점탄성층을 갖는 내다지보형 동흡진기의 최적설계

Optimal Viscoelastic Layered Dynamic Vibration Absorber

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요 약

구속 점탄성 층을 갖는 복합보를 동흡진기로 이용하는 문제를 다루었다. 이 동흡진기의 효과를 조사하기 위해 양단 고정 주진동 보에 설치했을 때의 변위전달율을 구하여 기존 균일 재진보 동흡진기 효과와 비교했다. 해석결과 최적 동흡진기를 구성하는 패러매터 값이 현재 활용되는 재질 측면에서 의미있는 범위내에 존재하고 있고 둘이상의 공진 진동수에서 전달율을 동시에 줄일 수 있었다.

이점으로 기존 제시된 균일보에 비해 점탄성층 복합보가 유리하게 이용될 수 있음을 확인했다.

ABSTRACT

The effectiveness of using a composite beam with constrained visco-elastic layer as a dynamic vibration absorber is investigated. The performance of this absorber is evaluated in terms of displacement transmissibility when applied to a primary beam with built-in ends and compared to that of the uniform beam absorber. The results of analysis and design show that it is possible to suppress simultaneoulsy the peak transmissibilities at two or more resonance frequencies and the optimal parameters are located within the available viscoelastic material properties.

I. INTRODUCTION

The dynamic vibration absorber, which has been developed and used successfully as an effective means for controlling the unwanted vibrations of various mechanical structures and equipments, has been an lasting study object for improving its performance and enlarging the spectrum of application. 1)

The efficiency of a dynamic vibration absorber is assessed in terms of the reduction level of transmissibility of primary vibrating system in the vicinity of resonance frequencies and the width of effective frequency band.

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For the system of multi-degrees of freedom or distributed parameter system, it may be of concern to suppress simultaneously the transmissibility at several resonance frequencies.

Snowdon performed extensive analyses for the effects of the lumped, parameter dynamic absorber, which is composed of mass, spring and viscous damper, to the primary system of beam. 2)

Jacquot designed an optimal dynamic absorber for the 1st mode vibration of a beam using approximation method 3) and the method was extended to the plate and shell system by Warburrton and Ayorinde. 4)

As a distributed parameter absorber, Snowdon considered the problem of implementing circular plate absorber with massive rim, to one degree of freedom and distributed primary system. 5)

And Jacquot presented a cantilever beam absorber attached to one degree of freedom primary system. 6)

Preceding studies considered the absorbers which are effective at only one resonance frequency.

Recently, Snowdon proposed a cruciform absorber that can be effectively tuned simultaneously to two resonance frequencies 7) and Yamaguchi used some viscoelastic materials for absorber beam and connecting block, so rigid body motion was permitted between primary system and absorber beam and so the absorber can control more than two resonance frequencies. 8)

But the absorber is less effective relatively at the 1st resonance frequency which generally is important in most vibration problems.

To synthesize the optimal dynamic absorber, optimal tuning ratio and damping factor must be determined, and the greater mass ratio between absorber and primary system requires the greater damping factor.

Generally, it is difficult to obtain the viscoelstic materials that satisfy completely all the requirements about stiffness and damping, and also the damping stiffness characteristics of viscoelastic materials largely depend on the operation frequency and environmental temperature.

So this study is interested in investigating the usefulness of the viscoelastic cored composite beam as a dynamic vibration absorber, which is used widely for the purpose of solving the vibration problems.

Through theoretic analysis, the effects of favorable dynamic damping characteristics of the viscoelastic-cored composite beam compared to homogeneous beam and the possible benefits over the homogenous beam when used as a dynamic absorber are investigated and discussed.

II. ANALYSIS

The vibrating system of a beam depicted in Fig.1 is considered for analyzing the properties of a composite beam dynamic absorber.

The main beam is clamped and excited by transverse displacement at both ends.

The absorber is attached to the center of the main beam with a rod of relatively stiff material within which no dynamics is assumed.

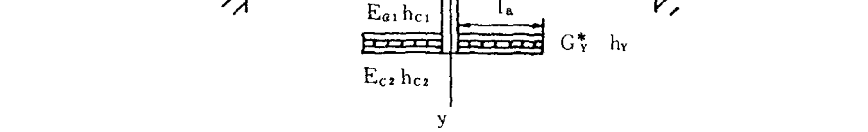

그림 1 Schemetic diagram of the vibrating system

The equation of motion of the main beam, when assuming Bernoully-Euler beam, is written as follows.

$$\frac{\partial^4 y}{\partial x^4}+\frac{m_P}{E_P I_P}\frac{\partial^2 y}{\partial t^2}=0 \tag{1}$$

Assume the solution

$$y(x,t)=Y(x)e^{i\omega t}$$

Then,

$$\frac{\partial^4 Y}{\partial x^4}=n^4 Y \tag{2}$$

where,

$$n^4=\frac{m_P\,\omega^2}{E_P I_P}$$

The solution of eq.(2) becomes

$$Y(x)=C_1\cosh nx+C_2\cos nx+C_3\sinh nx+C_4\sin nx \tag{3}$$

where, C_1, C_2, C_3 and C_4 constants are determined from the boundary conditions of the main beam.

The forced vibration problem can be treated as free vibration one considering the absorbing force F acting on the midpoint of the main beam as a boundary condition.

The main beam boundary conditions is written as

$$\left.\frac{\partial y}{\partial x}\right|_{x=0}=0$$

$$V(x)\Big|_{x=0}=-E_P I_P\left.\frac{\partial^3 y}{\partial x^3}\right|_{x=0}=\frac{F_o}{2}$$

$$y\,|_{x=a}=W_E e^{i\omega t}$$

$$\left.\frac{\partial y}{\partial x}\right|_{x=a}=0$$

where we can get F_o, by analyzing the equation of motion of the composite absorber.

Fig. 2 shows the geometric structure and coordinate system of the composite dynamic absorber.

The equation of motion of the composite beam, negleting the higher order effects like rotary inertia, i.e, assuming Bernoulli-Euler beam, is written as follows

$$\frac{\partial^6 w}{\partial \xi^6}-g(1+R)\frac{\partial^4 w}{\partial \xi^4}=\frac{1}{bD_t}\left(\frac{\partial^2 P}{\partial \xi^2}-gP\right) \tag{4}$$

where b is the width of the composite beam and P is the transverse load including the inertia force and

$$g=\frac{G_v^*}{h_v^2}\left(\frac{1}{E_{c1}h_{c1}}+\frac{1}{E_{c2}h_{c2}}\right)$$

; complex shear parameter

$$R=\frac{d^2}{D_t}\left(\frac{E_{c1}h_{c1}\,E_{c2}h_{c2}}{E_{c1}h_{c2}+E_{c2}h_{c2}}\right)$$

; nondimensional geometric parameter

$$d=\frac{1}{2}(h_{c1}+h_{c2})+h_v$$

; thickness

$$D_t=\frac{1}{12}(E_{c1}h_{c1}^3+E_{c2}h_{c2}^3)$$

; overall bending stiffness of the constraining elastic layer about the neutral axis

$$G_v^*=G_v(1+i\beta)$$

; complex shear modulus

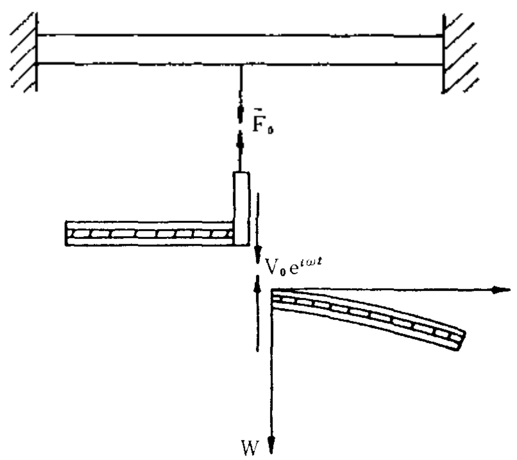

그림 2 Free body diagram

The composite beam is excited by the displacement y(0,t) transmitted directly from the primary system.

Treating this transmitted exciting displacement as a boundary condition, we can consider the problem as a free vibration one.

Putting $P=-m_a\dfrac{\partial^2 w}{\partial t^2}$, we get the following homogeneous equation.

$$\frac{\partial^6 w}{\partial \xi^6} - g(1+R)\frac{\partial^4 w}{\partial \xi^4} + \frac{m_a}{bD_t}\left(\frac{\partial^4 w}{\partial \xi^2 \partial t^2} - g\frac{\partial^2 w}{\partial t^2}\right) = 0 \qquad (5)$$

As both ends of the primary beam are forced by same harmonic excitation displacement, we can assume that the composite beam, which is excited by the transmitted displacement from the main beam, will show the synchronous motion with the primary system.

So we can put

$$w(\xi, t) = W(\xi)e^{i\omega t} \qquad (6)$$

From eq.(5) and eq.(6), we can get

$$\frac{\partial^6 w}{\partial \xi^6} - g(1+R)\frac{\partial^4 w}{\partial \xi^4} - w^2\frac{m_a}{bD_t}\left(\frac{\partial^2 w}{2\xi^2} - g w\right) = 0 \qquad (7)$$

The solution of eq.(7) can be put as

$$W(\xi) = a_1 \sin r_1\xi + a_2 \cos r_1\xi + a_3 e^{-r_2\xi} + a_4 e^{r_2\xi} + a_5 e^{r_3\xi} + a_6 e^{-r_3\xi} \qquad (8)$$

where, γ_i (i = 1, 2, 3) is the root of the following characteristic equation

$$r^6 - g(1+R)r^4 - w^2\left(\frac{m_a}{bD_t}\right)(r^2 - g) = 0 \qquad (9)$$

When we use relatively stiff material as the connecting rod, it is reasonable to consider that midpoint displacement of the main beam is transmitted directly to the absorber.

Then the undetermined constants of eq.(8) is obtained from the following boundary conditions (i=1,...,6)

$$W(0) = Y(0)$$

$$\frac{\partial^2 W(l_a)}{\partial \xi^2} = 0$$

$$\frac{\partial W(0)}{\partial \xi} = 0$$

$$\frac{\partial^4 W(l_a)}{\partial \xi^4} - \left(\frac{m_a\omega^2}{bD_t}\right)W(l_a) = 0$$

$$\frac{\partial^5 W(0)}{\partial \xi^5} - gR\frac{\partial^3 W(0)}{\partial \xi^3} = 0$$

$$\frac{\partial^5 W(l_a)}{\partial \xi^5} - g(1+R)\frac{\partial^3 W(l_a)}{\partial \xi^3} - \left(\frac{m_a W^2}{bD_t}\right)\frac{\partial W(l_a)}{\partial \xi} = 0$$

The shear force of an arbitary point of the composite beam can be get from the following equation. 10)

$$V(\xi, t) = \frac{D^t}{g}\left(-\frac{\partial^5 w}{\partial \xi^5} + g(1+R)\frac{\partial^3 w}{\partial \xi^3} - \frac{m_a\omega^2}{bD_t}\frac{\partial w}{\partial \xi}\right) \qquad (10)$$

From eq.(10), the shear force at the root position of the composite beam is written as follows.

$$V(0, t) = \frac{D_t}{g}\left(-\frac{\partial^5 W}{\partial \xi^5}\bigg|_{\xi=0} + g(1+R)\frac{\partial^3 W}{\partial \xi^3}\bigg|_{\xi=0} - \frac{m_a\omega^2}{bD_t}\frac{\partial W}{\partial \xi}\bigg|_{\xi=0}\right)e^{i\omega t} = V_0 e^{i\omega t} \qquad (11)$$

The vibration absorbing force acting on the midpoint of the main beam becomes

$$F_o = -2V_o e^{i\omega t} \qquad (12)$$

from the free body diagram of Fig. 2

Here we define the displacement transmissibility T between exciting displacement w and center displacement y(0,t) of the main beam as a performance index.

Representing F_0 with C_1 and C_2 coefficients of eq.(3)

$$F_o = k(C_1 + C_2)e^{i\omega t} \qquad (13)$$

we get the transmissibility in terms of k as follows

$$T = \left|\frac{\sinh(nl_P) + \sin(nl_P)}{\cosh(nl_P)\sin(nl_P) + \sinh(nl_P)\cos(nl_P) + k(1 - \cosh(nl_P)\cos(nl_P)/2E_P I_P n^3)}\right| \qquad (14)$$

Because the transmissibility includes implicitly the various interconnected parameters through the preceding many equations, we use the numerical trial and error method to determine the optimal parameters.

III. DESIGN EXAMPLE AND DISCUSSION

The transmissibility given as eq.(14) is governed by the matching conditions of physical and geometric parameters of the main and absorber beam.

As shown in the analysis, many parameters are related interdependently and so it is desirable to use nondimensional parameters for deriving general results.

But the dynamic characteristics of the composite beam absorber unfortunately do not allow it's possibility.

Therefore, we choose and fix the main beam system with the following parameters as an example and design the composite beam dynamic absorber accordingly and discuss the results.

$$m_P = 12.43\ (\mathrm{kg/m})$$
$$h_P = 3.5\times 10^{-3}\ (\mathrm{m})$$
$$E_P = 1.15\times 10^{11}\ (\mathrm{N/m^2})$$
$$b_P = 5\times 10^{-2}\ (\mathrm{m})$$
$$I_P = 1.79\times 10^{-7}\ (\mathrm{m^4})$$
$$l_P = 2\ (\mathrm{m})$$

As the primary system is subjected to symmetric modes, the mode numbers represented depict only the symmetric modes.

First, the effects of the absorber when used for suppressing only the 1st resonance is analyzed and the results is given in Fig.3.

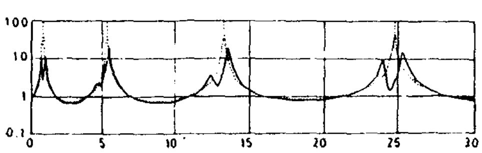

그림 3 Frequency response of the transmissibility for the composite beam absorber

The parameter values of the absorber used are

$$G_v = 2.73\times 10^{6}\ (\mathrm{N/m^2})$$
$$h_{c1} = 1.75\times 10^{-3}\ (\mathrm{m})$$
$$h_{c2} = 0.25\times 10^{-3}\ (\mathrm{m})$$
$$h_v = 1\times 10^{-3}\ (\mathrm{m})$$
$$E_a = 1.15\times 10^{11}\ (\mathrm{N/m^2})$$
$$m_a = 0.5325\ (\mathrm{kg/m})$$
$$b = 0.02\ (\mathrm{m})$$
$$l_a = 0.403\ (\mathrm{m})$$
$$\beta = 0.6$$

The physical and cross sectional geometric parameters of the absorber appear in terms of lumped form parameters g, R and D in eq.(4).

The tuning ratio, which gives the same value to both peaks of the transmissibility in the vicinity of the tuned frequency depends on the damping factor β of damping layer.

Of all the tuning ratios and damping factors satisfying above conditions, the one that give the smallest transmissibility is chosen as the optimal parameters.

Here, though the tuning of the absorber is done with fixed g. R, only by changing absorber length, the maximum transmissibility also can be controlled by modifying g, R parameters.

We decided g, R parameters that give the maximum loss fator.

Fig.4, which shows the case when using the uniform beam absorber, is given for comparision.

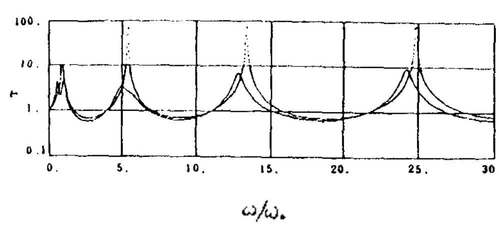

그림 4 Frequency response of the transmissibility for the uniform beam absorber

In the case of tuning only near to the 1st resonance, the composite beam absorber shows only similar performances to the uniform one.

But the composite absorber is favorable in point of the availability of the material that can fulfill the optimal condition.

Next, the possibility of simultaneous supprerssion of several resonances is examined.

For this, the relationships between shear modulus of the damping layer G_v and absorber length l_a with fixed young's modulus of the constraining layer, which tune the absorber to each resonance frequency of the main beam, is calculated and given in Fig.5.

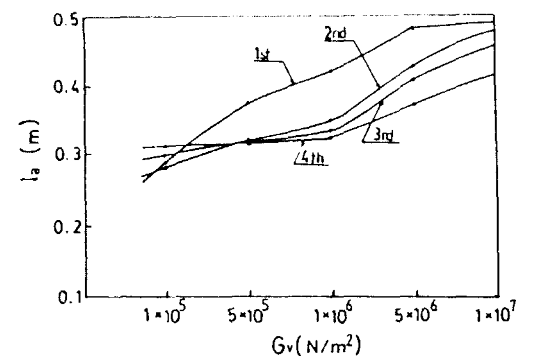

그림 5 Relationship between l and G when tuned separately to each main resonance.

The figure shows several points which can tune simultaneously two or three resonance frequencies.

One point at high G_v gives the absorber length which can tune simultaneously to 2nd, 3rd and 4th modes and other points at low G_v to 1st and also 2nd, 3rd or 4th. However, it is noted that there is no parameter set of G_v and l_a for this system to be tuned simulataneously more than two resonances including 1st mode.

Fig.6 represents the result of simultaneous tuning of the absorber to 2nd, 3rd and 4th resonance in the case of low G.

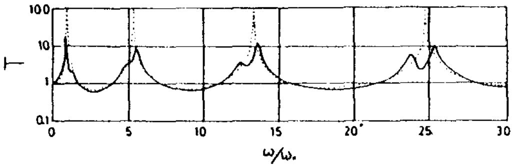

그림 6 Frequency response of the transmissibility when tuned simultaneously to 2 nd, 3 rd and 4 th main beam resonance

Even when we tune the parameters to 1st and 2nd mode, 3rd or higher resonance can be suppressed by the high damping effect.

So this composite beam absorber can be used for controlling simultaneously more than two resonances.

IV. CONCLUSION

The effectiveness of using the constrained damping layer composite beam as a dynamic vibration absorber is investigated.

Through theoretic analysis for the primary beam with built-in ends, the following conclusions are obtained.

(1) It is not easy to find the material satisfying the optimal condition for the parameters when using the uniform absorber, but for the composite beam absorber, it is relatively easy to satisfy the requirement by suitable structuring of constraining and damping layers.

(2) By controlling the frequency which gives the maximum loss factor, the absorber that is effective in several resonances can be designed.

(3) More than two resonance frequencies can be suppressed by tuning the absorber simultaneously to several resonance frequencies.

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