

# Improved Method for Calculating Radiation Admittance of a Rectangular Microstrip Patch Antenna

## (구형 마이크로스트립 안테나의 개선된 복사 어드미턴스 계산방법)

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### 要 約

본 논문에서는 균일한 유전체로 채워져 있고 윗면에 슬릿이 있는 평행평판 도파로의  $\pi$  등가 회로로부터 구형 마이크로스트립 안테나의 복사 컨덕턴스와 서셉턴스를 동시에 구하는 새로운 방법을 제시하였다. 계산된 결과를 기존의 이론치와 비교하였다. 또한 등가전송선로를 이용하여 반사손실(return loss) 및 공진주파수를 계산하여 실험치와 비교한 결과 양호한 특성을 얻었다.

### Abstract

In this paper, a new method which simultaneously gives the radiation conductance and susceptance of the rectangular microstrip patch antenna. This solution is derived from the load admittance of the equivalent  $\pi$ -network circuit of the slitted parallel-plate waveguide filled with homogeneous dielectric. Our theoretical results for the radiation admittance are compared with other theoretical results. Also, our theoretical return loss and resonant frequency which are calculated by use of the equivalent transmission line model are in fairly good agreement with the experimental results.

### I. Introduction

Microstrip antennas have the advantage of small size, light weight, ease of construction, and compatability with solid-state microwave devices. As practical applications of microstrip antenna increase, the need for more exact solutions is

growing. Although several approximate expressions for the radiation admittance of a rectangular microstrip patch antenna have been described by several authors, no rigorous expressions have been found yet. The aperture admittance for a parallel-plate waveguide into a half space[1] is commonly used as the radiation admittance of the rectangular microstrip patch antenna. Derneryd[2] and Bahl[3] analyzed the radiation admittance using the aperture theory and concept of the line extension  $\Delta l$ . Recently, Sengupta[4] obtained the radiation admittance by use of the image principle

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and the results for a semi-infinite parallel-plate waveguide radiating into free space.

In this paper, a new method is presented which simultaneously gives the radiation conductance and susceptance of the rectangular microstrip patch antenna. These calculations are derived from the equivalent  $\pi$ -network circuit [5],[6] of the slitted parallel-plate waveguide filled with homogeneous dielectric. Our computed results for the radiation admittance are compared with other theoretical solutions. Also, from the knowledge of the radiation admittance, return loss is calculated and compared with the measured result. Here the antenna is fabricated in Teflon substrate with relative dielectric constant  $\epsilon_r=2.6$  and thickness  $h=0.155\text{cm}$ . The experimental results are in fairly good agreement with the calculated results.

### II. Calculation of Radiation Admittance

The geometry of the slitted parallel-plate waveguide filled with homogeneous dielectric is shown in Fig.1(a), where the slit width  $2a$  is parallel to Y axis and the waveguide height is  $h$ .

When only a TEM wave whose electric field component has +X direction can propagate along +Z direction, the slitted parallel-plate waveguide shown in Fig.1(a) is represented by the equivalent  $\pi$ -network circuit [5],[6] shown in Fig.1(b).

In Fig.1(b), the TEM magnetic field reflection coefficient  $\Gamma_H$  is defined as the ratio of reflected to incident magnetic field at a reference plane located at  $z=-a$ , and the transmission coefficient  $T_H$  as the ratio of transmitted field at  $z=+a$  to incident field at the reference. Therefore  $\Gamma_H$  and  $T_H$  can be written as[5],[6]

$$\Gamma_H = \frac{H_{yI.ref}(z)|_{z=-a}}{H_{yI.inc}(z)|_{z=-a}} \quad (1)$$

$$T_H = \frac{H_{yI.tran}(z)|_{z=+a}}{H_{yI.inc}(z)|_{z=-a}} \quad (2)$$

where  $H_{yI.ref}(z)$ ,  $H_{yI.tnc}(z)$ , and  $H_{yI.tran}(z)$  are incident, reflected, and transmitted magnetic field in the region I, respectively.

The normalized series admittance  $\bar{Y}_1$ , shunt admittance  $\bar{Y}_2$ , and load admittance  $\bar{Y}_L$  of the equivalent  $\pi$ -network circuit are given by the following expressions[5],[6] in terms of the coefficients  $\Gamma_H$  and  $T_H$ :

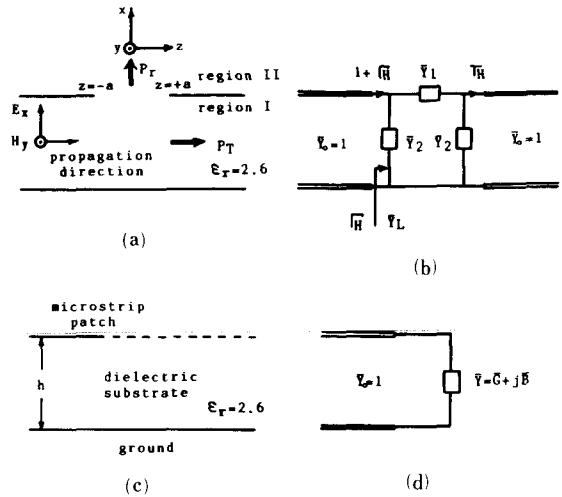


Fig.1. (a) Geometry of a slitted parallel-plate waveguide filled with homogeneous dielectric. (b) Equivalent  $\pi$ -network. (c) Reduced geometry of a radiating edge. (d) Equivalent circuit of the radiating edge in Fig.1(c).

$$\bar{Y}_1 = \bar{G}_1 + j \bar{B}_1 = \frac{(\bar{Y}_2 + 1)(\bar{Y}_L - \bar{Y}_2)}{2\bar{Y}_2 + 1 - \bar{Y}_L} \quad (3)$$

$$\bar{Y}_2 = \bar{G}_2 + j \bar{B}_2 = \frac{1 + \Gamma_H - T_H}{1 - \Gamma_H + T_H} \quad (4)$$

$$\bar{Y}_L = \bar{G}_L + j \bar{B}_L = \frac{1 + \Gamma_H}{1 - \Gamma_H} \quad (5)$$

- In Fig.1(a), 1(b), it was found[5],[6] that:
- 1) for  $ka > 0.1$ , the normalized load conductance  $\bar{G}_L$  and susceptance  $\bar{B}_L$  are constant, independently of the slit width, and  $\bar{G}_L$  is almost the same as  $\bar{G}_1$  for  $ka < 0.2$ . Here  $k$  is the propagation constant of the free space.
  - 2) for  $ka < 0.2$ , i.e., narrow slit width,  $\bar{G}_1$  is identical to the normalized radiation conductance of the microstrip radiating edge calculated by the method of Bahl[3].

When the ratio of the normalized transmitted power  $P_T$  to the normalized radiated power  $P_r$  approaches to an extremely small value, i.e., one percent, then the slit width  $2a$  is equivalent to

about  $0.56\lambda_g$ [7]. Here  $\lambda_g$  is the wavelength in the waveguide. In this case, the transmitted power is nearly negligible so that the geometry of the slitted parallel-plate waveguide shown in Fig.1(a) reduces to the geometry with the upper plate removed beyond  $z=+a$  in Fig.1(a), as shown in Fig.1(c). Therefore the load admittance in Fig.1(b) can be interpreted physically as the radiation admittance of the edge in Fig.1(d). In addition it has been well taken that two dimensional parallel-plate result is valid for the rectangular microstrip patch application. From the load admittance discussed above, we obtain the radiation admittance of the rectangular microstrip patch antenna.

### III. Theoretical and Experimental Results

#### 1. Radiation Admittance

Fig.2 shows the geometry of the rectangular microstrip patch antenna fed by a  $50\text{-}\Omega$  microstrip line, where the feed point location is X. The substrate used here is copper-clad 'CGP-512' with a dielectric constant of 2.6. The antenna dimensions are listed in Table 1.

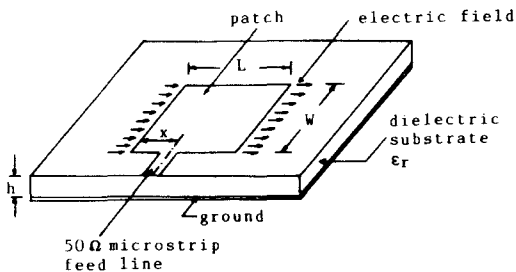


Fig.2. Geometry of a rectangular microstrip patch antenna.

Table 1. Specification of a rectangular microstrip patch antenna.

Width W [cm]	Length L [cm]	Dielectric constant $\epsilon_r$	Substrate thickness h [cm]
3.7	3.02	2.6	0.155

Teflon substrate : CGP-512

For the antenna given in Table 1, the radiation admittance computed by using the present method is compared with the theoretical results by the method of Harrington[1], Derneryd[2], and Sengupta[4] at 3GHz, respectively. Comparative results are given in Table 2. Our theoretical radiation admittance  $Y$  ( $Y=\bar{Y}\cdot Y_0$ :  $Y_0$  denotes the characteristic admittance), as mentioned in section II, is given by the load admittance  $Y_L$  in the region of  $ka>0.1$ .

Table 2. Comparison of the radiation admittance for the antenna specified in Table 1 at 3GHz.

Method	Radiation conductance G [mho]	Radiation susceptance B [mho]
Harrington <sup>1</sup>	$0.3082 \times 10^{-2}$	$0.7648 \times 10^{-2}$
Derneryd <sup>2</sup>	$0.1359 \times 10^{-2}$	$0.8829 \times 10^{-2}$
Sengupta <sup>4</sup>	$0.3083 \times 10^{-2}$	$0.9009 \times 10^{-2}$
Ours	$0.3259 \times 10^{-2}$	$1.4597 \times 10^{-2}$

As shown in Table 2, our computed radiation conductance is almost the same as Harrington's[1] and Sengupta's[4] theoretical results except for Derneryd's[2] theoretical result. But our computed radiation susceptance is much greater than other theoretical values[1],[2],[4].

In order to confirm the validity of our results for the radiation admittance, the theoretical results of the present method are compared with Chang's results[8] obtained by the Wiener-Hopf techniques and with Sengupta's results[4].

Fig.3 shows that the normalized radiation conductance  $\bar{G}$  and susceptance  $\bar{B}$  are plotted as a function of  $kh$  for  $\epsilon_r=2.56$ . In Fig.3, our results for  $\bar{G}$  and  $\bar{B}$  are in agreement with the theoretical results of Chang's[8], respectively. Also, it is observed that our normalized susceptance and Chang's values are much greater than Sengupta's theoretical values.

#### 2. Feed Point

The rectangular microstrip patch antenna shown in Fig.2 is represented by the equivalent transmission line model, as shown in Fig.4. Here X is a  $50\text{-}\Omega$  microstrip line feed point location.  $Y_0$  is the characteristic admittance of the microstrip patch.  $Y_{in}$  and  $Y$  are the input admittance

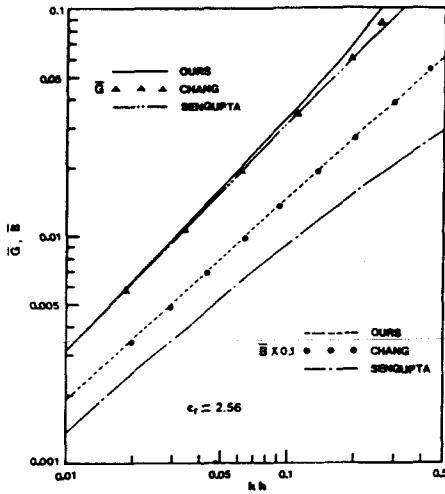


Fig.3. Normalized radiation admittance as a function of dimensionless thickness  $kh$ .

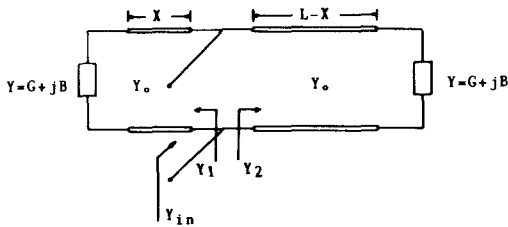


Fig.4. Equivalent circuit of the rectangular microstrip patch antenna.

and the radiation admittance of the rectangular microstrip patch antenna, respectively.

In Fig.4 the input admittance of the antenna  $Y_{in}$  is calculated by

$$Y_{in} = Y_1 + Y_2 = Y_0 \frac{Y + jY_0 \tan \beta X}{Y_0 + jY \tan \beta X} + Y_0 \frac{Y + jY_0 \tan \beta (L-X)}{Y_0 + jY \tan \beta (L-X)} \quad (6)$$

where  $\beta = \frac{2\pi}{\lambda_0} \sqrt{\epsilon_{eff}}$  is the propagation constant of the transmission line,  $\lambda_0$  is the free space wavelength, and  $\epsilon_{eff}$  is the effective dielectric constant.

The characteristic admittance  $Y_0$  [9] of the rectangular microstrip patch antenna is given by

$$Y_0 = \frac{W \alpha \sqrt{\epsilon_{eff}}}{\eta_0 h} \quad (7)$$

where

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left[ 1 + 10 \frac{h}{W} \right]^{-1/2}$$

$$\alpha = 1 + 1.393 \frac{h}{W} + 0.667 \frac{h}{W} \ell_n \left[ \frac{W}{h} + 1.444 \right]$$

and  $\eta_0$  is the intrinsic impedance of the free space.

From (6) and (7), it is seen that the input admittance of the antenna varies with the feed point location. Therefore, the feed point should be accurately chosen so as to provide a good match between the antenna and the feed line. A 50- $\Omega$  microstrip line feed point location can be determined by the input admittance locus for the feed point location.

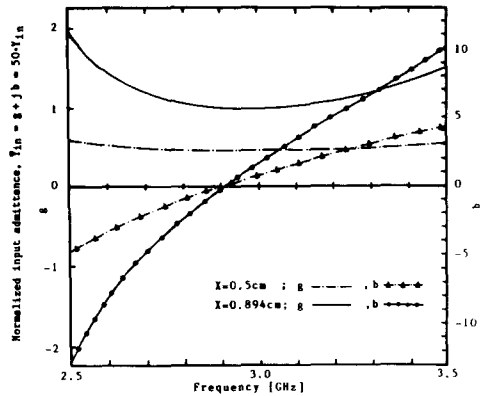


Fig.5. Normalized input admittance loci as a function of frequency for  $X=0.5\text{cm}$  and  $X=0.894\text{cm}$ .

Fig.5 shows the input admittance loci normalized to 50  $\Omega$ , i.e.,  $\bar{Y}_{in} = g + jb = 50 Y_{in}$ , as a function of frequency for different feed point location, 0.5 and 0.894cm, respectively. Here the detailed specification of this antenna is given in Table 1. It is observed that for the case of 0.894cm, when  $g=1$ , the frequency where the imaginary part of the normalized input admittance is nearly zero exists. This implies that for  $X=0.894\text{cm}$ , the input admittance of the antenna is

equal to the characteristic admittance of the feed line at the frequency. Also, in Fig.5 the resonant frequency is observed to be 2.910 GHz.

3. Return Loss

The return loss RL is defined as

$$RL = 20 \log |\Gamma_{in}| = 20 \log \left| \frac{1 - \bar{Y}_{in}}{1 + \bar{Y}_{in}} \right| \quad (8)$$

where  $\Gamma_{in}$  is the input reflection coefficient of the rectangular microstrip patch antenna.

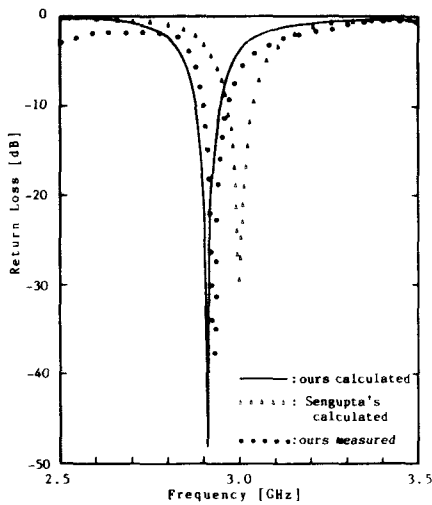


Fig.6. Measured and computed return losses as a function of frequency for the antenna specified in Table 1 at X=0.894cm.

In Fig.6, the computed and measured return losses are illustrated and compared with the theoretical results by the method of Sengupta[4]. The detailed specification of the antenna is given in Table 1. The feed point location proved to be 0.894cm. As shown in Fig.6, the experimental results are in better agreement with our theoretical results than Sengupta's theoretical results. Also, it is observed in Fig.6 that our computed and measured resonant frequency of the rectangular microstrip patch antenna are 2.910 and 2.937GHz, respectively, whereas the Sengupta's theoretical value is 3.000GHz. Therefore, the frequency discrepancy between our measured and calculated resonant frequency is smaller than that between

our measured and the Sengupta's theoretical value.

4. Resonant Frequency

From the resonant condition:  $Im(Y_{in})=0$ , the resonant frequency  $f_0$  of the rectangular microstrip patch antenna can be obtained by use of the following expression in terms of the radiation admittance:

$$\tan \left( \frac{2\pi f_0 L}{c} \sqrt{\epsilon_{eff}} \right) = \frac{2B Y_0}{B^2 + G^2 - Y_0^2} \quad (9)$$

where  $c$  is the light velocity of the free space.

For the comparison of the resonant frequency of the rectangular microstrip patch antenna for various values of  $W$ , our theoretical results, Sengupta's computed results[4] and the experimental results reported in [10] are shown in Table 3, from which the experimental results[10] are seen to be in better agreement with our computed values than Sengupta's calculated values[4].

Table 3. Calculated and measured values of the resonant frequency for rectangular microstrip patch antennas.

Width W [cm]	Length L [cm]	Resonant frequency, $f_0$ (MHz)		
		Measured	Calculated	
		Howell <sup>[10]</sup>	Sengupta <sup>[4]</sup>	Ours
4.100	4.140	2228	2248	2200
6.858	4.140	2200	2228	2179
10.800	4.140	2181	2216	2166

$h=0.1524\text{cm}$ ,  $\epsilon_r=2.5$

IV. Conclusion

This paper presents a new method which simultaneously gives the radiation conductance and susceptance of the rectangular microstrip patch antenna. This solution is derived from the load admittance of the equivalent  $\pi$ -network of the slitted parallel-plate waveguide filled with homogeneous dielectric. Our theoretical results for the radiation admittance are compared with other theoretical results. Also, applying the

radiation admittance in the equivalent transmission line model, we observed that our theoretical return loss and resonant frequency are in fairly good agreement with the experimental results. Further, it is concluded that these are more accurate than previous theoretical results[1]-[4].

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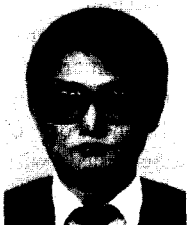
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