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Travel Time Modeling and Analysis for an Automated Work-in-process Carousel†

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Abstract

This paper deals with an automated storage carousel which handles work-in-process(WIP) such as small parts for kitting. The system has been used predominately for order picking applications. Throughput performance of the system can be measured by the inverse of the expected order picking time. Analytic models are developed for approximating the expected times under the "nearest-item" sequencing rule. The performance of the models are tested through computer simulation. The gap between the two is shown to be reasonably small.

1. Intorduction

One obvious advantage of storage carousels compared with conventional automated srorage/retrieval(S/R) systems is that they rotate the desired bin to the operator and thus save a considerable amount of time in traveling to search for required items. The carousel system has been frequently used for order picking where individual items on an order are sequentially picked by order pickers.

This paper considers such a horizontal carousel equipped with a S/R machine as shown in Figure 1.

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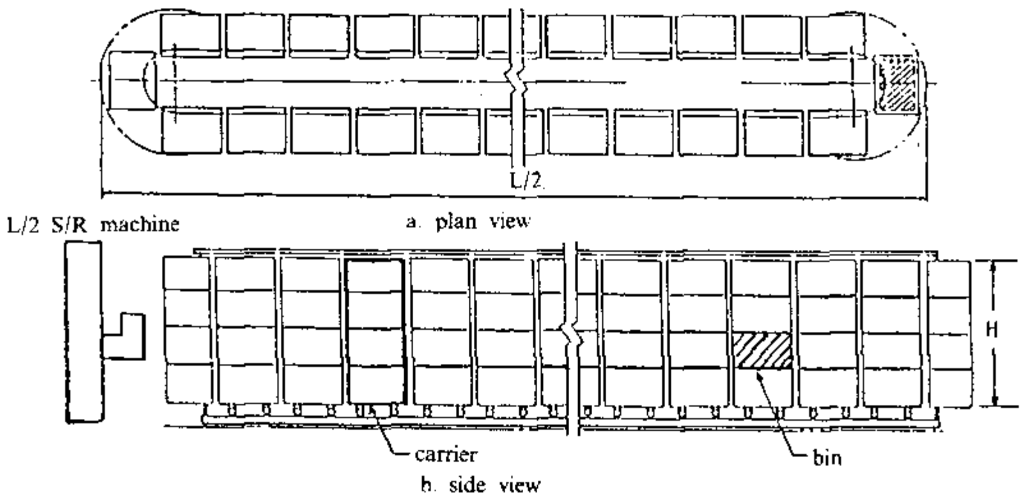

Figure 1. Schematic diagram of the carousel system.

In order to design the system, evaluation of system performance is an essential step. In general, the performance is measured by the throughput, the inverse of an expected time to pick an order. However, very little analytic work has been done on throughput evaluation of the carousel system. Mardix and Sharp[6] and Stern[7] studied that subject, but the systems considered were those operated by human order pickers.

To reduce the order picking time, items on an order have to be cleverly sequenced. The problem of optimally sequencing a given list of items is certainly the well-known traveling salesman problem which is NP-complete[3]. Since item sequencing must be done very frequently in real-world situations, and quite often, the sequences must be determined by a small computer, solution procedures have to be fast. In this regard, heuristics for the problem appear to be appropriate.

Bartholdi and Platzman[1] presented the "nearest-item" heuristic where starting from the input/output(I/O) point the nearest item is to be picked successively. For a unit load carousel, Han and McGinnis[4] also suggested the same heuristic to sequence retrieval and storage orders.

In this paper, based on the nearest-item heuristic, analytic expressions are derived for approximating the expected order picking time in the storage carousel served by a single S/R machine. The results obtained from the analytic models are compared with those from Monte Carlo simulation.

2. Assumptions

The followings are assumed throughout this paper.

1) A bidirectional carousel is served individually by a single S/R machine.
2) The conveyor length and height, its rotating speed, and the vertical speed of the S/R machine are known.
3) The I/O point is located at the bottom front corner of the carousel.

4) The S/R machine can move in a vertical direction while the casousel rotates(Chebyshev travel).
5) Each bin stores only one item type and is equal in size.
6) A randomized storage assignment rule is used.
7) Only one order can be picked at a time without utilizing information on the next order.

3. Evaluation of Order Picking Time

In order picking, an order consists of a number of items, all of which have to be retrieved before the next order occurs.

The total picking time of an order can be expressed as the sum of three components : (1) the travel time during which the carousel and the S/R machine are traveling, (2) the time during which both the carousel and the S/R machine is stopped for picking, and (3) the time to pickup an empty container at the I/O point and the time to deposit the loaded container. Due to the Chebyshev travel, element (1), the travel time, is determined by the maximum of the carousel rotation and the S/R machine movement times. For convenience, element (2) in the above is ignored in this study. A simple correction should be to add an appropriate constant which is a linear function of the number of items to the total time. Element (3), the pickup and the deposit time can be assumed to be constant for all cycles, each taking the same E time units.

To facilitate the analysis, we consider a continuous approixmation to the storage rack of the carousel, and divide the rack into two equal-sized surfaces at the I/O point. The result is a normalized surface as shown in Figure 2 where a cycle of picking an order consisting of 4 retrieval items is represented.

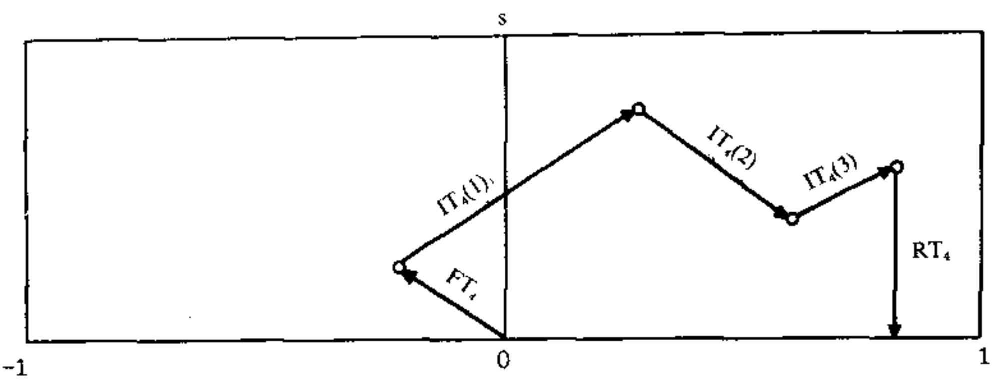

Figure 2. Representation of a cycle of order picking on a normalized surface.

Assuming $L/2v_x \geqq H/v_y$, the shape factor in the Figure becomes :

$s=(H/v_y)/(L/2v_x)$

where L=carousel rack length :

H=carousel rack height;

v_x=carousel rotating velocity; and

v_y=S/R machine movement velocity in the vertical direction.

Note that due to the normalization of the rack surface, all the time variables being addressed hereafter have no unit.

We use the following notation:

n =the number of items required on an order (i.e. order size);

e =normalized pickup(or deposit) time = $E/(L/2v_x)$;

FT_n =time to retrieve the first item following the nearest-item heuristic;

$IT_n(i)$ =i-th interleaving time between two consecutive retrieval points (see Figure 2), i= 1, ⋯, n−1;

IT_n =total interleaving time to perform an order of size n $=\sum_{i=1}^{n-1} IT_n(i)$;

RT_n =returning time which is the sum of the travel time to return to the I/O point after picking the last item and the time to deposit the loaded container; and

T_n =order picking time of an order of size n

$$= \begin{cases} FT_n+RT_n & \text{if } n=1, \\ FT_n+\sum_{i=1}^{n-1} IT_n(i)+RT_n & \text{if } n\geqq 2. \end{cases}$$

E(X) =expected value of a random variable X.

3-1 Single-Command Order Picking Time

For the simplest case, we will first evaluate the expected time of single-item order picking based on a statistical approach.

Recall that intermittent job operation is assumed, i.e. each order is assumed to be independently processed on a first-come-first-serve rule. In this case, the carousel conveyor can start to rotate as soon as the S/R machine begins to pickup an empty container, and continue to rotate while the machine travels from the I/O point to the first item location. Therefore, if we let the time required to pickup a container at the I/O point and move to a random point, (X, Y), be U, then the cummulative distribution function(cdf) of U becomes :

$$F(u)=Pr(U\leqq u)=Pr(|X|\leqq u)\ Pr(Y+e\leqq u).$$

Since for randomized storage the coordinate locations are assumed to be uniformly distributed,

$$Pr(|X|\leqq u)= \begin{cases} u & \text{for } 0\leqq u\leqq 1 \\ 1 & \text{for } u>1 \end{cases}$$

and

$$Pr(Y+e\leqq u)= \begin{cases} (u-e)/s & \text{for } e\leqq u\leqq s+e \\ 1 & \text{for } u>s+e \end{cases}$$

Thus, the cdf, F(u), and the probability density function(pdf), f(u), can be obtained as shown in Table 1. Then, the expected value of FT_1 is easily computed as

$$E(FT_1)=E(U)=\begin{cases} s^2/6+es/2+(1+e^2)/2 & 0\leqq e\leqq 1-s \\ s/2+e+(1-e)^3/6s & 1-s<e\leqq 1 \\ s/2+e & e>1 \end{cases}$$

Note that the results derived above are equivalent to those made by Lee and Hwang[5].

Next, since the returning time is equal to the return travel time plus the deposit time, the expected returning time is given by

$$E(RT_1)=s/2+e.$$

Tabele 1. pdf and cdf of U

Range of e	Range of u	F(u)	f(u)
$0\leq e\leq 1-s$	$e\leq u\leq s+e$	$(u-e)u/s$	$(2u\text{-}e)/s$
	$s+e<u\leq 1$	u	1
	$u>1$	1	–
$1-s<e\leq 1$	$e\leq u\leq 1$	$(u-e)u/s$	$(2u-e)/s$
	$1<u\leq s+e$	$(u-e)/s$	$1/s$
	$u>s+e$	1	–
$e>1$	$e\leq u\leq s+e$	$(u-e)/s$	$1/s$
	$u>s+e$	1	–

Finally, by the definition the expected single-command order picking time will be

$$E(T_1)=(FT_1)+E(RT_1)$$
$$=\begin{cases} s^2/6+(1+e)s/2+(1+e)^2/2 & 0\leqq e\leqq 1-s \\ s+2e+(1-e)^3/6s & 1-s<e\leqq 1 \\ s+2e & e>1 \end{cases} \qquad (1)$$

3-2 Dual-Command Order Picking Time

In this case, since two items are picked together by a trip of the S/R machine, a single interleaving between the two points occurs. Notice that due to the cylindrical form of the carousel rack, the interleaving time will be the shorter of the clockwise and the counterclockwise interleaving time.

Han and McGinnis[4] showed that given a sample of k random interleaving times, the smallest of them is a random variable, Z_k with pdf :

$$h(z_k)=\begin{cases} k(1-2z_k^2/s+z_k^3/s^2)^{k-1}(4z_k/s-3z_k^2/s^2) \text{ for } 0<z_k\leqq s \\ k(1-z_k)^{k-1} \text{ for } s<z_k\leqq 1 \end{cases}$$

And they evaluated the expected value of Z_k, $E(Z_k)$, numerically for different combinations of k and s.

The expected interleaving time under dual command, $E(IT_2)$, will then be

$$E(IT_2)=E(Z_1)=1/2+s^2/12 \tag{2}$$

Next, consider the expected first-item picking time, $E(FT_2)$, under the heuristic.

Given a sample of n random points the pdf of the smallest travel time from the I/O point, U_n, is obtained by using the results shown in Table 1 :

$$g(u_n)=n(1-F(u_n))^{n-1} f(u_n) \qquad u_n \geqq e$$

Then, the expected time of U_n can be obtained by

$$\begin{aligned} E(U_n) &= \int u_n g(u_n) du_n \\ &= e+(1-s-e)^{n+1}/(n+1)+\sum_{i=1}^{n+1} a_1(i) && 0 \leqq e \leqq 1-s \\ &\quad e+(s+e-1)^{n+1}/(n+1)s^n+\sum_{i=1}^{n+1} a_2(i) && 1-s < e \leqq 1 \\ &\quad e+s/(n+1) && e > 1 \end{aligned}$$

where $a_1(i)=c_i[(e^2/4s+s+e)^{(2i-1)/2}(1-s-e)^{n-i+1}-(e^2/4s)^{(2i-1)/2}]$;

$a_2(i)=c_i[(e^2/4s+(1-e)/s)^{(2i-1)/2}(1-(1-e)/s)^{n-i+1}-(e^2/4s)^{(2i-1)/2}]$; and

$c_i=[(\pi s)^{1/2}/2\Gamma(i+1/2)]\,[n!/(n-i+1)!]$

Now, the expected first-item picking time for an order of size n can be computed from the above result

$$E(FT_n)=E(U_n) \tag{3}$$

and thus, the expected time under dual command,

$$E(FT_2)=E(U_2) \tag{4}$$

Finally in the following, the expected returning time to the I/O point will be found. If we do not consider sequencing of items and pick them in a random manner, the expected returning time will be equal to $s/2+e$. Under the nearest-item heuristic, however, the result is not valid due to the fact that the second item to be retrieved tends to be located farther from the I/O point than the first.

Let W be a random variable to represent the return travel time to the I/O point, that is

$$W=RT_2-e \tag{5}$$

To derive the pdf of W, $r(w)$, we first consider the conditional distribution function of W,

$$R(w \mid u)=Pr(W \leqq w \mid U_2=u),$$

given the first item picking time, $U_2=u$. Using the result, the cdf, $R(w)$, and the pdf of W can be derived sequentially. The expected returning time, $E(RT_2)$, of interest will then be obtained from(5) :

$$E(RT_2)=E(W)+e \tag{6}$$

Three cases arise according to the value of e as in $f(u)$. To illustrate the procedure, consider the case of $0 \leqq e \leqq 1-s$.

For the randomized storage assignment rule, two retrieval points are assumed to be distributed uniformly on the rack surface. Also, when the location of the first item is specified, the second have to be placed outside the area which is confined by the first item and the I/O point. Hence,

$$R(w \mid u)=\begin{cases}(w-u(u-e))/(s-u(u-e)) & e \leqq u \leqq w+e \\ w(1-u)/(s-u(u-e)) & w+e<u \leqq s+e \\ w/s & s+e<u \leqq 1 \\ 1 & w>s\end{cases}$$

By the definition, the cdf becomes

$$\begin{aligned} R(w) &= \int R(w \mid u) f(u) du \\ &= \begin{cases}((3s-(s+e)^3)/3s^2)w+w(w+e)^3/3s^2 & 0 \leqq w \leqq s \\ 1 & w>s\end{cases}\end{aligned}$$

And the pdf will be

$$\begin{aligned} r(w) &= dR(w)/dw \\ &= (3s-(s+e)^3)/3s^2+(4w+e)(w+e)^2/3s^2 \text{ for } e \leqq w \leqq s\end{aligned}$$

Thus,

$$E(W)=\int w\ r(w)dw = s/2+s^3/10+es^2/4+e^2s/6$$

Finally from (6), we have the result

$$E(RT_2)=s/2+e+s^3/10+es^2/4+e^2s/6 \qquad 0 \leqq e \leqq 1-s \tag{7}$$

In the same way, the expected returning time can be obtained for both the cases of $1-s<e \leqq 1$ and $e>1$ as follows:

$$E(RT_2)=\begin{cases}2s/3+e-(4-15e+20e^2-10e^3+e^5)/60s^2 & \text{for } 1-s<e \leqq 1 \\ 2s/3+e & \text{for } e>1\end{cases} \tag{8}$$

Note that the expected order picking time under dual command is represented as

$$E(T_2)=E(FT_2)+E(IT_2)+E(RT_2) \tag{9}$$

Consequently, substituting the expressions (2), (4), (7) and (8) into (9) we have

$$E(T_2)=\begin{cases}s^2/4+(1+e)s/2+e^2/2+e+1 & \text{for } 0 \leqq e \leqq 1-s \\ s^2/12+s+2e+.5+(1-e)^3/6s & \text{for } 1-s<e \leqq 1 \\ s^2/12+s+2e+.5 & \text{for } e>1\end{cases} \tag{10}$$

3-3 Multi-Command Order Picking Time

Multi-command order picking involves a number of interleaving travels of the S/R machine. Analysis of the expected order picking time can be done by the extension of the previous dual-command analysis.

Since the number of interleaving operations is equal to $n-1$ when an order size is n, the expected total interleaving time is given by

$$E(IT_n)=\sum_{i=1}^{n-1} E(IT_n(i)) \tag{11}$$

The numerical evaluation of $E(Z_k)$ will be used to approximate the expected value of $IT_n(i)$, $i=1, \cdots, n-1$ via the following argument.

If we have just picked the first item that is located closest to the I/O point, the opportunity for choosing the second is limited to the remaining $n-1$ retrieval items. Thus, the corresponding expected value of $IT_n(1)$ can be approximated as $E(Z_{n-1})$. On choosing the next item to be picked, the opportunity is reduced by one, so the corresponding expected value of $IT_n(2)$ is $E(Z_{n-2})$. Continuing this process, the last interleaving will be made between locations of the remaining two items, and therefore, $E(Z_1)$ approximates the expected value of $IT_n(1)$.

Consequently, from expression(11) $E(IT_n)$ can be approximated as:

$$E(IT_n)=\sum_{i=1}^{n-1} E(Z_{n-i}) \tag{12}$$

Under the multi-command, estimating the expected returning time in a closed form expression is extremely difficult. Therefore, instead of deriving an exact value a very simple approximation curve is presented as follows:

It can be shown that the expected returning time under dual command is much longer than that under random sequencing. For instance, when $0 \leqq e \leqq 1-s$, the amount of excessive travel time, ERT, is given by

$$\begin{aligned} ERT &= E(RT_2)-(s/2+e) \\ &= s^3/10+es^2/4+se^2/6 \end{aligned}$$

However, the excessive time would probably approach to zero if the order size, n, is sufficiently large. Observing from expirical results that if $n \geqq 40$, ERT approaches to zero, we will approximate the expected returning time, $E(RT_n)$, for the multi-command case as a quadratic function of n:

$$E(RT_n)= \begin{cases} k_1(n-2)^2+k_2(n-2)+k_3 & \text{if } 2 \leqq n \leqq 40 \\ s/2+e & \text{if } n>40 \end{cases} \tag{13}$$

where $k_1=ERT/1444$, $K_2=-ERT/19$, and $k_3=E(RT_2)$

Figure 3 shows an approximation of $E(RT_n)$ where the 99% confidence interval based on the corresponding simulation results is depicted together. For the simulation, s and e were set equal to 1.0 and 0.2, respectively, and the sample size was 1000. Observe that the estimates from the approximation function are almost in the middle of the interval.

It follows that the expected picking time, $E(T_n)$, of an order of size n can be approximated by substituting (3), (12) and (13) into the following expression:

$$E(T_n)=E(FT_n)+E(IT_n)+E(RT_n).$$

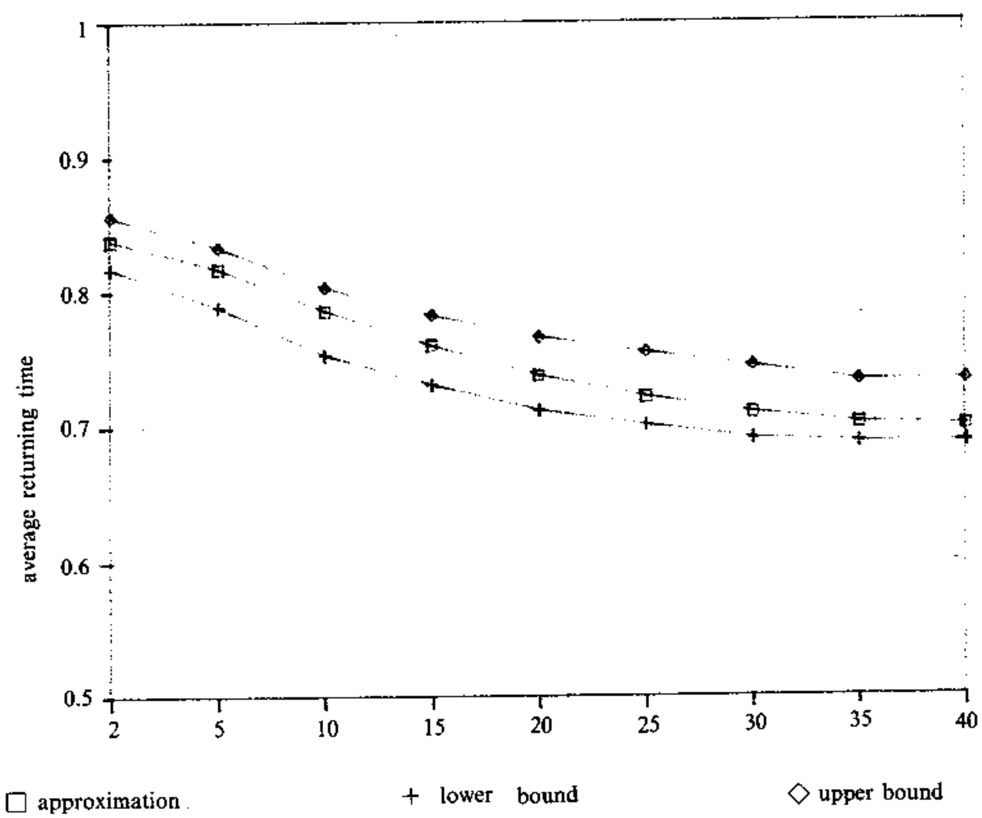

Figure 3. An approximation of $E(RT_n)$.

4. Comparison with Monte Carlo Simulation

To assess the accuracy of the analytic models developed in this study, the nearest-item heuristic was implemented and tested on an IBM PC/AT using Monte Carlo sampling. In the sampling procedure, item points were randomly chosen in a normalized surface. For each replication, n retrieval points included in an order were generated. Then, the heuristic was applied until n retrievals were made. For each combination of n, s, and e, a total of 1000 replications were taken.

The values of $E(T_n)$ from the expressions derived above are plotted in Figure 4 and 5 together with the results of simulation. In Figure 4, the effect of variation in the values of s is shown when e=0.2. Figure 5 shows the behavior of the models according to the variation in e, given s=1.0.

From the figures, it is observed that the performance of the analytic models displays a satisfactory result with the largest deviation being 1.06%. And their performance is shown to be relatively insensitive to shape factor and pickup(or deposit) time.

Note that the models appear to overestimate the order picking time as the order size increases. However, from the various computational results obtained during the study it is shown that the percent deviation does not seriously grow with the order, or rather, it remains almost constant in some range.

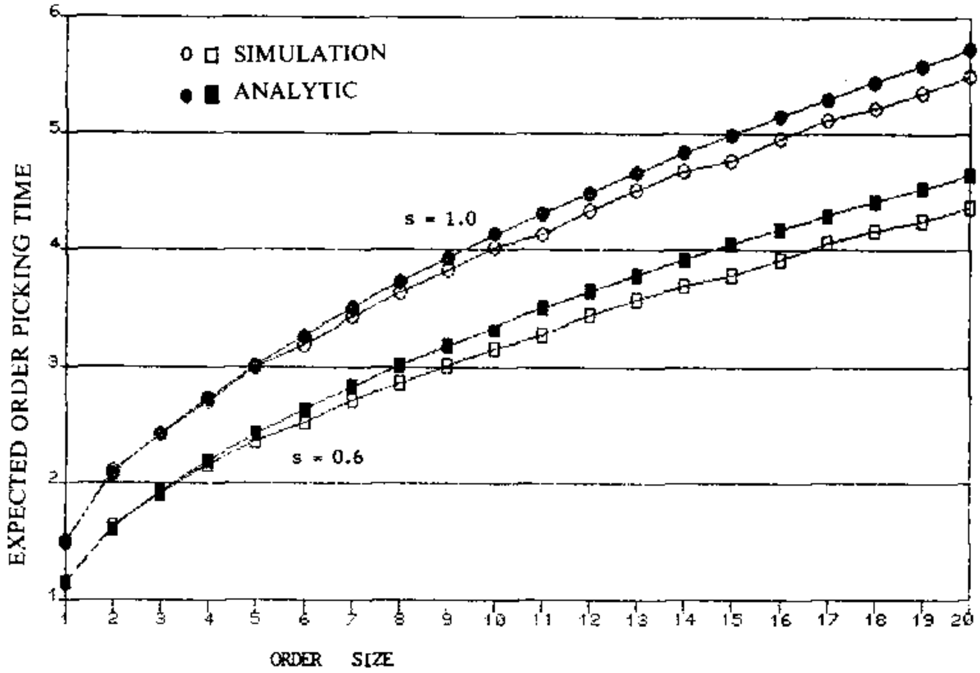

Figure 4. Effect of shape factor on the performance of the analytic model(e=0.2).

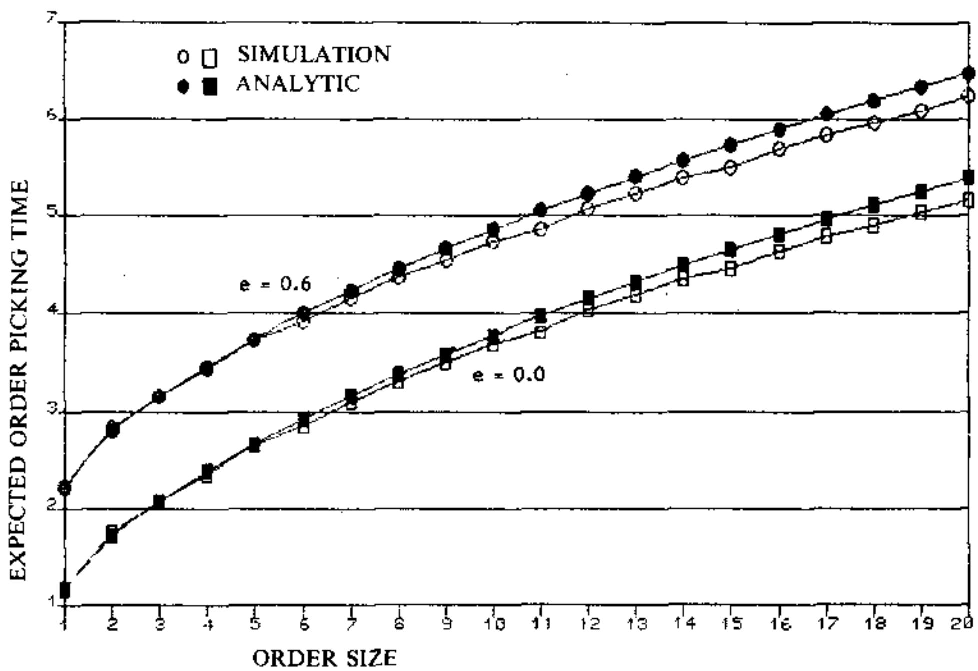

Figure 5. Effect of pickup(or deposit) time on the performance of the analytic model(s=1.0).

5. Conclusions

In this paper, an approach has been presented for evaluating the throughput performance of a carousel storage system. The approach is in most parts based on a statistical approach. To sequence items in a multi-command order, the "nearest-item" heuristic is employed. In real-life situations, the heuristic may be fairly well used to avoid the computational burden in finding an optimal routing sequence of the S/R machine.

It is of worth to note that the total expected time expressions for both single—and dual—command orders are completely equivalent to those under the random sequencing rule.

Although the analytic expressions are derived based on an approximation scheme in some parts, they may provide useful information in first-cut evaluation of order picking performance

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