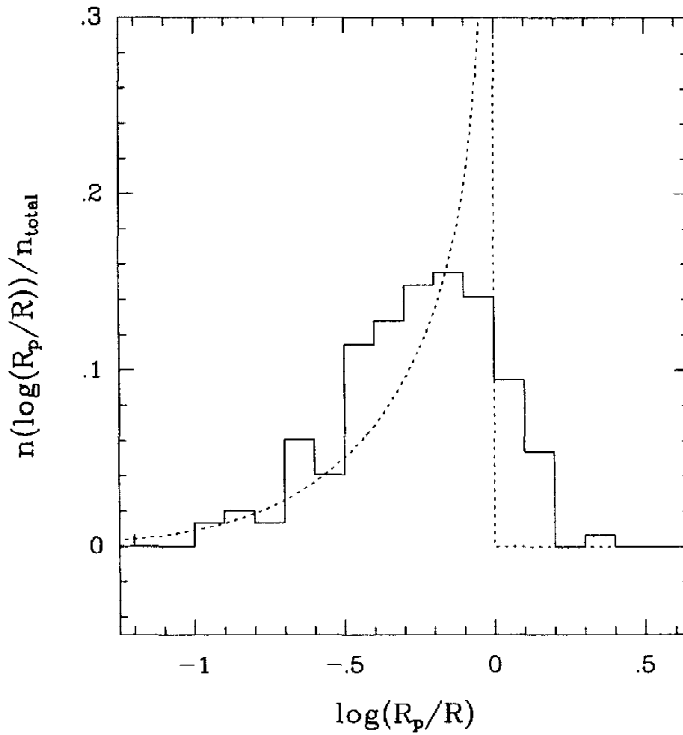**Fig. 2.** Distribution of R_p/R_G for the entire sample with R_G > 3 kpc. The dotted line is the theoretical distribution for a SIS potential assuming isotropic velocity distribution.

measurements and fitting the data to King models. Various other analyses showed that M/L of globular clusters have very small variations from a cluster to another with the midian value being about 1.7. Most photometric, spectroscopic and star count data set for globular clusters can be equally well fitted to multi-component King models, but the resulting M/L_V still lie between 1.5 and 3 (Pryor, McClure, Fletcher and Hesser 1988).

On the other hand, Richer and Fahlman (1989) found that there may be a large number of low mass stars ($m < 0.1M_\odot$) in a globular cluster M71 based on the extrapolation of mass function derived from deep CCD data. If this is a common phenomenon in globular clusters, the M/L is very poorly determined because of very little dynamical effects that these stars produce to the clusters (especially to the central parts where most of detailed observations are targeted), but M/L is still expected to be smaller than 3.

In summary, the multi-mass models tend to produce larger values for M/L compared to single-component ones, but more than a factor of 3 increase from the conventional value appears to be rather difficult to achieve. Thus we conclude that the varying M/L alone would not be sufficient to make the observations consistent with theory.

3. Distribution of R_p/R_G

One can obtain unbiased distribution of R_p/R_G using equation (4) from the observed data. Such distribution could be used to probe the Galactic potential if the velocity distribution of the clusters with respect to the Galaxy is known.

We have shown the frequency distribution of R_p/R_G as a histogram for the entire sample (excluding systems with $R_G < 3kpc$). The dotted line is a theoretical frequency distribution assuming isotropic velocity distribution in an SIS potential. Both curves are normalized such that the total area becomes unity.

It is clear that the observed result is consistent with the theoretical expectation for an SIS potential except near $R_p/R_G = 1$. The broadening near $R_p/R_G = 1$ can be attributed to the observational errors. The estimated error of $\epsilon (\ln r_t) \approx 0.1$ appears to be suffient to make such broadening. This is a significant impovement from IHW who found that their data deviate appreciably from the theoretical curve in the sense that there are too many clusters whose $R_p/R_G > 1$. Again such an improvement is achieved by the reduction of tidal radius.

The similarity between the observed data and theoretical curve does not necessarily mean that the galactic potential is that of an SIS and the velocity distribution is isotropic.

The extent of galactic halo with r^{-2} density distribution is not well established. While the timing argument for our Galaxy and M31 indicates very large halo (larger than about 100 kpc, e.g., Binney and Tremaine 1987), the kinematical data for globular clusters and satellite systems imply smaller halo (about 40 kpc; e. g., Little and Tremaine 1987). Our sample contains systems whose R_G reach up to more than 100 kpc, but the Galactic mass distribution at that distance becomes very uncertain because the number of systems decrease rapidly with distances. In addition, one may expect that the velocity distribution could change at large distances. If the orbits of clusters are predominantly radial at large distances while those in the inner parts is isotropic, the observed

distribution of R_p/R_G could still be consistent with a much smaller halo model(i. e., $R_{Halo} <$ 40kpc).

The kinematical data may contain information regarding the velocity distribution of clusters. If the ambiguity in velocity distribution is eliminated, the Galactic mass distribution can be much better constrained. This subject, which is beyond the scope of this paper, will be treated in the forthcoming papers.

IV. SUMMARY AND CONCLUSION

Our analysis of tidal densities of entire globular clusters and dwarf spheroidals implies that either current esimates for the M/L_V are too small by a factor of more than 3 or the tidal radii are overestimated by about a factor of 1.5 but significant increase of M/L from conventional value of 1.7 is very unlikely. Systematic reduction of tidal radii has been suggested by Lee and Ostriker (1986) based on the theortical investigation of dynamical evolution of globular clusters in a steady galactic field.

By changing the definition. of the tidal radii described above, the observed distribution of $R_p/$ R_G as well as ρ_{tid} versus R_G relationship become more consistent with the theoretical expectations for clusters if the Galactic potential is close to that of an SIS and the velocity distribution of clusters with respect to the Galactic rest frame. This does not necessarily mean that the Galactic mass distribution is similar to SIS since the the data may still be consistent with the smaller halo model (Little and Tremaine 1987), if the orbits become more radial as the Galactocentric distance increases. Thus it would be useful to incorporate with the kinematic data in order to eliminate ambiguities in velocity distribution of clusters.

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