

Density-Order Index Rule for Stock Location in a Distribution Warehouse[†]

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Abstract

This paper deals with the problem of space allocation of items within a warehouse. Recognizing the importance of weights associated with material handling, mathematical models are developed for two cases, out-and-back selection and storage retrieval interleaving. It is proved that the density order index rule we proposed generates an optimal solution for the first model. An example problem solved with the pairwise interchange method indicates that the rule is also fairly efficient for the second model. The proposed rule is compared with other assignment rules of warehouse space such as COI rule, space and popularity.

1. INTRODUCTION

Material handling is major activities in warehouse operations and represents between 15 and 70 percent of the total cost of a manufactured product. Depending on the type of warehouse, it is reported that 30 to 40 percent of warehouse labor costs are incurred by the picking operation [3]. Therefore, the decision on stock locations in a distribution warehouse is an important concern in terms of the material handling cost and the work-load of the order picker.

Several papers addressed the stock location problem in a warehouse with the objective of assigning items to storage locations to minimize materials handling cost. Heskett [5] proposed a criterion, which he called the Cube per Order Index(COI) rule. The COI for an item is simply the quotient of the space requirement and the order frequency for the item. Under the COI rule, items are assigned to locations in increasing order of COI such that those with lower COI are placed closer to the shipping area. Harmatuck [4] showed the

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optimality of the COI rule for at least one linear programming formulation of the problem. Kallina and Lynn [6] summarized the overall problems related to stock location and reported the experience with the application of the COI rule.

All studies mentioned did not consider the effect of the weights on material handling costs and assumed that the cost of moving an order of any item from a particular location to the shipping area depends only on the distance, regardless of the item type handled. However, in real situations, the weights of items, and order picking equipment such as cart, could be a critical factor in material handling costs. From the view point of human safety, the importance of weights becomes more prominent when the handling operations are carried out manually.

Recently, we proposed a criterion, called the density order index (DOI) rule [6] which integrates the weight into the COI rule. This paper intends to briefly introduce the concept of the DOI rule and investigate the efficiency when storage and retrieval requests are interleaved. The rest of the paper is organized as follows. First, a linear programming model is presented in which the work load, i.e., [distance] \times [weights of items and order picking equipment] is minimized. Second, the DOI is shown to generate an optimal solution to the model. And then, a quadratic programming model is developed where storage and retrieval transactions are interleaved. Finally, through an example problem, we evaluate the efficiency of the DOI rule by comparing the total cost of the initial solution obtained by the DOI rule with those by the pairwise interchange method.

2. LINEAR PROGRAMMING MODEL FOR SIMPLE OUT-AND-BACK

The LP model is developed with the following assumptions.

- (1) An order consists of a quantity of a single item which is retrieved by means of simple "out-and-back" selection procedure.
- (2) The space required for storing n items may be allocated among m locations in any manner.
- (3) The cost of moving an order is proportional to the workload imposed, i.e., [distance traveled] \times [weight] ^{α} , where α is a nonnegative constant.
- (4) The work-load required to pick an order at location is constant and thus can be ignored.
- (5) There is a limit on volume but not on weight in each storage location.
- (6) Replenishment of stock within the storage area occurs separately from order picking.

The following notations are used throughout this paper.

α = constant which represents the effects of weight on the materials handling cost, $\alpha \geq 0$.

c = the estimated cost per [distance traveled] \times [weight]

m = the number of storage locations in the warehouse

n = the number of items to be stored in the warehouse

WT = the weight of order picker (weight)

S_i = the average order size for item i (units)

F_i = the number of orders per period for item i (time⁻¹)

DD_i = the number of demand periods for item i for which space must be allocated (time)

CU_j = the space required to store one unit of item i (length³/unit)

WT_i = the weight of one unit of item i (weight/unit)

CAP_k = the capacity of storage location k (length³)

D_{ku} = the distance from storage location k to storage location u (length)

o denotes the shipping area and the locations are ordered so that

$$D_{ok} \leq D_{o, k+1} \quad k, u = 1, 2, \dots, m-1$$

X_{ki} = decision variable which denotes the number of units of item i assigned to storage location K . (units in real number)

The objective of the LP model is to minimize order retrieving costs per period. The average number of orders of item i filled from location k per period is

$$\begin{aligned} & \{\text{number of orders of item } i \text{ per period}\} \\ & \times \{\text{fraction of item } i \text{ stored in location } k\} \end{aligned}$$

$$\begin{aligned} & = F_i X_{ki} / (S_i DD_i F_i) \\ & = \frac{X_{ki}}{S_i DD_i} \end{aligned} \quad (1)$$

As already stated, we assume simple "out-and-back" selection procedure [7] and each order for an item can be picked by a single trip. In case if an order is expected to be too large in size to be handled by one pick, we can replace the order estimate for the item with an estimate of the total number of picks for the item. Thus, equation (1) represents the average number of visits to location k for the retrieval of item i per period.

The expected cost of a round trip per period for the retrieval of item i from location k can be decomposed into two elements. One is the cost of movement of the order picking equipment from the shipping area to location k for access to item i and can be expressed as

$$c \cdot D_{ok} (WT)^\alpha \frac{X_{ki}}{S_i DD_i} \quad (2)$$

The other is the cost of movement of the items picked and the equipment from location k to the shipping area for the retrieval of item i and can be expressed as

$$c \cdot D_{ok} (WT + S_i WT_i)^\alpha \frac{X_{ki}}{S_i DD_i} \quad (3)$$

With equations (2) and (3), the problem can be formulated as an LP model as the followings.

$$\text{Min. } TC_s = \sum_{k=1}^m \sum_{i=1}^n \frac{c \cdot D_{ok} \{ (WT)^\alpha + (WT + S_i WT_i)^\alpha \}}{S_i DD_i} X_{ki} \quad (4)$$

$$\text{s.t. } \sum_{i=1}^n CU_i X_{ki} \leq CAP_k \quad k = 1, 2, \dots, m \quad (5)$$

$$\sum_{k=1}^m X_{ki} = S_i DD_i F_i \quad i = 1, 2, \dots, n \quad (6)$$

$$X_{ki} \geq 0 \quad k = 1, 2, \dots, m; i = 1, 2, \dots, n$$

Constraint (5) represents the capacity restriction on location k and (6) assures the maximum inventory level for item i . Harmatuck [4] expressed the total expected handling cost per period as

$$TC_s = 2 \cdot \sum_{k=1}^m \sum_{i=1}^n \frac{COST_k}{S_i DD_i} X_{ki}$$

, where $COST_k$ is the cost of moving an order of any item from location k to the shipping area.

3. DENSITY ORDER INDEX(DOI) RULE

Define the density, DST_i , for item i as follows

$$DST_i = \frac{(WT)^\alpha + (WT + S_i WT_i)^\alpha}{S_i CU_i}$$

With DST_i , define the Density Order Index(DOI) for each item as

$$\begin{aligned} DOI_i &= \frac{DST_i}{DD_i} \\ &= \frac{(WT)^\alpha + (WT + S_i WT_i)^\alpha}{S_i DD_i CU_i} \quad i=1, 2, \dots, n \end{aligned} \quad (7)$$

Note that the larger either the density (DST_i) or the turnover rate ($1/DD_i$) is, the greater DOI_i becomes. In [4], the cube-per-order index for item i , COI_i was defined as $S_i DD_i CU_i$. Hence, when either $\alpha=0$ or $S_i WT_i$ is same for each item, the DOI Rule in the next paragraph produces the same result as the COI rule does.

〈Density Order Index Rule〉

All items are ranked based on their DOI defined as equation (7) such that the item with the highest index being ranked first. Then, the storage area layout is planned as the followings :

The highest index item goes to location 1, closest to the shipping area, using up as much space required to accommodate the periods demand target of the item. If not enough space is available in location 1, the amount left over goes to location 2. On the other hand, if any empty space remains in location 1, the next highest index item is also placed in location 1 in the appropriate amount up to the capacity of location 1. This process continues until all items have been placed in their proper locations, successively further away from the shipping area.

Briefly speaking, under the DOI rule, items are assigned to storage locations in decreasing order of DOI, those with higher DOI being placed closer to the shipping area.

One of major characteristics of the DOI rule is described in the following theorem.

Theorem 1.

The layout obtained by the DOI Rule is an optimal layout to the LP model represented by equations (4), (5) and (6).

⟨proof⟩

To prove the optimality of the DOI Rule, it is only necessary to show that a violation of the DOI Rule will never lower the cost. Assume that item i is located in location k and item j is located in location u according to the DOI Rule, where $DOI_i \geq DOI_j$ and $D_{ou} \geq D_{ok}$. Then for each cubic foot of item i and item j exchanged between locations k and u , the cost (DTC_s) incurred by the exchange is.

$$\begin{aligned} DTC_s &\approx c \cdot D_{ou} \frac{(WT)^{\alpha} + (WT + S_j WT_j)^{\alpha}}{S_i DD_i CU_i} - c \cdot D_{ou} \frac{(WT)^{\alpha} + (WT + S_j WT_j)^{\alpha}}{S_j DD_j CU_j} \\ &+ c \cdot D_{ok} \frac{(WT)^{\alpha} + (WT + S_j WT_j)^{\alpha}}{S_j DD_j CU_j} - c \cdot D_{ok} \frac{(WT)^{\alpha} + (WT + S_j WT_j)^{\alpha}}{S_i DD_i CU_i} \\ &= c \cdot (D_{ou} - D_{ok})(DOI_i - DOI_j) \geq 0 \end{aligned}$$

Since $DTC_s \geq 0$, a violation of DOI Rule will never make the costs smaller.

[Q.E.D]

The major merit of this rule is the simplicity of application compared with ordinary LP algorithm such as the simplex method.

To illustrate the application of the DOI Rule and the effects of α , consider an example with the data shown in Table 1.

Table 1. The data for an example problem

| Location | D_{ok} | CAP_k | Item | DD_i | S_i | F_i | CU_i | WT_i |
|-------------------------|----------|---------|------|--------|-------|-------|--------|--------|
| 1 | 10 | 300 | 1 | 2.0 | 7 | 25 | 1.0 | 1.0 |
| 2 | 15 | 300 | 2 | 2.0 | 6 | 7 | 1.5 | 2.0 |
| 3 | 20 | 500 | 3 | 4.0 | 5 | 6 | 1.0 | 5.0 |
| 4 | 25 | 500 | 4 | 1.0 | 15 | 2 | 2.5 | 6.0 |
| 5 | 30 | 400 | 5 | 2.0 | 2 | 50 | 3.0 | 1.5 |
| $c = \$1$ and $WT = 10$ | | | 6 | 0.5 | 30 | 4 | 2.0 | 2.0 |
| | | | 7 | 3.0 | 10 | 5 | 1.5 | 10.0 |

[unit of distance is meter, unit of weight, kilogramme and unit of period, week]

Table 2. The ranks according to COI_i and DOI_i

| Item | $\alpha : 0.0$ | | 0.5 | | 1.0 | | 2.0 | | | |
|------|----------------|------|---------|------|---------|------|---------|------|-------|---|
| | COI_i | Rank | DOI_i | Rank | COI_i | Rank | DOI_i | Rank | | |
| 1 | 14.0 | 2 | 0.14 | 2 | 0.52 | 2 | 1.93 | 5 | 27.8 | 6 |
| 2 | 18.0 | 3 | 0.11 | 3 | 0.44 | 4 | 1.78 | 7 | 32.4 | 5 |
| 3 | 20.0 | 4 | 0.10 | 4 | 0.45 | 3 | 2.25 | 4 | 66.3 | 4 |
| 4 | 37.5 | 6 | 0.05 | 6 | 0.35 | 6 | 2.93 | 1 | 269.3 | 2 |
| 5 | 12.0 | 1 | 0.17 | 1 | 0.56 | 1 | 1.92 | 6 | 22.4 | 7 |
| 6 | 30.0 | 5 | 0.07 | 5 | 0.38 | 5 | 2.67 | 2 | 166.7 | 3 |
| 7 | 45.0 | 7 | 0.04 | 7 | 0.30 | 7 | 2.67 | 3 | 271.1 | 1 |

TABLE 2 shows the computed values of DOI_i of equation (7) and the rank of each item under a given α ($\alpha=0.0, 0.5, 1.0$ and 2.0). According to the COI rule, the item with lowest value is ranked first and goes in location 1 which is closest to the shipping area. Note that, with $\alpha=0$, $DOI_i=2/COI_i$ and the DOI rule generates the same rankings as those by the COI rule. It can be observed that the effects of α become substantial as α becomes larger. For instance, item 5 is ranked first according to the COI rule but ranked sixth in the DOI rule with $\alpha=1$. Also, item 4 occupies sixth rank in the case of the COI rule whereas first rank in the DOI rule with $\alpha=1$. These significant effects of α are expected since item 4 is considerably heavier compared to item 5 and thus needs greater work-load in order to be moved by a unit distance. The X_{ki} 's obtained from the COI and DOI rules are listed in Table 3. The additional cost incurred by neglecting the weight effects can be significant. Let X_{DOI} be the solution obtained by the DOI rule and X_{COI} by the COI rule. The additional costs as the results of deviating from the DOI Rule become 74.49% of $TC_s(X_{DOI})$ for $\alpha=2$, 8.16% for $\alpha=1$ and 0.06% for $\alpha=0.5$ (see Table 3). Generally, the extent of the additional cost would increase as α increases though it depends on the data used.

Table 3. Optimal stock locations

| | $\alpha=0$ | $\alpha=0.5$ | $\alpha=1.0$ | $\alpha=2.0$ |
|---|----------------|----------------|---------------|----------------|
| optimal locations : X_{DOI} (others are 0) | $X_{15}=100$ | $X_{15}=100$ | $X_{14}=30$ | $X_{14}=30$ |
| | $X_{25}=100$ | $X_{25}=100$ | $X_{16}=60$ | $X_{17}=150$ |
| | $X_{31}=350$ | $X_{31}=350$ | $X_{17}=70$ | $X_{22}=40$ |
| | $X_{32}=84$ | $X_{32}=20$ | $X_{21}=60$ | $X_{23}=120$ |
| | $X_{33}=24$ | $X_{33}=120$ | $X_{23}=120$ | $X_{26}=60$ |
| | $X_{43}=96$ | $X_{42}=64$ | $X_{27}=80$ | $X_{31}=350$ |
| | $X_{44}=30$ | $X_{44}=30$ | $X_{31}=290$ | $X_{32}=44$ |
| | $X_{46}=60$ | $X_{46}=60$ | $X_{35}=70$ | $X_{35}=28$ |
| | $X_{47}=139.3$ | $X_{47}=139.3$ | $X_{42}=73.3$ | $X_{45}=166.7$ |
| | $X_{57}=10.7$ | $X_{57}=10.7$ | $X_{45}=130$ | $X_{55}=5.3$ |
| | | $X_{52}=10.7$ | | |
| A : $TC_s(X_{DOI})$ | 3371.6 | 13812.4 | 62451.1 | 1826405.0 |
| B : $TC_s(X_{COI})$ | 3371.6 | 13820.9 | 67548.9 | 3186942.0 |
| $\% = 100 \times (B - A) / A$ | (0.00%) | (0.06%) | (8.16%) | (74.49%) |

4. QUADRATIC PROGRAMMING MODEL FOR INTERLEAVING

In this section, a quadratic programming (QP) model is developed to represent the material handling costs in case of dual command operations. That is, "storage retrieval interleaving" is adopted as scheduling policy. The order picker interleaves a pair of storage and retrieval requests in each order picking operation as frequently observed in the tool room operations in a heavy machine manufacturing industries. For instance, the order picker places the jig and fixture back in storage which was taken out of the tool storage room

previously and on his way back picks up a tool requested by some machining department.

The round trip cost of a dual command cycle consists of the cost of travel from the shipping area to the storage location and the expected cost of interleaving and return travel to the shipping area. Given that a storage request of item i occurs at location k , the expected cost of travel to retrieve an item and back to the shipping area is

$$c \cdot \sum_{j=1}^n P(j) \sum_{u=1}^m \{WT^a D_{ku} + (WT + S_j WT_j)^a D_{ou}\} P(u/j) \quad (8)$$

, where $P(j)$ is the probability that retrieval request is for item j and $P(u/j)$ is the conditional probability that item j is retrieved from storage location u . $P(j)$ can be estimated by the ratio of the number of retrieval requests per period for item j , to the total number of retrieval requests for all items in a period. That is,

$$P(j) = F_j / FF \quad j = 1, 2, \dots, n \quad (9)$$

, where $FF = \sum_{j=1}^n F_j$

Similarly, $P(u/j)$ can be defined as the ratio of the number of retrieval requests per period for item j in location u , to the number of retrieval requests per period for item j . That is,

$$P(u/j) = \frac{x_{uj} / (S_j DD_j)}{F_j} = \frac{x_{uj}}{S_j DD_j F_j} \quad u = 1, 2, \dots, m \quad (10)$$

Using these relationships, the total handling cost (TC_d) of dual command operations in a period can be expressed as

$$TC_d = c \cdot \sum_{k=1}^m \sum_{l=1}^n \frac{x_{kl}}{S_l DD_l} [(WT + S_l WT_l)^a D_{ok} + \sum_{j=1}^n P(j) \sum_{u=1}^m \{WT^a D_{ku} + (WT + S_j WT_j)^a D_{ou}\} P(u/j)] \quad (11)$$

Substituting (9) and (10) for $P(j)$ and $P(u/j)$ in equation (11), it reduces to

$$TC_d = c \cdot \sum_{k=1}^m \sum_{l=1}^n \frac{x_{kl}}{S_l DD_l} [(WT + S_l WT_l)^a D_{ok} + \frac{1}{FF} \sum_{j=1}^n \sum_{u=1}^m \frac{x_{uj}}{S_j DD_j} \{WT^a D_{ku} + (WT + S_j WT_j)^a D_{ou}\}] \quad (12)$$

Rearranging equation (12) into linear and quadratic terms yields the following equation of matrices.

$$TC_d = CX + X^t QX$$

, where $X_{(mn \times 1)} = (x_{11}, x_{12}, \dots, x_{mn})^t$

$$C_{(1 \times mn)} = (c_{11}, c_{12}, \dots, c_{mn})$$

$$Q_{(mn \times mn)} = \begin{bmatrix} q_{11,11} & q_{11,12} & \dots & q_{11,mn} \\ q_{12,11} & q_{12,12} & \dots & q_{12,mn} \\ \vdots & \vdots & \ddots & \vdots \\ q_{mn,11} & q_{mn,12} & \dots & q_{mn,mn} \end{bmatrix}$$

$$c_{ki} = c \cdot \frac{(WT + S_i WT_i)^\alpha}{S_i DD_i} D_{ok}$$

$$q_{kl, w} = c \cdot \frac{2 WT^\alpha D_{ku} + (WT + S_i WT_i)^\alpha D_{ok} + (WT + S_j W_j)^\alpha D_{ou}}{2 FF(S_i DD_i)(S_j DD_j)}$$

$k, u = 1, 2, \dots, m; i, j = 1, 2, \dots, n$

Also, constraints (5) and (6) can be rewritten by

$$AX = b$$

, where A is the $[(m+n) \times mn]$ matrix of constraints and b is the $[(m+n) \times 1]$ vector of right hand side.

From the above representations, the QP model under dual command operation is

$$\begin{aligned} \text{Min. } & TC_d = CX + X^t QX \\ \text{s.t. } & AX = b \\ & X \geq 0 \end{aligned} \quad (13)$$

For $\alpha > 0$, the symmetric matrix Q is not necessarily either positive definite or semidefinite. Therefore, it is not an easy task to obtain a gloval optimal solution.

One approach to find a lower cost solution is the application of the pairwise interchange method on some initial solution. Let X^0 be an initial solution or assignment vector with $x_{ki}^0 > 0$, $x_{uw}^0 > 0$, $k < u$ and $i < j$. Also, let ΔX denote the change in X^0 incurred by interchanging a unit volume of item i in k and with the same volume of item j in u. Then ΔX is mn by 1 coulmn vector whose elements are given as the followings :

$$\begin{aligned} & -1/CU_i \text{ for } ki^{\text{th}} \text{ element, } 1/CU_j \text{ for } kj^{\text{th}}, \\ & 1/CU_i \text{ for } ui^{\text{th}}, -1/CU_j \text{ for } uj^{\text{th}} \text{ and } 0 \text{ for all others.} \end{aligned}$$

Suppose i in k and j in u are interchanged with the amounts equivalent to z units of volume. Then the new assignment vector X^1 can be denoted by

$$X^1 = X^0 + z \cdot \Delta X^0 \text{ with } 0 \leq z \leq \min\{x_{ki}^0 CU_i, x_{uw}^0 CU_j\}$$

to maintain the feasibility of X^1 . The total cost change, DTC_d , due to this interchange becomes

$$\begin{aligned} DTC_d &= TC_d(X^1) - TC_d(X^0) \\ &= (CX^1 + X^{1t} QX^1) - (CX^0 + X^{0t} QX^0) \\ &= C(X^0 + z \cdot \Delta X) + (X^0 + z \cdot \Delta X)^t Q(X^0 + z \cdot \Delta X) - CX^0 - X^{0t} QX^0 \\ &= z \cdot C \Delta X + 2 \cdot z \cdot \Delta X^t QX^0 + z^2 \cdot \Delta X^t Q \Delta X \end{aligned}$$

The value of z which minimizes DTC_d may be either $\frac{-C \Delta X - 2 \cdot \Delta X^t QX^0}{2 \cdot \Delta X^t Q \Delta X} > 0$, $\min\{x_{ki}^0 CU_i, x_{uw}^0 CU_j\}$ or 0.

Following the above procedure, we can improve an initial solution obtained by either the DOI rule or the COI rule until the total cost can be no further decreased, i.e., $DTC_d \geq 0$ for all possible interchanges. Even though the pairwise interchange method does not guarantee a least total cost design, it has been frequently used for the development of

heuristic solutions of location problems, i.e., CRAFT in computer-aided layout [2].

To illustrate the pairwise interchange procedure, we solve the QP model with the data of TABLE 1. The results are listed in TABLE 4 and 5 where.

X_{COI-P} = the assignment vector obtained by the pairwise interchange procedure using X_{COI} as the initial solution,

X_{DOI-P} = the assignment vector obtained by the pairwise interchange procedure using X_{DOI} as the initial solution and

N = the number of iterations required to reach X_{COI-P} or X_{DOI-P} from X_{COI} or X_{DOI} , respectively.

Table 4. The number of iterations required

| N | $\alpha=0$ | $\alpha=0.5$ | $\alpha=1.0$ | $\alpha=2.0$ |
|--------------------------|------------|--------------|--------------|--------------|
| X_{DOI} to X_{DOI-P} | 0 | 2 | 3 | 0 |
| X_{COI} to X_{COI-P} | 0 | 3 | 8 | 10 |

Table 5. Stock locations by pairwise interchange procedure

| | $\alpha=0$ | $\alpha=0.5$ | $\alpha=1.0$ | $\alpha=2.0$ |
|---|------------------|------------------|------------------|------------------|
| $X_{DOI-P} = X_{COI-P}$ (others are 0) | $X_{15} = 100$ | $X_{11} = 180$ | $X_{14} = 30$ | $X_{14} = 30$ |
| | $X_{25} = 100$ | $X_{15} = 120$ | $X_{17} = 150$ | $X_{17} = 150$ |
| | $X_{31} = 350$ | $X_{25} = 100$ | $X_{22} = 40$ | $X_{22} = 40$ |
| | $X_{32} = 84$ | $X_{31} = 170$ | $X_{23} = 120$ | $X_{23} = 120$ |
| | $X_{33} = 24$ | $X_{32} = 20$ | $X_{26} = 60$ | $X_{26} = 60$ |
| | $X_{43} = 96$ | $X_{35} = 100$ | $X_{31} = 350$ | $X_{31} = 350$ |
| | $X_{44} = 30$ | $X_{42} = 64$ | $X_{32} = 33.3$ | $X_{32} = 44$ |
| | $X_{46} = 60$ | $X_{44} = 30$ | $X_{35} = 33.3$ | $X_{35} = 28$ |
| | $X_{47} = 139.3$ | $X_{46} = 60$ | $X_{45} = 166.7$ | $X_{45} = 166.7$ |
| | $X_{57} = 10.7$ | $X_{47} = 139.3$ | $X_{52} = 10.7$ | $X_{55} = 5.3$ |
| | $X_{57} = 10.7$ | | | |
| A : $TC_d(X_{DOI-P})$ | 3942.3 | 18676.5 | 88715.9 | 3288089.0 |
| B : $TC_d(X_{DOI})$ | 3942.3 | 18762.2 | 89122.3 | 3288089.0 |
| $\% = 100 \times (B - A) / A$ | (0.00 %) | (0.46 %) | (0.46 %) | (0.00 %) |
| C : $TC_d(X_{COI-P})$ | 3942.3 | 18676.5 | 88715.9 | 3288089.0 |
| D : $TC_d(X_{COI})$ | 3942.3 | 18784.6 | 107089.1 | 6093795.0 |
| $\% = 100 \times (D - C) / C$ | (0.00 %) | (0.58 %) | (20.71 %) | (85.33 %) |

TABLE 4 shows the number of iterations carried out by the pairwise interchange method until it stops improving each initial solution. It requires 10 interchange operations on X_{COI} to obtain X_{COI-P} when $\alpha=2$. It is interesting to find that X_{COI-P} becomes identical with X_{DOI-P} for each α tested. Almost no improvement is made on X_{DOI} in terms of the total cost which indicates the efficiency of the DOI rule. For instance, the pairwise interchange method decreases $TC_d(X_{DOI})$ only by 0.46% for each case of $\alpha=0.5$ and 1 and none for $\alpha=2$. The pairwise interchange method make substantial improvement on the initial layout generated by COI rule, i.e., 85.33% for $\alpha=2$ and 20.71% for $\alpha=1$.

The above observations suggest that the DOI rule is fairly effective for obtaining an assignment vector and the pairwise interchange procedure is an efficient tool for finding a heuristic solution in the QP problem.

6. CONCLUSIONS

The decision on stock location in a warehouse can be very important in terms of the materials handling cost. Human safety should be incorporated into the cost, particularly when the handling operations are carried out manually as it frequently done in small and medium industries in developing countries. In this regards, we propose the DOI rule based on the weight, space requirement and order frequency of item. The DOI rule generated an optimal stock locations in the LP model developed for the Heskett problem in a distribution warehouse. It is shown that the weights of items have significant effects on stock locations. Also, the DOI rule is shown to be cost effective even in dual command operations since the pairwise interchange method achieves a very limited improvement on the initial layout generated by the DOI rule. Thus, in addition to the COI rule, the DOI rule appears another practical solution technique due to its simplicity and easiness to use.

Another question of interest is what is the proper values of α . For this, we applied the simple linear regression to the data developed for fatigue allowance such as those from International Labor Office(ILO) and weight factor included in the objective rating method developed by Mundel [8].

For the data of ILO, we obtain $\alpha=1.72347$ and for the data of Mundel, α varies from 1.26787 to 1.58118 which is increasing as the ratio of the elemental time involved in weight lifting to the cycle time increases. Thus, we suggest that appropriate value of α is 1.5.

REFERENCES

1. Bartholdi, J.J. and Platzman, L.K., "Heuristic based on Spacefilling Curves for Combinatorial Problems in Euclidean Space", *Management Science*, Vol. 34, 1988.
2. Francis, R.L. and White, J.A., *Facility Layout and Location: An Analytical Approach*, Prentice-Hall, Englewood Cliffs, New Jersey, 1974.
3. Gross, G., "Picking Method May Provide Key to Lower Cost Warehouse Plan", *Industrial Engineering*, June, 1981.
4. Harmatuck, D. J., "A Comparison of Two Approaches to Stock Location", *The Logistics and Transportation Review*, Vol. 12, 1976.
5. Heskett, J. L., "Cube-Per-Order Index-A Key to Warehouse Stock Location", *Transportation and Distribution Management*, Vol. 3, 1963.
6. Hwang, H., "A Stock Location Rule in a Distribution Warehouse", paper presented at the International Conference on Comparative Management, Taipei, Taiwan, May, 1988.
7. Kallina, C. and Lynn, J., "Application of the Cube-Per-Order Index Rule for Stock Location in Distribution Warehouse", *Interfaces*, Vol. 7, 1976.
8. Mundel, E. M., *Motion and Time Study: Improving Productivity*, Prentice-Hall, Englewood Cliffs, New Jersey, 1978.