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STRONGLY SEMIPRIME ALTERNATIVE RINGS WITH $xy^2x = yx^2y$

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Let R be an alternative ring, i.e. a nonassociative ring in which $x^2y = x(xy)$ and $yx^2 = (yx)x$ for all x and y in R. R is said to be *strongly semiprime* if $xRx = (0)$ implies $x = 0$. We wish to establish the following

Theorem. *Let R be a strongly semiprime alternative ring. If R satisfies the identity*

$$xy^2x = yx^2y \tag{1}$$

then R is associative and commutative.

R. Awtar in [1] showed that a semiprime associative ring with $xy^2x - yx^2y$ central for all its elements x and y is commutative.

Lemma. *If R is a strongly semiprime alternative ring with (1), then R has no nilpotent element $\neq 0$.*

In the rest of this paper we use freely Artin's theorem and the Moufang identities ([2], pp 35–36).

Proof. Suppose to the contrary that there exist a nilpotent element $a \neq 0$ in R. Let m be the smallest integer ≥ 1 such that $a^{m+1} = 0$ but $a^m \neq 0$. Then $(a^m)^2 = 0$ and $a^m r^2 a^m = 0$ for all r of R by (1). It follows that for all r and s of R

$$(a^m r)(s a^m) + (a^m s)(r a^m) = 0. \tag{2}$$

Since $(a^m r a^m)(a^m s a^m) = a^m\{r[a^m(a^m s a^m)]\} = a^m\{r[(a^m)^2(s a^m)]\} = 0$, we have $(a^m R a^m)^2 = (0)$. Let $a^m r a^m$ be an element of $a^m R a^m$ and set $b = a^m r a^m$. We note $b^2 = 0$ and show $bRb = (0)$. For any ele–

ment s of R, $bsb = (a^m r a^m)[s(a^m r a^m)] = a^m \cdot r\{a^m[s(a^m r a^m)]\} = a^m \cdot r\{(a^m s a^m)(r a^m)\} = a^m \cdot r\{-(a^m r)[(s a^m) a^m]\}$ by (2). Since the last expression is 0, we have $bsb = 0$. This implies $0 = b = a^m r a^m$. Since r is an arbitary element of R, $a^m = 0$ which is a contradiction to the way of choosing m.

Proof of Theorem. First let us show that R is commutative. Let

a and b be two elements of R and S the subring generated by a and b. By Artin's theorem, S is associative. Since S contains no nilponent element $\neq 0$, $vSv = (0), v \in S$ implies $v = 0$. S is now a semiprime associative ring with (1). It follows from the result of R. Awtar [1] that S is commutative. Hence $ab = ba$ and R is commutative. It then follows from Lemma 8 ([2], p 142) that $(a,b,c)^2 = 0$ for any elements a, b, c in R. By Lemma we have $(a,b,c) = 0$ and thus R is associative.

References

[1] R. Awtar, *A remark on the commutativity of certain rings*, Proc. Amer. Math. Soc. 41(1973), 370-372.

[2] K.A. Zhevlakov, A.M. Slin'ko, I.P. Shestakov, A.I. Shirshov, *Rings that are nearly associative*, Academic Press, 1982.

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