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Gyro Drift Model Using Structure Function and Effect on Control System Performance

(Structure Function을 사용한 Gyro Drift의 등가모델과 제어시스템에 끼치는 영향의 연구)

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要 約

본 논문은 gyro의 drift에 관한 일반적 등가회로를 처음에 발진기의 위상안정을 규정화 하기 위하여 개발되었던 structure function 방법을 사용하여 분석하였다. 이 방법을 사용함으로서 임의의 order의 확정성, 내지 불확정성 성격의 Gyro drift가 쉽게 규정화되고 또 측정될 수 있음을 보였다. 그리고, drift의 power spectal density와 structure function과의 관계도 분명히 하였다. 마지막으로 이 방법을 이용하여 drift가 제어시스템에 끼치는 영향을 분석함이 매우 용이하게 됨을 보였다.

Abstract

This paper addresses modeling of the gyro drift by using the structure function approach which has been originally developed for characterization of the oscillator phase noise. It is shown that by using this approach, an arbitrary order of random and deterministic gyro drift processes can be characterized and easily measured. The relationship between the drift power spectral density and structure function is clarified. It is also shown that this approach simplifies analysis of the effect of drift on the control system performance.

I. Background

Gyro has been used successfully in many aerospace applications for decades. It has also been known that the gyro output exhibits unwanted response called drift[1],[3],[4],[5]. In general, drift is the determining factor contributing to gyro accuracy and reduction of unwanted error torques (and thus unwanted drift) is a primary design goal for a high-performance gyro. By using various manufacturing techniques, the amount of drift has been steadily reduced. Currently, a good quality (and thus expensive) gyro has only a fraction of a degree per hour drift performance. However, there are applications where the system price/performance tradeoff is at a premium and the use of economically viable gyro is highly desirable. In this case, the designer has to accept reasonably priced gyros with fairly high amount of drift. Even with

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gyros having a good drift specification, ultimate system performance due to drift can be analyzed only by an accurate drift model.

Gyro drift model has been studied since 1950s and a number of papers have been published on this subject [2], [3], and [6]. Basically, these papers have identified certain random phenomenon out of gyro drift and have suggested a statical characterization of the drift. However, these models were basically *ad hoc* models for specific applications and thus were not general enough to describe the gyro drift behavior completely. In this paper, we will adopt a more unified approach called the structure function and show the gyro drift model developed in this way can be used for performance evaluation of the systems using gyros.

II. Gyro Drift Model Using Structure Functions

For modeling purposes, we note the difference between the drift *rate* process and drift *angle* process. Drift rate process is the rate of drift per unit time and the units are given in degrees (or radians) per second. Drift angle process denotes the amount of drift in a specified time interval and is formally given as an integration of the drift rate process and the units are in terms of absolute degrees (or radians).

The drift *rate* process, $\Omega(t)$, in gyro can be generally modeled as

$$\Omega(t)=a_1+a_2t+a_3t^2+a_4t^3+\cdots+\dot{\psi}(t) \qquad (1)$$

where a_1 represents coefficient of the constant drift rate component (or mean drift rate), a_2 is coefficient of the linear drift rate component, a_3 is for acceleration drift rate component, etc., and $\dot{\psi}(t)$ represents the random part of the drift rate process. In fact, the coefficients a_i's, i=1,2,3, ...are also random variables with certain statistical distributions.

The drift *angle* process, $\Phi(t)$, in gyro is simply given by the integral of Eq.(1)

$$\Phi(t)=\int_{-\infty}^{t}\Omega(t)\,dt=a_1t+a_2t^2/2+a_3t^3/3+a_4t^4/4+\cdots+\psi(t) \qquad (2)$$

where $\psi(t)$ is now the random drift angle process. In what follows, we will use both Eq.(1) and Eq.(2) extensively.

Thus, the drift model described by either Eq.(1) or Eq.(2) consists of a polynomial drift part and random drift part. Basically, polynomial part is a kind of deterministic process (within the range of random variables a_1, a_2,...).

The gyro drift behavior described by either Eq.(1) or Eq.(2) strongly suggests an analogy to the oscillator noise. For example, the drift rate process corresponds to the oscillator frequency noise and the drift angle process corresponds to the oscillator phase noise. The main difference between gyro model defined above and the oscillator phase noise is that the gyro drift does not have any periodic carrier term; however, this is only minor difference in terms of modeling. In fact, expressions similar to Eq.(1) and Eq.(2) were adopted in [1] and [5] for drift analysis but were not complete. By using the analogy between gyro drift and oscillator noise, we can make use of various advanced theories established for the description of oscillator noise.

In this report, we will take the so called *Structure Function* approach that has been succesfully used to model the oscillator instability [7], [8],[9],[10]. The theory of structure function is well established [7],[8] so that here we will mainly show how it can be applied to the gyro drift model. The following are main advantages for using Structure function. (i) Structure functions can overcome the signularity problems near f=0 area for $1/f^{\nu}$ type of noise where $\nu \geqslant 1$. (ii) It is basically a time-domain approach and thus the parameters will generally lead to the easily measurable quantities such as the mean square drift angle during a given interval or the increments thereof. (iii) Moreover, the structure function can be shown to be uniquely related to the power spectral density (PSD) of the drift process. Since it is difficult to measure the PSD of the drift process directly, we can start form the measurement of the structure function and obtain the corresponding PSD.

We first define the increment process. The first drift angle increment process is defined by

$$\Delta\Phi(t,\tau)=\Phi(t+\tau)-\Phi(t) \qquad (3)$$

Now, the N-th increment process, for N > 1, is defined recursively as

$$\Delta^N\Phi(t,\tau)=\Delta^{N-1}(\Delta\Phi(t,\tau)) \qquad (4)$$

and the N-th structure function of phase instability is given by

$$D_{\Phi}^{(N)}(\tau) \triangleq \epsilon\{[\Delta^N \Phi(t, \tau)]^2\} \quad (5)$$

where ϵ denotes taking expectations.

A simple calculation shows the $D_{\Phi}^{N}(\tau)$ is related to the autocorrelation of the first increment $R_{\Delta\Phi}(k\tau) = \epsilon[\Delta\Phi(t,\tau)\Delta\Phi(t+k\tau, \tau)]$ by [7]

$$D_{\Phi}^{N}(\tau) = \sum_{k=-(N-1)}^{N-1} (-1)^k \binom{2(N-1)}{N-1+k} R_{\Delta\Phi}(k\tau) \quad (6)$$

It can also be shown that if $S_{\dot\psi}(\omega)$ is the two sided Power Spectral Density (PSD) of the stationary frequency process $\dot\psi(t)$, then

$$R_{\Delta\Phi}(k\tau) = 4\int_{-\infty}^{\infty} \exp(-jkw\tau)\frac{S_{\dot\psi}(\omega)}{\omega^2} \sin^2\left(\frac{\omega\tau}{2}\right)\frac{d\omega}{2\pi} \quad (7)$$

Using Eq.(7) in Eq.(6) and making some staightforward manipulation, one finally obtains

$$\begin{aligned} D_{\Phi}^{(N)}(\tau) &= \tau^{2N}\epsilon[a_N^2] + D_{a}^{N}(\tau) \\ &= \tau^{2N}\epsilon[a_N^2] + 2^{2N}\int_{-\infty}^{\infty} \sin^{2N}\left(\frac{\omega\tau}{2}\right)\frac{S_{\dot\psi}(\omega)}{\omega^2}\frac{d\omega}{2\pi} \end{aligned} \quad (8)$$

The first term on the right hand side of Eq.(8) is related to the long term drift in Eq.(1) or Eq.(2) and the second term is due to the random drift $\psi(t)$ process. This is also the key equation to relate the time-domain quantity $D_{\Phi}^{(N)}(\tau)$ to the frequency domain quantity (PSD) $S_{\dot\psi}(\omega)$ of drift.

It is interesting to note that for $M > N$, Eq.(8) reduces to

$$\begin{aligned} D_{\Phi}^{(M)}(\tau) &= D_{\Phi}^{(N)}(\tau) \\ &= 2^{2M}\int_{-\infty}^{\infty} \sin^{2M}\left(\frac{\omega\tau}{2}\right)\frac{S_{\dot\psi}(\omega)}{\omega^2}\frac{d\omega}{2\pi} \end{aligned} \quad (9)$$

From this equation, we know that the structure function of order $M > N$ is solely determined by $S_{\dot\psi}(\omega)$.

The first increment Structure function of the drift angle is simply the Mean Square (MS) jitter accumulated in a given time interval τ and is given by

$$D_{\Phi}^{(1)}(\tau) = \epsilon[(\Phi(t+\tau) - \Phi(t))^2] \quad (10)$$

and this is also called the Mean Square Drift Angle (MSDA) [6] during the time interval τ. For simple gyro drift modeling, measurement of $D_{\Phi}^{(1)}(\tau)$ is adequate. Furthermore, $D_{\Phi}^{(1)}(\tau)$ can be readily measurable.

For the first increment structure function, Eq.(8) simplifies to

$$D_{\Phi}^{(1)}(\tau) = \tau^2\epsilon[a_1^2] + 4\int_{-\infty}^{\infty} \sin^2\left(\frac{\omega\tau}{2}\right) S_{\psi}(\omega)\frac{d\omega}{2\pi} \quad (11)$$

where we assumed $a_2 = a_3 = a_4 = \cdots = 0$ and utilized the relationship between $S_{\psi}(\omega)$ and $S_{\dot\psi}(\omega)$ as given by

$$S_{\psi}(\omega) = \frac{S_{\dot\psi}(\omega)}{\omega^2}$$

From references [2] and [6], we can identify at least two random processes assoicated with gyro drift. The first one, $\psi_1(t)$, is usually called the random walk process and has the power spectral density

$$S_{\psi_1}(t) = \frac{\sigma_1^2}{\omega^2} \quad (12)$$

This process is essentially the white drift rate process with flat power spectral density of σ_1^2 The second process, $\psi_2(t)$, is a stationary random process and is characterized by the power spectral density

$$S_{\psi_2}(\omega) = \frac{2\sigma_2^2\alpha}{\omega^2 + \alpha^2} \quad (13)$$

where $1/\alpha$ denotes the correlation time of the process and σ_2^2 is the variance of the process.

If we substitute $\psi(t) = \psi_1(t) + \psi_2(t)$ into Eq.(11) and evaluate the integral, we obtain,

$$D_{\Phi}^{(1)}(\tau) = \tau^2\epsilon[a_1^2] + \sigma_1^2\tau + 2\sigma_2^2(1 - \exp(-\alpha\tau)) \quad (14)$$

where the second term is due to $\psi_1(t)$ and the third term is due to $\psi_2(t)$. Note also that this is the same result as was obtained in [6].

If we can identify any other random drift power spectral density function, then the total MSDA is simply the sum of MSDA of the individual processes. For example, with the addition of the arbitrary $S_{\psi_3}(\omega)$ process, $D_{\Phi}^{(1)}(\tau)$ can be obtained by

$$D_{\Phi}^{(1)}(\tau)=\tau^2\varepsilon[a_1^2]+\sigma_1^2\tau+2\sigma_2^2(1-\exp(-\alpha\tau))+4\int_{-\infty}^{\infty}\sin^2\left(\frac{\omega\tau}{2}\right)S_{\psi_3}(\omega)\frac{d\omega}{2\pi} \tag{15}$$

It is highly probable that the drift contains the processes whose PSD's can be expressed by more than the second order poles, or deterministic processes with $a_i \neq 0$ for $i \geqslant 2$. Higher order structure function can be useful for identifying and measuring these higher order random or deterministic drift processes. For example, the second order structure function can be reprsented by

$$D_{\Phi}^{(2)}(\tau)=\varepsilon[(\Delta^2\Phi(t,\tau))^2]=\varepsilon[(\Phi(t+2\tau)-2\Phi(t+\tau)+\Phi(t))^2] \tag{16}$$

If $a_2 \neq 0$, then $D_{\Phi}^{(1)}(\tau)$ is time dependent. Also, if $S_{\psi}(t)$ contains 4-th order pole at the origin, then $D_{\Phi}^{(1)}(\tau)$ is not bounded. However, $D_{\Phi}^{(2)}(\tau)$ can be shown to be time independent for $a_2 \neq 0$ and for up to 4-th order pole PSD since

$$D_{\Phi}^{(2)}(\tau)=\tau^4\varepsilon[a_2^2]+8\int_{-\infty}^{\infty}\sin^4\left(\frac{\omega\tau}{2}\right)S_{\psi}(\omega)\frac{d\omega}{2\pi} \tag{17}$$

The measurement of the second order structure function is suggested by Eq.(16). We simply take an ensemble of drifted angle measurements at times of $t+2\tau$, $t+\tau$, and t and take expectations according to the formula. Third or higher order structure functions can also be measured in this way. For a more detailed algorithm, see reference [7].

Measurement of higher order structure function can identify the existence of various components of the drift model shown in Eq.(1) or Eq.(2). For serious research into the problem of drift behavior, we recommend to collect measurement data on the drift and obtain up to N-th order structure functions to completely characterize the drift process. Once drift is accurately modeled, its effect on the loop can be analyzed by using the techniques described in the next section.

Finally, we want to note that the estimation of the drift PSD $S_{\psi}(\omega)$ from the structure function can be done by using the established algorithms. For more details on this matter, please refer reference [7].

III. Analysis of Drift Effect on Control Systems

Control systems make frequent use of the gyro as a sensing element. In particular, three axis gyros are used in many control systems such as the stable gyro platform, attitude stabilizing loop, etc. In this section, we analyze the effect of gyro drift on the control system performance by using the model developed in the previous section.

For this purpose, it is convenient to consider separately the effect due to deterministic (polynomial) drift and the effect due to random drift as the physical mechanism governing the two components of the drift are different. Let us denote that $\Phi(t)=\Phi_D(t)+\Phi_R(t)$ where $\Phi_D(t)$ is the deterministic part and $\Phi_R(t)$ is the random part. Then from Eq.(2), $\Phi_R(t)=\psi(t)$ and

$$\Phi_D(t)=a_1t+a_2t^2/2+a_3t^3/3+a_4t^4/4+\cdots$$

The effect of deterministic drift can be easily evaluated by using the linear system theory. We can represent the deterministic drift angle input, $\Phi_D(s)$, by using the Laplace transform as

$$\Phi_D(s)=a_1\frac{1!}{s^2}+a_2\frac{1!}{s^3}+a_3\frac{2!}{s^4}+a_4\frac{3!}{s^5}+\cdots \tag{18}$$

Then the loop output, $Y_D(s)$, due to $\Phi_D(s)$ is simply given by

$$Y_D(s)=\Phi_D(s)\,H(s) \tag{19}$$

where H(s) is the Laplace transform of the system transfer functions $H(\omega)$ between the drift input and the loop output with ω replaced by s. For a typical application, the gyro control system configuration is given by the closed loop feedback architecture between command input and response output, where drift input can be considered as a disturbance input. For example, in shipboard antenna pointing control system, the input command moves the antenna boresight to the desired direction and the drift input is the major source of error between input and output. The steady state response of the output due to drift can be obtained by using the final value theorem, which states that

$$\lim_{t\to\infty} Y_D(t)=\lim_{s\to 0} s\,Y_D(s)=m_{Y_D} \tag{20}$$

where m_{Y_D} denotes the mean since the error due to deterministic drift corresponds to the mean error out of the total error. Thus it can be said the system error output is strongly dependent on the order of the loop. Also, similar equations can be established for drift rate process $\Omega(t)$ as well.

Now we consider the effect due to random drift. For random drift processes, an appropriate way to represent them in the control loop is the PSD. Fortunately, PSDs can be obtained from the measurement of the structure function as mentioned in Sec. II.

From the system theory, we know that the PSD of the system output, $S_{Y_R}(\omega)$, for the input random process, in this case $S_\psi(\omega)$, can be expressed by

$$S_{Y_R}(\omega) = S_\psi(\omega) \mid H(\omega) \mid^2$$

or the system output MS angular jitter due to gyro drift can be obtained by

$$\sigma^2_{Y_R} = \int_{-\infty}^{\infty} S_{Y_R}(\omega)\, d\omega \qquad (21)$$

Finally, the total MS system output error due to both deterministic and random drift, σ^2_Y, is given by combining Eq.(20) and Eq.(21)

$$\sigma^2_Y = \sigma^2_{Y_R} + m^2_{Y_D}$$

From these observations, it can be said that due to the presence of the strong drift components near f=0 with random drift processes such as $S\psi_1(t)$ and $S\psi_2(t)$ [see Eq.(12) and Eq.(13)] and also with deterministic process, it is desirable for the system transfer function $H(\omega)$ to have a zero at the origin, to minimize σ_{Y_R} component.

IV. Discussions

In previous sections, we have seen that the gyro drift can be characterized by using the structure function approach. By using this approach, we believe that the ultimate characterization of the random gyro process $\psi(t)$ is possible and we can extract the last tiny measure of performance out of the systems that utilize gyros.

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