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Direction-of-Arrival Estimation in Broadband Signal Processing : Rotation of Signal Subspace Approach

(광대역 신호 처리에서의 도래각 추정 : Rotation of Signal Subspaces 방법)

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要 約

본 논문은 rotation of subspaces 개념을 이용한 도래각 추정 방법을 제시한다. 이 방법은 여러 응용분야에서 대두되는 부공간들(subspaces)의 각도 및 거리와 밀접한 관련이 있다. 먼저 최소 자승을 이용하여 한 부공간을 다른 부공간으로 변환 시켜주는 최적 변환 행렬을 구하기 위한 효율적인 방법을 유도하고 이를 이용하여 다중 광대역 신호들(인코히어런트, 부분적인 코히어런트와 완전한 코히어런트의 혼합신호들)의 도래각을 추정한다.

대표적인 응용으로, 잡음의 배열 스펙트럼 밀도 행렬이 변하지 않는 액티브 시스템(e. g. sonar system) 경우에 성능을 높이기 위하여 효율적인 ROSS(rotation of signal subspaces) 알고리듬을 제안한다. ROSS 알고리듬은 Wang-Kaveh's CSS-focusing 방법에서 사용하는 예비처리와 공간 필터링을 필요로하지 않는 장점이 있으며 일반적인 모든 배열 안테나에도 적용될 수 있다.

시뮬레이션 결과, 제안된 새로운 알고리듬이 CSS focusing 방법 및 Forward-Backward Spatial Smooothed MUSIC 보다 높은 성능을 가짐을 알 수 있었다.

Abstract

In this paper, we present a method which is based on the concept of the rotation of subspaces. This method is highly related to the angle (or distance) between subspaces arising in many applications. An effective procedure is first derived for finding the optimal transformation matrix which rotates one subspace into another as closely as possible in the least squares sense, and then this algorithm is applied to the solution to general direction-of-arrival estimation problem of multiple broadband plane waves which may be a mixture of incoherent, partially coherent or coherent. In this typical application, the rotation of signal subspaces (ROSS) algorithm is effectively developed to achieve the high performance in the active systems for the case in which the noise field remains invariant with the measurement of the array spectral density matrix (or data matrix). It is not uncommon to observe this situation in sonar systems. The advantage of this technique is not to require the preliminary processing and spatial prefiltering which is used in Wang-Kaveh's CSS focusing method. Furthermore, the array's geometry is not restricted. Simulation results are presented to illustrate the high performance achieved with this new approach relative to that obtained with Wang-Kaveh's CSS focusing method for incoherent sources and forward-backward spatial smoothed MUSIC for coherent sources including the signal eigenvector method (SEM).

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I. Introduction

In many applications, the principal angles and the distance between two subspaces are considered to find out the relationships between subspaces. For example, one may take an interest in finding the intersection of null spaces (or ranges) of given rectangular matrices and the rotation of subspaces, and so forth. The problem related to the rotation of subspaces, the so-called orthogonal procrustes problem, is to find the orthogonal transformation matrix which transforms a given matrix into another given matrix so that the sum of squares of the residual matrix is minimized. The solution to this problem has been first proposed by Green [5]. Green presented a solution to somewhat less general formulation of the orthogonal procrustes problem which has more stringent restrictions in requiring that the matrices are of full column rank. To overcome this restrictions, Schönemann proposed an alternate approach which can be applicable to the matrices which are of less than full rank. In this approach, a matrix of Lagrange multipliers is used to solve the problem which is composed of real matrices[8].

In this paper, however, a more general method is proposed for solving the least squares problem by using Singular Value Decomposition (SVD). Unlike the Schönemann method, this method is applicable to both real and complex matrices. As a typical application of this algorithm, the direction-of-arrival estimation problem is herein considered to demonstrate the effectiveness in solving the spatially close broadband plane waves.

It is often the case that in broadband signal transmission, the individual narrowband SNR's are not sufficient for a resolution of closely spaced sources. Thus, Wang and Kaveh showed that high resolution direction-of-arrival estimation for multiple broadband sources was achieved by using frequency domain averaging with transformation matrix [10,11]. In this approach, preliminary processing and spatial prefiltering are required to estimate an initial angle of the source directions and separate the groups of sources under the assumption that a knowledge of the neighborhoods of these initial angles is sufficient to effect the advantage of broadband signal processing. Furthermore, Cadzow has recently proposed the signal eigenvector method (SEM) which has a superior performance relative to MUSIC in conjunction with Wang-Kaveh's CSS focusing method[1]. The signal enhancement approach has been also proposed to improve the performance for resolving the closly spaced broadband plane waves [7]. When multi groups of plane waves impinge on an array of sensors, however, those approaches need at least a good estimator for the initial angle estimates of each group of sources.

The rotation of signal subspace (ROSS) algorithm proposed in this paper is developed without appealing to preprocessing and spatial prefiltering. The main difficulty in developing signal subspace processing for broadband sources arises from the fact that the signal subspace at one frequency is different from that at another frequency. Thus, the proposed approach constructs a common signal subspace by using a unitary transformation matrix which represents the rotation of signal subspace at the other frequency into a single one. This technique is applicable to the situations in which the noise field remains invariant while the signal field changed in the direction of arrival of sources. Under this circumstances, the proposed method provides the effective direction-of-arrival estimation while Wang-Kaveh's method takes the several degradation such as bias and poor resoution without spatial prefiltering.

II. Signal Model Formulation

A standard approach to signal processing problems entails formulating a model for the process that gives rise to the data being analyzed. For the direction-of-arrival problem, the underlying plane wave direction vectors are obtained by modeling the delay pattern across the array's sensors. Let us now consider an array of M omnidirectional sensors which is receiving N incident plane waves traveling in directions $\underline{\kappa}_1$, $\underline{\kappa}_2$, ..., $\underline{\kappa}_N$ with associated envelope signals $f_1(t)$, $f_2(t)$, ..., $f_N(t)$ where $\underline{\kappa}$ designates real 3 x 1 unit vector. Appealing to the principle of superposition, the m^{th} array sensor is specified as

$$X_m(\omega) = \sum_{n=1}^{N} F_n(\omega)\, e^{j\phi_n}\, e^{j(\omega_o+\omega)\tau_{n(m)}} \quad \text{for } 1 \le m \le M \qquad (1)$$

where ω_o is a center frequency, ω is a baseband frequency and $F_n(\omega)$ and ϕ_n denote the Fourier transform of the n^{th} envelope signal $f_n(t)$ and phase angle, respectively. The parameter $\tau_n(m)$ corresponds to the delay which the n^{th} plane wave is received at m^{th} sensor relative to the origin of three space and is given by

$$\tau_n(m) = \underline{\kappa}_n \cdot \underline{z}_m / c \quad \text{for } 1 \le m \le M \text{ and } 1 \le n \le N \qquad (2)$$

where $\underline{z}_m$ is a 3 x 1 vector representing the location of m^{th} sensor, $\underline{\kappa}_n \cdot \underline{z}_m$ denotes the standard vector inner product in real three space and c is a propagation velocity. These delay parameters are fundamental to the direction-of-arrival problem since they characterize the combined array geometry-incident plane wave direction-of-arrival information.

In the analysis to follow, a vector space approach is taken for analyzing the direction-of-arrival problem and forming an algorithm for its solution. With this objective in mind, let us express the relationship (1) in the M x 1 spectral snapshot vector format

$$\underline{x}(\omega) = [X_1(\omega), X_2(\omega), \cdots, X_M(\omega)]^T \qquad (3)$$

The lower case letter $x(\omega)$ (instead of $X(m)$) is here used to designate a vector quantity in keeping with standard vector notation and the prime symbol (^{T}) denotes vector transposition. From relationship (1) it is seen that these sensor signals are linear combination of the spectral envelope signal terms $F_n(\omega) \cdot \exp(j\phi_n)$. It is then convenient to express this linear combination as

$$\underline{x}(\omega) = S(\omega)\underline{f}(\omega) \qquad (4)$$

In this expression, $\underline{f}(\omega)$ is the N x 1 spectral envelope vector as given by

$$\underline{f}(\omega) = [F_1(\omega) \cdot \exp(j\phi_1), F_2(\omega) \cdot \exp(j\phi_2), \cdots, F_N(\omega) \cdot \exp(j\phi_N)]^T \qquad (5)$$

The M x N composite steering matrix $S(\omega)$ appealing in representation (4) plays a prominent role in the development to follow and its columns are composed of the M x 1 steering vectors

$$\underline{s}(\omega, \underline{\kappa}_n) = [e^{j(w+w_o)\tau_{n(1)}}, e^{j(w+w_o)\tau_{n(2)}}, \cdots, \cdots, e^{j(w+w_o)\tau_{n(M)}}]^T \qquad (6)$$

The n^{th} steering vector is associated with the n^{th} incident plane wave and is a function of the delay terms $\tau_n(m)$ which from expression (2) are seen to be dependent on both the array's geometry and the plane waves direction vector $\underline{\kappa}_n$. It is for this reason that the steering vector is expressed as an explicit function of the plane wave's direction vector.

In most practical applications, the measured sensor signals are corrupted by environmental noise and instrumentation error. If this corruption enters in an additive fashion, the measured sensor signals are modeled as

$$\underline{x}(\omega) = S(\omega)\underline{f}(\omega) + \underline{\eta}(\omega) \qquad (7)$$

in which the m^{th} element of the M x 1 noise vector $\underline{\eta}(\omega)$ designates the Fourier transform of the m^{th} sensor noise signal $\eta_m(t)$.

A particularly insightful and useful tool for estimating the number of incident plane waves and their direction-of-arrivals is provided by the second order statistics of the array's spectral snapshot vector. It is assumed that the plane wave signals and additive noises are zero mean widesense stationary complex-valued random processes which are pairwise uncorrelated. The spatial spectral density matrix $P_x(\omega)$ is formally specified by

$$P_x(\omega) = E\{\underline{x}(\omega)\underline{x}(\omega)^*\} \qquad (8)$$

where 'E' and '*' designate the expected value and complex conjugate transposition operators, respectively.

Upon the substitution of expression (7) into the relationship (8), the spectral density matrix is expressed as

$$P_x(\omega) = S(\omega)\, P_f(\omega)\, S(\omega)^* + \sigma^2(\omega)\, P_\eta(\omega) \qquad (9)$$

The N x N envelope spectral density matrix $P_f(\omega)$ appearing in this expression is given by

$$P_f(\omega) = E\{\underline{f}(\omega)\underline{f}(\omega)^*\} \qquad (10)$$

and characterizes the second order statistical relationship between the N incident plane wave envelope signals taken as pairs at frequency ω. The N envelope signals are said to be pairwise noncoherent at frequency ω if the rank of $P_f(\omega)$

equals N. On the other hand, if the rank of $P_f(m)$ is less than N, this indicates that a subset of the envelope signals are coherent at frequency ω. The M x M noise spectral density matrix $\sigma^2(\omega) P_\eta(\omega)$ appearing in expression (9) is specified by

$$\sigma^2(\omega) P_\eta(\omega) = E\{\underline{\eta}(\omega)\underline{\eta}(\omega)^*\} \quad (11)$$

In what is to follow, it is assumed that the noise spectral density matrix $p_\eta(\omega)$ is known and has been normalized so that its trace equals M but that the noise power level $\sigma^2(\omega)$ is unknown.

III. Generalized Eigenanalysis

In recent years, eigenvalue-eigenvector decomposition has become an important mathematical analysis for direction-of-arrival estimation problem. Using the definition of the generalized eigen-characterization of the matrix pencil ($P_x(\omega)$, $p_\eta(\omega)$), we have

$$P_x(\omega)\underline{e}_m(\omega) = \lambda_m(\omega) P_\eta(\omega)\underline{e}_m(\omega)$$
$$\text{for } m = 1, 2, \cdots, M \quad (12)$$

or

$$P_x(\omega) E(\omega) = P_\eta(\omega) E(\omega) \Lambda(\omega) \quad (13)$$

where

$$E(\omega) = [\underline{e}_1(\omega),\ \underline{e}_2(\omega),\ \cdots,\ \underline{e}_M(\omega)]$$
$$\Lambda(\omega) = \mathrm{diag}[\lambda_1(\omega), \lambda_2(\omega), \cdots, \lambda_M(\omega)],$$
$$\lambda_1(\omega) \geq \lambda_2(\omega) \geq \cdots \geq \lambda_M(\omega) \geq 0$$

and the spectral density matrices $P_x(\omega)$ and $p_\eta(\omega)$ are the MxM Hermitian positive semidefinite matrices. The $\lambda_m(\omega)$ scalars are called eigenvalues while the $\underline{e}_m(\omega)$ are M x 1 (generalized) eigenvectors. The eigenvectors are normalized and have the following properties, that is,

$$P_\eta(\omega)\text{-orthogonal}: \underline{e}_1(\omega)^* P_\eta(\omega)\underline{e}_m(\omega) = \delta(1-m)$$
$$P_x(\omega)\text{-orthogonal}: \underline{e}_1(\omega)^* P_x(\omega)\underline{e}_m(\omega) = \lambda_m(\omega)\delta(1-m)$$

where the Kronecker delta sequence $\delta(1\text{-}m)$ equals one for 1= m and is otherwise zero [9]. From this analysis, a fundamental theorem is now given.

Theorem 1. Let the MxN composite steering matrix $S(\omega)$ have rank N, the N x N envelope spectral density matrix $P_f(\omega)$ have rank K where $K \leqslant N$, and the M x M noise spectral density matrix $P\eta(m)$ have rank M. It then follows that the eigenvalues as specified in relationship (12) are distributed as

$$\lambda_1(\omega) \geq \lambda_2(\omega) \geq \cdots \geq \lambda_K(\omega) > \lambda_{K+1}(\omega) = \cdots = \lambda_M(\omega) = \sigma^2(\omega) \quad (14)$$

A proof of this theorem is readily obtained from the substitution of Eq.(9) into Eq.(13) and the fact that $P_f(\omega)$ has rank K.

IV. Rotation of Subspaces

Let us first consider the rotation of one dimensional subspace in three dimensions. It is shown from figure 1 that a subspace Y_b can be rotated into a subspace Y_a with the orthogonal transformation T since both sets of points can be made collinear where Y_a and Y_b are one-dimensional subspace spanned by a vector $\underline{a}$ and $\underline{b}$, respectively. It is noted that T can be selected so that $\|\underline{a}-T\underline{b}\|_2$ is minimized and T preserves the norm of vector.

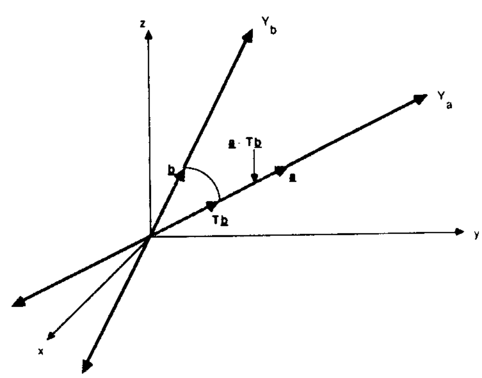

Fig.1. Rotation of one-dimensional subspace into the other one in three dimensional space.

This concept allows us to extend our geometric intuition to higher dimensions. The possibility that a subspace can be rotated into the other one is examined in the following theorem.

Theorem 2. Let A, B denote given matrices contained in the vector space $C^{m \times n}$ of all m xn complex matrices. Then the optimal unitary transformation matrix T which minimizes $\| A-TB \|_F$ subject to $T^*T=TT^* = I$, can be given by

$$T = VU^*$$

where $\| \cdot \|_F$ is Euclidean or Frobenius norm of matrix which is used as the distance measure, and

$$BA^* = U \Sigma V^*$$

$$U = [\underline{u}_1, \underline{u}_2, \cdots, \underline{u}_m], \quad V = [\underline{v}_1, \underline{v}_2, \cdots, \underline{v}_m]$$

$$\Sigma = \mathrm{diag}[\sigma_1, \sigma_2, \cdots, \sigma_m] \text{ where } \sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_m \geq 0$$

in which the σ_k 's are real nonnegative singular values which are ordered in the monotonically nonincreasing fashion $\sigma_k \geqslant \sigma_{k+1}$ and the $\underline{u}_k$ and $\underline{v}_k$ are the corresponding m x 1 orthonormal left and right singular vectors, respectively [4].

The proof is given in [6] and it is to be noted that T is not unique for the rank deficient matrix. Based on the theorem 2, the following observation can be made.

- The mapping $T: R(B) \rightarrow R(A)$ represents a rotation of the range of B into that of A as closely as possible, possibly preceded by a reflection in some hyperplane.
- If A is equal to B, then T is the identity matrix. The simple proof is shown in that V is equal to U since BA^* is Hermitian positive semidefinite matrix.
- If B is equal to identity matrix, then this problem is established as the orthogonalization of A.

It is to be noted in this optimization problem that the normalized error $\| A-TB \|_F / \| A-B \|_F$ may be relatively large even though R(A) matches R(TB) perfectly. This case results from the fact the unitary transformation matrix T provides the invariant norm, that is, $\| TB \|_F = \| B \|_F$.

The relationships between the normalized error and the matching of subspaces can be clearly explained by considering the distance (or angle) between two subspaces in terms of the orthonormal basis of subspace [6].

From the preceding discussion, it is apparent that the unitary transformation matrix can be optimally selected in order to rotate one subspace into the other one. We shall now consider whether this idea is effectively applicable to a signal subspace approach employed widely for direction-of-arrival estimation of multiple broadband plane waves.

V. Rotation of Signal Subspaces

The fundamental least-squares problem can now be formulated.

Least Squares Problem: Given the composite steering matrix $S(\omega_c)$ and $S(\omega_j)$, develop a procedure for selecting $T(m_c, m_j)$ which minimizes $\| S(\omega_c) - T(\omega_c, \omega_j) S(\omega_j) \|_F$, for j=1,2, ..., J, where J is the number of frequency subbands and ω_c is a frequency at which a common signal subspace is constructed. It is assumed that the number of incident plane waves N is unknown but that the plane wave direction-of-arrivals are such as to ensure that the MxN composite steering matrix $S(\omega)$ has full rank N.

To solve this least-squares problem, $S(\omega_c)$ and $S(\omega_j)$ should be known to select $T(\omega_c, \omega_j)$ for j= 1,2,... , J. Those matrices, however, are unknown to the processor. The focusing method, which has been proposed by Wang and Kaveh, is derived by using preliminary processing and spatial prefiltering in order to estimate an initial angle of source direction and separate the groups of sources. Specifically, two drawbacks of CSS focusing method are considered as follows

(1) it requires preliminary processing which can provide at least a good estimate of the direction-of-arrival before it can be applied.

(2) spatial prefiltering is required when some group of sources are far from the initial estimates obtained from preliminary processing

We shall now propose an effective procedure which doesnot require any preprocessing in order to find the transformation matrix $T(\omega_c, \omega_j)$. Furthermore, this is applicable to general array geometries.

Rotation of Signal Subspaces Algorithm

It is first assumed that the MxN composite steering matrix $S(\omega)$ has rank N and the NxN envelope spectral density matrix $P_f(\omega)$ has rank K, where $K \leqslant N$.

Corollary 3. Let $\underline{d}(\omega_c)$ and $\underline{d}(\omega_j)$ be given noise-free spectral snapshot vectors at frequency ω_c and ω_j, respectively. Then an optimal transformation matrix which minimizes $E\{\| \underline{d}(\omega_c) - T(\omega_c, \omega_j)\, \underline{d}(\omega_j) \|_F^2\}$ subject to $T(\omega_c, \omega_j)^* T(\omega_c, \omega_j) = I_{M\times M}$ can be expressed as

$$T(\omega_c, \omega_j) = V(\omega_c, \omega_j)\, U(\omega_c, \omega_j)^*$$

where

$$E\{\underline{d}(\omega_j)\, \underline{d}(\omega_c)^*\} = U(\omega_c, \omega_j) \textstyle\sum(\omega_c, \omega_j)\, V(\omega_c, \omega_j)^*$$

$$\underline{d}(\omega_k) = S(\omega_k)\, \underline{f}(\omega_k)$$

in which $I_{M\times M}$ denotes the MxM identity matrix. The proof is readily illustrated in[6]. It follows from the corollary 3 that

The constraint of the unitary matrix $T(\omega_c, \omega_j)$ is used to preserve the sensor power at frequency ω_j (i.e, $\|T(\omega_c, \omega_j)\, \underline{d}(\omega_j)\|_2 = \|\underline{d}(\omega_j)\|$). $T(\omega_c, \omega_j)$ can be considered as the transformation matrix achieved by the rotation of

(1) N-dimensional signal subspace for N non-coherent (or partially coherent) sources.
(2) K-dimensional signal subspace for some coherent sources (rank of $P_f(m) = K < N$).
(3) 1-dimensional signal subspace for perfect coherent sources.

The effect of ROSS algorithm is that of transforming random vector $\underline{x}(\omega_j)$ into the random vector $\underline{y}(\omega_c, \omega_j)$ where

$$\underline{y}(\omega_c, \omega_j) = T(\omega_c, \omega_j)\, \underline{x}(\omega_j) \qquad (15)$$

Taking the average of the weighted spectral density matrix of the random vector $\underline{y}(\omega_c, \omega_j)$, $P_y(\omega_c)$ is expressed as

$$P_y(\omega_c) = \sum_{j=1}^{J} w_j E\{\underline{y}(\omega_c, \omega_j)\, \underline{y}(\omega_c, \omega_j))_j)^*\} \qquad (16)$$

where w_j is a normalized weight proportional to j^{th} frequency band's SNR. Substituting Eq. (15) into Eq. (16),

$$P_y(\omega_c) = \sum_{j=1}^{J} w_j T(\omega_c, \omega_j)\, P_x(\omega_j)\, T(\omega_o, \omega_j)^* \quad (17)$$

Since transformed vector $\underline{y}(\omega_c, \omega_j)$ is assumed to theoretically belong to the signal subspace at frequency ω_c which is the column space of $S(\omega_c)$, $\underline{y}(\omega_c, \omega_j)$ corrupted by noise can be modelled as follows

$$\underline{y}(\omega_c, \omega_j) = S(\omega_c)\, \underline{a}(\omega_c, \omega_j) + T(\omega_c, \omega_j)\, \underline{\eta}(\omega_j) \qquad (18)$$

where $\underline{a}(\omega_c, \omega_j)$ is the Nx1 random vector which is determined by $T(\omega_c, \omega_j)$ and $\underline{f}(\omega_j)$. Upon the substitution of expression (18) into expression (16)

$$P_y(\omega_c) = \sum_{j=1}^{J} w_j S(\omega_c) E\{\underline{a}(\omega_c, \omega_j)\, \underline{a}(\omega_c, \omega_j)^*\} S(\omega_c)^* + \sum_{j=1}^{J} w_j T(\omega_c, \omega_j) E\{\underline{\eta}(\omega_j)\, \underline{\eta}(\omega)^*\} T(\omega_c, \omega_j)^* \qquad (19)$$

Rewriting relationship (19) under the assumption that the sensor noise has the same power in the all frequencies.

$$P_y(\omega_c) = S(\omega_c)\, P_{ay}(\omega_c)\, S(\omega_c)^* + \sigma^2 P_{\eta y}(\omega_c) \qquad (20)$$

where

$$P_{ay}(\omega_c) = \sum_{j=1}^{J} w_j E\{\underline{a}(\omega_c, \omega_j)\, \underline{a}(\omega_c, \omega_j)^*\}$$

$$P_{\eta y}(\omega_c) = \sum_{j=1}^{J} w_j T(\omega_c, \omega_j)\, P_\eta(\omega_j)\, T(\omega_c, \omega_j)^*$$

Since expression (20) has the same array model form as expression (9) at the frequency band ω_c, a generalized eigenanalysis of the matrix pencil $(P_y(\omega_c), P\eta y(\omega_c))$ is now made as follows

$$P_y(\omega_c)\, \underline{e}_m(\omega_c) = \lambda_m(\omega_c)\, P_{\eta y}(\omega_c)\, \underline{e}_m(\omega_c) \quad \text{for } m = 1, 2\cdots, M \qquad (21)$$

Upon the substitution of expression (20) into the relationship (21) ,it follows from Theorem 1. that

$$S(\omega_c)\, P_{ay}(\omega_c)\, S(\omega_c)^* = 0 \quad \text{for } K+1 \le m \le M$$

$$S(\omega_c)\, P_{ay}(\omega_c)\, S(\omega_c)^* = (\lambda_m(\omega_c) - \sigma^2)\, P_{\eta y}(\omega_c)\, \underline{e}_m(\omega_c) \quad \text{for } 1 \le m \le$$

Thus, it is clear that the signal eigenvectors associated with the K largest eigenvalues satisfy a linear relationship of the form

$$P_{\eta y}(\omega_c)\underline{e}_k(\omega_c) = \sum_{n=1}^{N} \alpha_k(n)\, \underline{s}(\omega_c, \underline{\kappa}_n) \quad \text{for } 1 \le k \le K \tag{22}$$

where $\underline{s}(\omega_c, \underline{\kappa}_n)$ is the Mx1 steering vector as specified in expression (6). The relationship (22) is a basic idea of the signal eigenvector method (SEM) and a general solution procedure is given in [1,2].

VI. Practical Considerations

Let us consider a set of sampled array sensor signals

$$x_m(1),\ x_m(2), \cdots x_m(L), \quad \text{for } 1 \le m \le M \tag{23}$$

with these samples being made over a interval of T seconds so that the uniform sampling rate is L/T samples per second. Using this raw data, it is then desired to estimate the array spectral desnity matrix $P_x(\omega)$. To effect this estimate, let these array sensor samples be subdivided into q equal subintervals each of duration $\Delta T = T/q$ seconds.

If ΔT is sufficiently large and decomposed components $\underline{x}(\omega_j)$ are uncorrelated, the array's spectral density matrix $P_x(\omega_j)$ of j^{th} frequency ω_j is then approximately given by

$$P_x(\omega_j) \cong q^{-1} \sum_{k=1}^{q} \underline{x}^k(\omega_j)\, \underline{x}^k(\omega_j)^* \quad \text{for } 0 \le j \le N_T - 1 \tag{24}$$

in which $m_j = 2\pi j / N_T$ [1, section IX]. It is noted that under the assumption that $\underline{x}(\omega_j)$ is a zero mean normal random vector, the maximum likelihood estimate of $P_x(\omega_j)$ is given by this expression [3]. In using ROSS algorithm to estimate the unitary transformation matrix, the set of narrowband components for which the SNR is deemed sufficiently large is used. Let ω^1, ω^2, ... , ω^J designate the set of frequencies within the signal bandwidth. The array's spectral density matrix estimates are given by

$$P_x(\omega^j) \cong q^{-1} \sum_{k=1}^{q} \underline{x}^k(\omega^j)\, \underline{x}^k(\omega^j)^* \quad \text{for } 1 \le j \le J \tag{25}$$

To estimate the transformation matrix for the case in which the noise field remains invariant with the measurement of array data matrix in the active system, the spectral snapshot vector is measured as

$$\underline{d}^k(\omega^j) = \underline{x}^k(\omega^j) - \underline{\eta}^k(\omega^j) \tag{26}$$

In a similar fashion to expression (25), $P_d(\omega_c, \omega^j) = E(\underline{d}(\omega^j)\, \underline{d}(\omega_c)^*)$ can be estimated as

$$E\{\underline{d}(\omega^j)\, \underline{d}(\omega_c)^*\} \cong q^{-1} \sum_{k=1}^{q} \underline{d}^k(\omega^j)\, \underline{d}^k(\omega_c)^* \tag{27}$$

in which J is the number of frequency bands to be analyzed.

VII. Simulation Experiments

To examine the effectiveness of the proposed ROSS algorithm relative to Wang-Kaveh's focusing method, forward-backward spatial smoothed MUSIC[12] and signal eigenvector method (SEM) in conjunction with focusing method, two cases are considered. One is for the case of three perfect coherent sources incident on a linear equally spaced array while the other is for resolving three incoherent sources incident on a linear unequally spaced array. In testing focusing method, initial angle is assumed to form the transformation matrix and no spatial prefiltering is used in order to process under the same conditions as that of ROSS algorithm. Furthermore, the transformation matrix of focusing method is treated as the diagonal form of a matrix since it is empirically found to perform much better than the other matrix [11]. The signal considered is the zero mean stationary bandpass white Gaussian random process of which a center frequency is ω_o= 100 Hz and bandwidth is 40 Hz. The sensor noise vector $\underline{\eta}(t)$ is taken as a complex valued additive bandpass white Gaussian process which has the same bandwidth as signal. Its components have identical variances and are statistically independent of the envelope signals. The uniform sampling frequency is chosen to be 80 Hz and the numbers of snapshots is taken to be L=3840 (i.e., T=48 sec.). The total observation time is divided into q=30 segments with each segment (ΔT=1.6 sec.) being decomposed into J=33 narrowband components within the signal bandwidth. A frequency ω_c for constructing a common signal subspace is chosen to be a center frequency (i.e., $\omega_c = \omega_o$). To obtain a measure of statistical repeatability, ten independent generations of the spatial spectral density matrix estimates are made and the bearing estimates are plotted in superimposed fashion for both cases.

Case 1: 3 perteat coherent sources.

In this example, the array is taken to be linear and composed of M=16 uniformly spaced sensors with a sensor spacing $d=c\pi/\omega_o$ where ω_o is a center frequency and c is a propagation velocity. The sensor noise variance is selected so that the two signal-to-noise ratio(SNR) levels of 0 dB and -8 dB is obtained. The seven subarrays are taken with the each subarray's size being ten in order to use a spatial smoothed MUSIC. In employing SEM, the effective rank of the modified spectral density matrix $P_y(\omega_c)$ is selected to be one. The initial angle is estimated as 79.5° for focusing method. The resultant bearing estimates are shown in figure 2. It is shown from figure 2 that forward-backward spatial smoothing with focusing method is unable to resolve consistantly the three incident coherent plane waves at 0 dB and -8 dB. Although not shown, this approach is unable to resolve three perfect coherent sources in a high SNR setting since the source at 66.5° has an undesirable effect of resolution of two spatially close sources. In the SEM with focusing method, a relatively lage bias is made at 78° in even high SNR and some of ten trial runs shows poor resolution at SNR -8 dB. On the other hand, it is seen that the proposed algorithm is able to effectively detect three coherent sources at each of SNR levels without appealing to initial angles. To compare the performance in terms of sampled bias, sampled standard deviation (STD), and mean squared error (MSE), thirty statistically independent trials are made. It is seen in Table 1 that the paper's method provides better performance than the other two methods.

Case 2:3 incoherent sources

The linear unequally spaced array is taken to be composed of M=11 sensors with a sensor spacing unit $d=2c\pi/3\omega_o$ The sensors are located in

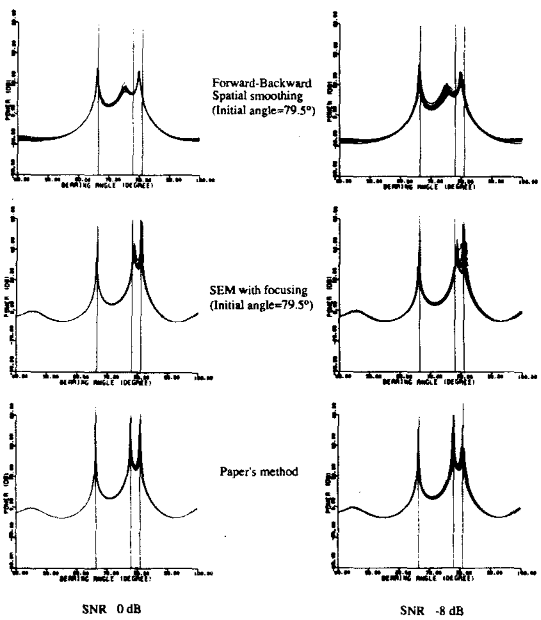

Fig.2. Ten statistically independent superimposed bearing estimates for three perfectly coherent incident plane waves at bearing angles of 66.5°, 78° and 81°. These estimates were obtained using a 16 element linear uniformly spaced array that had a spacing of $d=\lambda/2$.

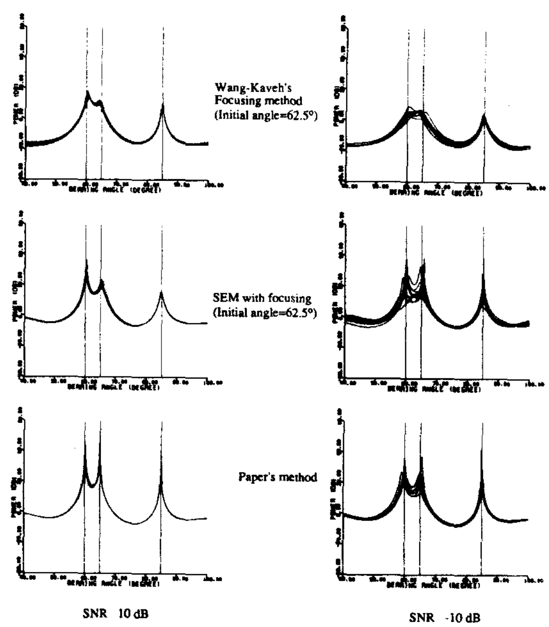

Fig.3. Ten statistically independent superimposed bearing estimates for three incoherent incident plane waves at bearing angles of 60°, 65° and 85°. These estimates were obtained using a 11 element linear unequally spaced array that had a spacing of $d=\lambda/3$.

$$\begin{cases} z_1=[-2.5,\ 0.0]^T, & z_2=[-0.6,\ 0.0]^T \\ z_3=[0.0,\ 0.0]^T, & z_4=[2.1,\ 0.0]^T \\ z_5=[3.5,\ 0.0]^T, & z_6=[5.6,\ 0.0]^T \\ z_7=[6.3,\ 0.0]^T, & z_8=[7.1,\ 0.0]^T \\ z_9=[8.4,\ 0.0]^T, & z_{10}=[10.2,\ 0.0]^T \\ z_{11}=[11.7,\ 0.0]^T \end{cases}$$

The simulation is performed with the two SNR of 10 dB and -10 dB. The effective rank of modified spectral density matrix is estimated to be three for both focusing method and ROSS algorithm and initial angle is estimated as 62.5° for Wang-Kaveh's focusing method. Upon comparing the bearing estimates shown in figure 3, Wang-Kaveh's method fails to detect consistantly two spatially close plane waves at SNR -10 dB while the proposed algorithm is better than SEM with focusing in terms of resolution and bias at SNR -10 dB. Table 2 is also given to examine the comparison of performance after thirty independent trials.

VIII. Conclusions

ROSS algorithm proposed herein has the desirable attributes of providing high resolution performance without appealing to preliminary processing and spatial prefiltering when this approach is applied to the situation in which the noise field remains invariant with the measurement of array snapshot vector. The proposed approach is applicable to general array geometries. Its performance has been found to be at least as good as that of focusing method and especially better in terms of bias and resolution when some sources are not in the neighborhood of the other groups of sources. The method which can be applicable to general situation (e.g., passive systems) is currently under study.

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Table 1. Comparison of Performance at SNR 0dB (Coherent Sources).

Degree	66.5			78			81		
Method	BIAS	STD	MSE	BIAS	STD	MSE	BIAS	STD	MSE
(A)	−0.3473	0.0639	0.1247	—	—	—	—	—	—
(B)	−0.1626	0.0435	0.0283	0.7027	0.1090	0.5058	0.0999	0.1828	0.0434
(C)	−0.0862	0.0608	0.0111	−0.0667	0.0990	0.0142	0.0957	0.1340	0.0271

Table 2. Comparison of Performance at SNR 10dB (Incoherent Sources)

Degree	60			65			85		
Method	BIAS	STD	MSE	BIAS	STD	MSE	BIAS	STD	MSE
(A)	0.4680	0.1890	0.2548	—	—	—	0.0055	0.0299	0.0009
(B)	0.3694	0.1462	0.1579	0.4957	0.1935	0.2833	−0.1167	0.0347	0.0148
(C)	0.2249	0.1057	0.0618	−0.0737	0.0624	0.0093	0.0235	0.0335	0.0017

(Note) The symbol "—" indicates the value below resolution threshold

(A) Forward-Backward Spatial Smoothing (in Table 2)
Wang-Kaveh's CSS Focusing Method (in Table 3)
(B) Signal Eigenvector Method in conjunction with Focusing method
(C) Paper's Method

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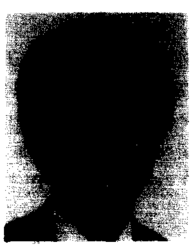

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