# A Two-Phase Pressure Drop Calculation Code Based on A New Method with a Correction Factor Obtained from an Assessment of Existing Correlations

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기존 상관관계식들의 평가를 통해 얻은 수정계수를 사용하는 새로운 방법에 기초한 2상류 압력강하 계산코드

> 전문헌 • 오재근 한국과학기술원 (1988, 11, 25 접수)

#### Abstract

Ten methods of the total two-phase pressure drop prediction based on five existing models and correlations have been examined for their accuracy and applicability to pressurized water reactor conditions. These methods were tested against 209 experimental data of local and bulk boiling conditions: Each correlations were evaluated for different ranges of pressure, mass velocity and quality, and best performing models were identified for each data subsets. A computer code entitled 'K-TWOPD' has been developed to calculate the total two-phase pressure drop using the best performing existing correlations for a specific property range and a correction factor to compensate for the predicted error of the selected correlations. Assessment of this code shows that the present method fits all the available data within  $\pm 11\%$  at a 95% confidence level compared with  $\pm 25\%$  for the existing correlations.

요 약

5개의 기존모델과 상관관계식에 기초한 10가지의 2상류 전체 압력강하 예측방법을 그 정확도와 가압경수로 운전 조건하에서의 적용도를 함께 검증하였다. 이 10가지 방법을 209개의 국부 및 전체비등 조건하의 실험치에 대해서 검증하였다. 각 상관관계식을 다른 범위의압력과 질량속도 및 건조도에 대해서 평가하여 각각의 적은 데이터 군(Subsets)에 가장 잘 맞는 모델을 찾아냈다. 주어진 상태량(Property) 범위에 대해 가장 잘 맞는 기존 상관관계식과 그 상관관계식이 가진 예측오차를 보정하기 위한 수정계수를 사용하여 2상류 전체 압력강하를 계산하는 'K-TWOPD'라고 명명한 전산코드를 개발하였다. 이 전산코드의 평가 결과를 보면 기존 상관관계식의 실험치와의 오차범위는 대체로 ±25%이상인데 비하여, 본 연구에서 제시한 방법은 사용한 모든 데이터를 95%의 확신도에서 ±11% 범위이내로 실험치와 일치하고 있다.

#### 1. Introduction

Although the average exit enthalpy of a current PWR core under normal operation does not exceed the saturation value, bulk boiling can occur in a hot channel. Moreover, in the event of a reactor transient, such as a small or a large break LOCA, bulk boiling can occur throughout the core. For both steady and transient conditions it is necessary to predict the pressure loss as accurately as possible for a two-phase mixture since pressure drop not only influences flow distribution and pumping power requirements but also the hot channel is limiting from DNB considerations.

Various approaches for two-phase flow pressure drop predictions have been proposed during the past two decades. The existing two-phase pressure drop models can be classified into: (a) homogeneous models [1–4], (b) separated flow models [5–6], and (c) mass flow correction models [7–8]. However, these existing models still have the following inherent drawbacks for application: (1) limited accuracy(e.g., the accuracy of the most widely used Martinelli-Nelson method has been quoted as  $\pm 35\%$ ), (2) limited applicable range of properties (such as pressure, mass velocity, and quality), and (3) often complicated and cumbersome numerical procedures (or the use of tables and figures) is required to obtain correction factors.

Idsinger et al. [9] compared a large number of existing correlations against measured data for conditions representing transient as well as steady-state BWR operation, and showed that the correlation which had the least RMS error overall for the normal BWR operation conditions is the Armand-Treschev Correlation [10]. Also, they reported that the correlations which have shown the best overall performance were (a) homogeneous theory, (b) the Thom [6], and (c) the Baroczy model [7].

The purpose of this paper is: first, to present the results of an assessment of existing correlations for their applicability to PWR conditions; and secondly, to present the outlines of a computer code entitled 'K-TWOPD' for two-phase pressure drop calculation based on the best performing existing models for specific property ranges and a correction factor to compensate for the predicted errors of selected correlations [1–8].

# 2. Summary of Existing Two-Phase Pressure Drop Correlations Examined

The existing pressure drop correlations may be classified into 3 broad categories: (1) the homogeneous model, (2) the separated flow model, and (3) the mass flow correction methods based on the homogeneous or separated flow models. A brief description of those models, that are selected for assessments of their applicability to PWR conditions and their accuracy of pressure drop predictions, is given here to aid in understanding and for convenience in discussion.

## The Homogeneous Model

The total static pressure gradient evaluated from the homogeneous model can be represented by the following expression [11]:

$$(\frac{dp}{dz}) = (\frac{dp}{dz}F) + (\frac{dp}{dz}a) + (\frac{dp}{dz}z)$$
 (1)

where

$$-(\frac{dp}{dz}F) = \frac{2f_{TP}G^2V_f}{D_e} \left[1 + x(\frac{V_{ig}}{V_f})\right]$$
 (2)

$$-\left(\frac{dp}{dz}a\right) = G^{2}\left[V_{ig}\frac{dx}{dz} + x\frac{dV_{g}}{dp}\left(\frac{dp}{dz}\right)\right] \tag{3}$$

$$-\left(\frac{\mathrm{d}p}{\mathrm{d}z}z\right) = \frac{g \sin \theta}{V_{\mathrm{f}}\left[1 + x\left(\frac{V_{\mathrm{fg}}}{V_{\mathrm{f}}}\right)\right]} \tag{4}$$

All the terms in Eqs. (1)–(4) are defineable except the two-phase friction factor  $f_{\rm TP}$ . To use the homogeneous model it is necessary to apply a suitably defined single-phase friction factor to two-

phase flow. A number of different approaches have been made to the definition of this two-phase friction factor: (1) Owens [1] has assumed that the friction factor  $f_{TP}$  is equal to that which would have occurred had total flow been assumed to be all liquid. (2) Another approach is to evaluate the friction factor  $f_{TP}$  using a mean two-phase viscosity  $\mu$  in the normal friction factor relationships. The three different forms of the relationship between  $\mu$  and the quality x proposed by McAdams et al. [2], Cicchitti et al. [3], and Dukler et al. [4] are

$$\frac{1}{\mu} = \frac{x}{\mu_g} + \frac{(1-x)}{\mu_f}$$
 (McAdams et al.) (5)

$$\mu = x \mu_g + (1 - x) \mu_i \text{ (Cicchitti et al.)}$$

$$\mu = \rho \left[ x V_g \mu_g + (1 - x) V_t \mu_t \right]$$
 (Dukler et al.) (7)

Thus, the methods proposed by Owens [1], McAdams et al. [2] and Dukler et al. [4] fall into the category of the homogeneous model.

# The Separated Flow Model

The total static pressure gradient as evaluated from the separated flow model can be represented by substitution of the following equations into Eq. (1):

$$-(\frac{dp}{dz}F) = \left[\frac{2f_{to}G^{2}V_{t}}{D_{t}}\right] \phi^{2}_{to}$$
 (8)

$$-\left(\frac{\mathrm{dp}}{\mathrm{dz}}a\right) = G^{2} \frac{\mathrm{d}}{\mathrm{dz}} \left[ \frac{x^{2}V_{a}}{\alpha} + \frac{(1-x)^{2}V_{t}}{(1-\alpha)} \right]$$
(9)

$$-\left(\frac{\mathrm{dp}}{\mathrm{dz}}z\right) = g \sin \theta \left[\alpha \rho_s + (1-\alpha)\rho_t\right]$$
 (10)

In order to apply Eqs.(8)–(10) it is necessary to develop expressions for the two-phase multiplier ( $\phi^2$ <sub>16</sub>) and the void fraction ( $\alpha$ ) in terms of the independent flow variables. As in the case of the homogeneous model a number of different approaches have been made to obtain  $\phi^2$ <sub>16</sub> and  $\alpha$ : (1) Martinelli-Nelson [5] presented values of  $\phi^2$ <sub>16</sub> as a function of mass quality and pressure (e.g., Fig. 2.4 and Table 2.2 in Ref. 11) and also values of  $\alpha$  as a function of mass quality x with pressure as parameter (e.g., Fig. 2.6 in Ref. 11). (2) Thom [6] proposed an alternative set of consistent

values for the two-phase frictional multiplier ( $\phi^2_{\text{fo}}$ ) as a function of pressure and mass quality (e.g., Table 2.3 in Ref.11). Also Thom [6] proposed to fit mass quality-void fraction curves of the type

$$\alpha = \frac{\gamma x}{1 + x(\gamma - 1)} \tag{11}$$

to their new data (Table 2.3 in Ref. 11) in which the slip factor  $\gamma$  is a constant at any given pressure.

# Mass Flow Correction Methods for Use with the Homogeneous or Separated Flow Models

Attempts to correct existing models for the influence of mass velocity on the friction multiplier  $\oint {}^2$ <sub>10</sub> have been published by Baroczy [7] and by Chisholm [8]: (1) The method of calculation proposed by Baroczy [7] employs two separate sets of curves. The first of these is a plot of the two-phase frictional multiplier  $\oint {}^2$ <sub>10</sub> as a function of a physical property index  $[(\mu_{\rm f}/\mu_{\rm g})^{0.2}(V_{\rm f}/V_{\rm g})]$  with mass quality x as parameter for a reference mass velocity of 1356 Kg/m²-s (e.g., Fig.2.12 and Table 2.5 in Ref.11). The second is a plot of a correction factor  $\Omega$  expressed as a function of the same physical property index for mass velocities of 339, 678, 2812, and 4068 Kg/m²-s with mass quality as parameter (e.g., Fig.2.13 in Ref. 11).

Thus.

$$(\frac{dp}{dz}F)_{TP} = \frac{2f_{lo}G^2V_i}{D_e} \phi^2_{lo} (G = 1356)^{\Omega}$$
 (12)

(2) Chisholm [8] has shown that the Lockhart-Martinelli [12] two-phase multiplier can be transformed with sufficient accuracy for engineering puroses to

$$\phi^{2}_{to} = 1 + (\Gamma^{2} - 1) \left[ Bx^{(2-n/2} + x^{2-n} \right]$$
 (13)  
where

$$\Gamma = \left[ -\frac{\left(\frac{dp}{dz}F\right)_{90}}{\left(\frac{dp}{dz}F\right)_{10}} \right]^{0.5}$$
 (14)

n=exponent in Blasius' relation for friction factor, and recommend values of B as a function of  $\Gamma$  and G (Tables 2 in Ref. 8).

# Summary of Correlations Examined

A summary of the correlations selected for assessments of their applicability to PWR conditions and their accuracy of pressure drop predictions is shown in Table 1. This table also shows that a different combination of frictional terms and

void fraction models were used in the 5 fundamental methods examined. This approach is taken becuase the use of different friction factor and void fraction models can obviously affect the pressure drops predicted by the same separated flow models, in particular.

Table I. A Summary of Two-Phase Pressure Drop Correlations Examined

Correlation Category (Authors) :	ID No.	Ref. No.	Frictional Terms (f <sub>TP</sub> or ${\phi^2}_{fo}$ )	Void Fraction Models
Homogeneous (Owens)	1	1	$f_{TP} = f_{fo}$	Homogeneous
Homogeneous (McAdams et al.)	2	2	Eq.5	Homogeneous
Homogeneous (Cicchitti et al.)	3	3	Eq.6	Homogeneous
Homogeneous (Dukler et al.)	4	4	Eq.7	Homogeneous
Separated Flow (Martinelli-Nelson)	5	5	Fig.2.4 in Ref.11	Fig.2.6 in Ref.11 Table 2.3 in Ref.11
Separated Flow (Thom)	6	6	Table 2.3. in Ref.11	Eq.11 Table 1 in Ref.6
Mass Flow Correction (Baroczy)	7	7	Fig.2.12 in Ref.11 Fig.2.13 in Ref.11	Fig.2.6 in Ref.11 Table 2.3 in Ref.11
Mass Flow Correction (Baroczy)	8	7	Fig.2.12 in Ref.11 Fig.2.13 in Ref.11	Eq.11 Table 1 in Ref.6
Mass Flow Correction (Chisholm)	9	8	Eq. 13 (or Eq.26 in Ref. 8)	Fig.2.6 in Ref.11 Table 2.3 in Ref.11
Mass Flow Correction (Chisholm)	10	8	Eq. 13 (or Eq.26 in Ref. 8)	Eq.11 Table 1 in Ref.6

# 3. Assessment of Existing Correlations and Discussions

### Methods Used to Evaluate Existing Correlations

For an objective evaluation and judgement of the relative superiority between the existing correlations it is necessary to define a quantitative parameter that can either measure or reveal the accuracy of each correlation. The most important parameters used to judge the relative superiority of a model in the present work is the fractional error, which is defined as the ratio of the error of the quantity to the true value of the quantity

$$\epsilon = \frac{(\Delta P)_{\text{corr}} - (\Delta P)_{\text{exp}}}{(\Delta P)_{\text{exp}}}$$
(15)

where  $\epsilon$  is the fractional error of the predicted value of the correlation,  $(\Delta P)_{corr}$  is the total amount of two-phase pressure drop calculated from the correlation, and  $(\Delta P)_{exp}$  is the total amount of two-phase pressure drop obtained from the experimental data which is assumed to be the true value.

In order to compare the accuracy of the predicted values  $(\Delta P)_{con}$  of the correlations for groups of data, the mean error  $\epsilon$ , root-mean-square (RMS) error  $\epsilon_{\rm RMS}$ , and standard deviation (SD) of the error from the mean  $\sigma_{\epsilon}$  were also computed using the following equations.:

$$\epsilon = \sum_{i=1}^{N} \frac{\epsilon_{i}}{N} = \frac{\epsilon_{1} + \epsilon_{2} + \dots + \epsilon_{N-1} + \epsilon_{N}}{N}$$
(16)

$$\epsilon_{\text{RMS}} = \left[ \sum_{i=1}^{N} \frac{\epsilon^{2}_{i}}{N} \right]^{\frac{1}{2}} \cdots (17)$$

$$\sigma_{\epsilon} = \left[\frac{1}{N} \sum_{i=1}^{N} (\epsilon_{i} - \epsilon)^{2}\right]^{\frac{1}{2}} = \left[(\epsilon_{RMS})^{2} - \epsilon^{2}\right]^{\frac{1}{2}}$$
(18)

Roughly speaking, the correlation that produces a smaller absolute values of  $\epsilon$ ,  $\epsilon_{RMS}$ , and  $\sigma_{\epsilon}$  of the predicted values ( $\Delta P$ )<sub>corr</sub> may be considered as a better correlation.

Experimental Data Used to Assess Existing Cor-

### relations

The experimental data represented several geometries (e.g., tube, annulus, rectangular channels, and rod array) and had the following property ranges:

Pressure(P) : 8.5–16.5 MN/m²

Mass Velocity(G) : 406.8–8000 Kg/m²-s

Exit Quality(x) : subcooled to 60%

Equivalent Diameter : 4.83–38.10 mm

All the data were subdivided into 14 subsets based on the property and flow conditions to study how each correlations behave for different ranges of pressure, mass velocity, and quality. The property/flow condition groupings combined data of similar pressure ranges, quality ranges, and mass velocity ranges. Table II shows the property and flow condition ranges from which 14 data subsets were formed.

Table II. The Ranges of Physical Properties and Flow Conditions Used to Form Data Subsets

Pressure	Mass Flux  Kg/m²-s	Quality	Number of Data	ID No. of Subsets
	G<1356	x<0.03 0.03 <u>&lt;</u> x<0.10 x <u>&gt;</u> 0.10	14 13 32	1 2 3
P <u>&lt;</u> 13.79 (P <u>&lt;</u> 2000 psia)	1356 <u>&lt;</u> G <u>&lt;</u> 2712	x<0.03 0.03<0.10 x>0.10	28 10 16	4 5 6
	G <u>&gt;</u> 2712	x<0.03 x≥0.03	22 12	7 8
P <u>&gt;</u> 13.79 (P <u>&gt;</u> 2000 psia)	G<1356	x<0.03 0.03 <x<0.10 x&gt;0.10</x<0.10 	19 25 21	9 10 11
	1356 <u>&lt;</u> G<2712	x<0.03 x≥0.03	30 14	12 13
	G <u>&gt;</u> 2712	All Region	22	14

## Calculation of Two-phase Pressure $Drop(\Delta P)_{corr}$

The total amount of two phase pressure drop from each correlation,  $(\Delta P)_{corr}$  in Eq.(15), is obtained by stepwise integration of Eqs. (1)–(4) and Eqs. (8)–(10). To reduce the uncertainty associated with the two-phase pressure drop correlations the local and bulk boiling regions were subdivided into a finite number of smaller differential control volumes and thermodynamic prop-

erties for each control volumes were found as a function of the temperature computed from a heat balance equation for the given region and system pressure. The whole process of computation for  $(\Delta P)_{corr}$  has been performed by the computer code entitled 'K-TWOPD'. The flow diagram of this code is given in Fig 1. Input data required for the 'K-TWOPD' to calculate  $(\Delta P)_{corr}$  and the sample result are shown in Table III.

Table III. Input Data For K-Twopd Code and Sample Results

Input Data	:Model ID no.		1
	Pipe Length	(m)	0.146
	Heat Flux	$(W/m^2)$	1.81x10 <sup>6</sup>
	De	(m)	0.0045
	Inlet Temperature	(degree C)	310.11
	Mass Flux	(Kg/m <sup>2</sup> -sec)	
	System Pressure	(Pa)	$1.013x10^8$
	e/De		0.0002
	Inclination Angle	(degree)	90.0
Output:	Void Departure Position		0.0396 m
	Saturation Position		0.162 m
	Bulk Boiling Position		0.247 m

Beginning Point (m)	Interval	Quality	dPf (Pa)	dPa (Pa)	dPg (Pa)
0.00	0.0396	0.00	26268.8	2895.8	2620.0
0.0396	0.0427	0.22	29647.2	16271.5	2688.9
0.0792	0.0427	0.66	30957.2	16340.4	2551.0
0.122	0.0427	1.09	32336.1	16340.4	2482.1
0.165	0.0427	1.53	33646.1	16340.4	2413.1
0.204	0.0427	1.97	34887.2	16340.4	2275.3
0.247	0.0335	2.61	29923.0	32060.4	1792.6
0.280	0.0335	3.47	31922.5	32060.4	1654.7
0.314	0.0335	4.32	33990.9	32060.4	1585.8
0.347	0.0335	5.18	35990.3	32198.2	1516.8
0.384	0.0335	6.03	37989.8	32198.2	1378.9
			357559.1	245175.5	23028.3

Total Pressure Drop =  $6.26 \times 10^5$  (Pa)

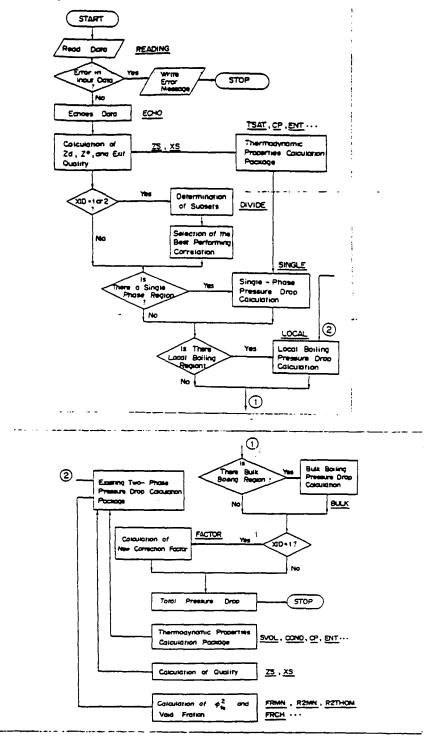


Fig.1 Flowchart of the Method for Evaluating a Single and Two-Phase Pressure Drop by the K-TWOPD Code.

# Comparison of Existing Correlations with Data

Table IV gives the results of the overall evaluation of existing correlations using 209 experimental data points: This table gives the mean (  $\varepsilon$  ), the RMS (  $\varepsilon$   $_{\text{RMS}}$ ), the standard deviation (  $\sigma$  ·) of the error, and the confidence limits of correlations at a 95% probability level. The correlations

are identified by the numbers indicated in Table I. The correlation errors appearing in Table IV refer to the uncertainty in the two-phase pressure drop based on data and the discrepancy between data and correlations. The errors are obtained by Eqs. (15)–(18).

Table IV. Results of Overall Evaluation of Existing Correlations

				Confidenc	e Limits(%)
Model ID No.	Mean Error	RMS Error	S.D. of Error	Upper	Lower
1	-0.07445	0.12507	0.10049	9.08616	-23.97585
2	-0.08043	0.13257	0.10538	9,29221	-25.37895
3	-0.08508	0.13367	0.10309	8.45094	-25.46660
4	-0.09164	0.13903	0.10455	8.03537	-26.36278
5	-0.01396	0.14710	0.14644	22.69316	-25.48523
6	-0.09435	0.14458	0.10955	8.58529	-27.45583
7	-0.06085	0.10657	0.08749	8.30697	-20.47734
8	-0.05257	0.10995	0.09657	10.62806	-21.14264
9	-0.05155	0.09934	0.08493	8.81569	-19.12494
10	-0.04379	0.10469	0.09509	11.26289	-20.02135

Table V. Mean Errors, RMS Errors, and Standard Deviations of the Error of Existing Correlations for Data Subsets

2   3   4   5   6   7   8   9   10		model										
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			1	2	3	4	5	6	7	8	9	10
1   E_RMS   0.0573   0.0573   0.0573   0.0575   0.0587   0.0575   0.0575   0.0575   0.0575   0.0575   0.0597   0.0599   0.0775   0.0595   0.0597   0.0595   0.0591   0.0597   0.0640   0.0534   0.05975   0.0597   0.059	100		-0.0427	-0.0436	-00467	-0.0450	-0.0374	-0.0458	-0.0410	-0.0313	-0.0425	-0.0324
Get/c         0.0370         0.0369         0.0359         0.0352         0.0415         0.0490         0.0404         0.0534         0.0377         0.053           E         P. D.1169         -0.1179         -0.1202         -0.1284         0.0174         0.0691         -0.0714         0.0385         -0.0774         0.0615         0.0015         0.1031         0.1071           T         0.0622         0.0687         0.0695         0.0700         0.0771         0.1666         0.0999         0.07724         0.103           3         EMAS         0.2522         0.2331         -0.2460         -0.2994         -0.1054         -0.0851         0.0931         -0.0653         -0.0675           σ <sub>E</sub> 0.0835         0.0812         0.0213         0.0160         0.0808         0.1432         0.0955         0.0051         0.0931         0.0199         0.0109         0.0100         0.0100         0.0177         0.0776         -0.0853         -0.0853         -0.0853         -0.0853         0.0081         0.0199         0.0100         0.0177         0.0080         0.0177         0.0081         0.01199         0.1019         0.0119         0.0000         0.0000         0.0000         0.0000         0.0000         0.00	1					i						0.0629
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	-					1			1		l .	0.0536
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$					-0.1207	-0.1238		-0.0976	-0.0741	-0.0385	-0.0734	
σ <sub>t</sub> 0.0682         0.0687         0.0695         0.0700         0.0778         0.1095         0.0661         0.0939         0.0724         0.106           ε         -0.2212         -0.2311         -0.2584         -0.2594         -0.1054         -0.2566         -0.1199         -0.0981         -0.0681         -0.081           σ <sub>t</sub> 0.0830         0.0818         0.0806         0.0808         0.1432         0.0851         0.0851         0.0931         -0.1228         0.119           4         ε <sub>RMS</sub> 0.1092         0.1093         -0.1044         0.1096         0.1070         0.1181         0.1087         0.0142         0.1097         -0.0853         -0.0898         -0.0155         -0.0674         0.0676         0.0677         0.0756         -0.0987         -0.0853         -0.0898         -0.0154         -0.0164         -0.1096         0.1070         0.1181         0.1087         0.0164         0.1146         0.1141         0.1016         0.0757         0.0664         0.0757         0.0673         0.0672         0.0684         0.0673         0.0674         0.0264         0.0560         0.0840         0.0684         0.0673         0.0167         0.0664         0.0553         0.0510         0.0631	2		j				ļ				i	0.1073
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	_							ł .			i	0.1005
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$			-0.2212	-0.2311	-0.2460	-0.2594	-0.1054	-0.2456	-0.1199	-0.0981	-0.0685	-0.0470
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	3	-	1	í			1	l				0.1183
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			0.0830	0.0618	0.0406	0.0808	0.1432	0.0985	0.0851	0.0931	0.1019	0.1046
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		₹	-0.0858	-0.0859	-0.0864	-0.0870	-0.0756	-0.0987	-0.0853	-0.0898	-0.0858	-0.0903
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	4	ERMS	0.1092	0.1093	0.1094	0.1096	0.1070	0.11\$1	0.1087	0.1142	1	0.1140
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			0.0676	0.0675	0.0672	0.0666	0.0757	0.0648	0.0673	0.0705	0.0662	0.0695
σ̄ <sub>E</sub> 0.0453         0.0479         0.0469         0.0479         0.0568         0.0535         0.0510         0.0637         0.0480         0.062           ₹         -0.0559         -0.0742         -0.1044         -0.1334         0.1583         -0.1432         -0.1046         -0.0901         -0.0995         -0.082           6         € <sub>RMS</sub> 0.07789         0.0912         0.1160         0.1421         0.2161         0.1546         0.1095         0.1095         0.1228         0.113           σ <sub>E</sub> 0.0557         0.0531         0.0506         0.0488         0.1471         0.0583         0.0604         0.0623         0.0720         0.073           7         € <sub>RMS</sub> 0.1156         0.1156         0.1156         0.1156         0.1157         0.0731         0.0971         0.0924         -0.0841         -0.0841         0.0654         0.0809         0.0774         0.0612         -0.0924         -0.0841         -0.0811         -0.0115         0.1168         0.1156         0.1156         0.1156         0.1156         0.0715         0.0715         0.0215         0.0216         0.0217         0.0216         0.2000         -0.0922         -0.0990         -0.1861         -0.0916         0.0		ξ	-0.0069	-0.0135	-0.0167	-0.0291	0.0620	-0.0588	0.0187	0.0107	0.0026	-0.0053
\$\begin{array}{c c c c c c c c c c c c c c c c c c c	5	E <sub>RMS</sub>	0.0459	0.0497	0.0498	0.0560	0.0840	0.0866	0.0543	0.0646	0.0481	0.0653
6 ε <sub>RMS</sub> 0.0789 0.0912 0.1160 0.1421 0.2161 0.1546 0.1208 0.1095 0.1228 0.1135 σ <sub>E</sub> 0.0557 0.0531 0.0506 0.0488 0.1471 0.0583 0.0604 0.0623 0.0720 0.0720  Ē -0.0873 -0.0873 -0.0875 -0.0900 -0.0730 -0.0945 -0.0886 -0.0984 -0.0831 0.095 7 ε <sub>RMS</sub> 0.1156 0.1156 0.1156 0.1157 0.1215 0.1168 0.1157 0.1181 0.1160 0.116 σ <sub>E</sub> 0.0758 0.0757 0.0755 0.0743 0.0971 0.0586 0.0744 0.0654 0.0809 0.070  Ē -0.0399 0.0330 0.0157 -0.0054 0.0900 -0.0928 -0.0990 -0.1360 0.0601 0.0991  ε̄ ε <sub>RMS</sub> 0.1276 0.1215 0.1088 0.1004 0.2971 0.1411 0.1334 0.1564 0.1138 0.135 σ <sub>E</sub> 0.1212 0.1169 0.1077 0.1002 0.2197 0.1063 0.0893 0.0771 0.0955 0.081  Θ ε̄ <sub>RMS</sub> 0.0506 0.0506 0.0507 0.0509 0.0495 0.0588 0.0496 0.0539 0.0499 0.055 σ <sub>E</sub> 0.0406 0.0405 0.0404 0.0402 0.0406 0.0470 0.0399 0.0492 0.0384 0.041  Ε̄ -0.0375 -0.0381 0.0787 0.0783 0.0792 0.0994 0.0744 0.0851 0.0754 0.084 σ <sub>E</sub> 0.0778 0.0781 0.0787 0.0783 0.0792 0.0994 0.0744 0.0851 0.0754 0.084 σ <sub>E</sub> 0.1134 -0.1155 0.0869 0.0839 0.1397 0.1639 0.0994 0.0744 0.0851 0.0772 0.084  Ε̄ -0.1134 -0.1155 0.0819 0.0839 0.0792 0.0994 0.0744 0.0851 0.0754 0.084 σ <sub>E</sub> 0.1373 0.1379 0.1389 0.1397 0.1639 0.1725 0.1538 0.1586 0.1487 0.1539 0.1595 0.1535 0.155 σ <sub>E</sub> 0.1373 0.1379 0.1389 0.1397 0.1639 0.1725 0.1538 0.1586 0.1487 0.153  Ε̄ -0.0837 -0.0837 -0.0841 -0.0846 -0.0724 -0.0944 -0.0734 -0.0913 0.0955 0.1535 0.155 σ <sub>E</sub> 0.0772 0.0702 0.0699 0.1099 0.1099 0.1189 0.1073 0.1146 0.1070 0.114 σ <sub>E</sub> 0.0702 0.0702 0.0699 0.1099 0.1099 0.1189 0.1073 0.1146 0.1070 0.114 σ <sub>E</sub> 0.0703 0.0815 0.0833 0.0752 0.0992 0.0618 0.0990 0.0827 0.0809 0.0724 0.0641  Ε -0.0837 -0.0899 0.0902 -0.0902 0.0906 0.1005 0.1243 0.1351 0.1100 0.1013		σε	0.0453	0.0479	0.0469	0.0479	0.0568	0.0635	0.0510	0.0637	0.0480	0.0651
σ <sub>E</sub> 0.0557         0.0531         0.0566         0.0483         0.1471         0.0583         0.0604         0.0623         0.0720         0.0772           E         -0.0873         -0.0873         -0.0875         -0.0900         -0.0730         -0.0486         -0.0984         -0.0831         -0.097           7         ε <sub>RMS</sub> 0.0156         0.1156         0.1156         0.1156         0.1156         0.1167         0.1215         0.1168         0.1157         0.1181         0.1160         0.116           σ <sub>E</sub> 0.0399         0.0330         0.0157         -0.0054         0.2000         -0.0928         -0.0990         -0.1360         -0.0618         -0.0991           ε̄ ε <sub>RMS</sub> 0.1276         0.1215         0.1088         0.1004         0.2971         0.1411         0.1340         -0.0618         -0.0991           ε̄ ε <sub>RMS</sub> 0.1276         0.1215         0.1088         0.1004         0.2971         0.1441         0.1334         0.1564         0.1138         0.133           ε̄ ε <sub>RMS</sub> 0.1212         0.1169         0.1077         0.0102         0.2197         0.1063         0.0893         0.0771         0.0959         0.0959         0.0959         0.0		£	-0.0559	-0.0742	-0.1044	-0.1334	0.1583	-0.1432	-0.1046	-0.0901	-0.0995	-0.0850
E         -0.0873         -0.0873         -0.0875         -0.0900         -0.0730         -0.0945         -0.0826         -0.0984         -0.0831         -0.097           σ <sub>E</sub> 0.0758         0.0757         0.0755         0.0743         0.0971         0.0586         0.0744         0.0654         0.0809         0.076           E         0.0999         0.0330         0.0157         -0.0054         0.2000         -0.0928         -0.0990         -0.1360         -0.0618         -0.091           E         0.0399         0.0330         0.0157         -0.0054         0.2000         -0.0928         -0.0990         -0.1360         -0.0618         -0.091           E         0.0216         0.1215         0.1021         0.1004         0.2971         0.1411         0.1334         0.1564         0.1138         0.13           σ <sub>E</sub> 0.1212         0.1169         0.1077         0.1002         0.2197         0.1063         0.0893         0.0771         0.0955         0.081           σ <sub>E</sub> 0.0306         0.0506         0.0507         0.0509         0.0495         0.0582         0.0496         0.0539         0.0492         0.0539         0.0492         0.0539         0.0492 <th< th=""><th>6</th><th>ERMS</th><th>0.0789</th><th>0.0912</th><th>0.1160</th><th>0.1421</th><th>0.2161</th><th>0.1546</th><th>0.1208</th><th>0.1095</th><th>0.1228</th><th>0.1131</th></th<>	6	ERMS	0.0789	0.0912	0.1160	0.1421	0.2161	0.1546	0.1208	0.1095	0.1228	0.1131
7 ε <sub>RMS</sub> 0.1156 0.1156 0.1156 0.1157 0.1215 0.1168 0.1157 0.1181 0.1160 0.116 σ <sub>ε</sub> 0.0758 0.0757 0.0755 0.0743 0.0971 0.0686 0.0744 0.0654 0.0809 0.076  E 0.0999 0.0330 0.0157 -0.0054 0.2000 -0.0922 -0.0990 -0.1360 -0.0618 -0.091  E ε <sub>RMS</sub> 0.1276 0.1215 0.1082 0.1004 0.2971 0.1411 0.1334 0.1564 0.1138 0.132 σ <sub>ε</sub> 0.1212 0.1169 0.1077 0.1002 0.2197 0.1063 0.0893 0.0771 0.0955 0.081  E -0.0302 -0.0303 -0.0307 -0.0312 -0.0283 -0.0354 -0.0295 -0.0721 -0.0318 -0.029 σ <sub>ε</sub> 0.0406 0.0506 0.0507 0.0509 0.0495 0.0582 0.0496 0.0539 0.0499 0.054 σ <sub>ε</sub> 0.0406 0.0405 0.0404 0.0402 0.0406 0.0470 0.0399 0.0492 0.0384 0.041  E -0.0375 -0.0381 -0.0393 -0.0407 -0.0216 -0.0271 -0.0140 0.0128 -0.0199 0.064 σ <sub>ε</sub> 0.0778 0.0781 0.0787 0.0795 0.0763 0.0956 0.0731 0.0841 0.0727 0.084  E -0.1134 -0.1155 -0.1192 -0.1227 -0.0705 -0.1054 -0.0415 0.1799 0.1553 0.1553 σ <sub>ε</sub> 0.1373 0.1379 0.1389 0.1397 0.1639 0.1725 0.1538 0.1586 0.1487 0.159  E -0.0836 -0.0837 -0.0841 -0.0846 -0.07724 -0.0913 -0.0784 -0.0913 -0.0778 0.0702 0.0599 0.1099 0.009		σε	0.0557	0.0531	0.0506	0.0488	0.1471	0.0583	_ 0.0604	0.0623	0.0720	0.0746
σ <sub>E</sub> 0.0758         0.0757         0.0755         0.0743         0.0971         0.0886         0.0744         0.0854         0.0809         0.078           E         0.0399         0.0330         0.0157         -0.0054         0.2000         -0.0928         -0.0990         -0.1360         -0.0618         -0.091           E         0.1276         0.1215         0.1082         0.1004         0.2971         0.1411         0.1334         0.1564         0.1138         0.13           σ <sub>E</sub> 0.1212         0.1169         0.1077         0.1002         0.2197         0.1063         0.0893         0.0771         0.0955         0.089           9         ε <sub>RMS</sub> 0.0506         0.0506         0.0507         0.0509         0.0495         0.0528         0.0496         0.0539         0.0499         0.0496         0.0539         0.0499         0.0496         0.0539         0.0492         0.0496         0.0539         0.0499         0.0492         0.0349         0.0549         0.0549         0.0406         0.0470         0.0399         0.0492         0.0344         0.0412         0.0406         0.0492         0.0466         0.0539         0.0752         0.0744         0.0813         0.0199		E	-0.0873	-0.0673	-0.0675	-0.0900	-0.0730	-0.0945	-0.0886	-0.0984	-0.0831	-0.0929
\$\begin{array}{c c c c c c c c c c c c c c c c c c c	7	<sup>E</sup> R⊁(S	1	0.1156		0.1167	0.1215	0.1168	0.1157	0.1181	0.1160	0.1164
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		σε	0.0758	0.0757	0.0755	0.0743	0.0971	0.0686	0.0744	0.0654	0.0809	0.0700
σ <sub>E</sub> 0.1212         0.169         0.1077         0.1002         0.2197         0.1063         0.0893         0.0771         0.0955         0.081           ε         -0.0302         -0.0303         -0.0307         -0.0312         -0.0223         -0.0225         -0.0221         -0.0312         -0.0234         -0.0295         -0.0221         -0.0318         -0.02         -0.02         -0.0496         0.0599         0.0496         0.0582         0.0496         0.0539         0.0499         0.05         0.05         0.0582         0.0496         0.0539         0.0499         0.05         0.05         0.0582         0.0496         0.0539         0.0492         0.0384         0.049         0.05         0.05         0.0492         0.0492         0.0384         0.049         0.05         0.05         0.0339         0.0492         0.0384         0.049         0.05		Ŧ	1	1	1	-0.0054	0.2000	-0.0928	-0.0990	-0.1360	-0.0618	-0.0988
\$\begin{array}{c c c c c c c c c c c c c c c c c c c	3		ì	Į.	l	l .	1		0.1334	<b>F</b>	0.1138	0.1328
9 ε <sub>χMS</sub> 0.0506 0.0506 0.0507 0.0509 0.0495 0.0588 0.0496 0.0539 0.0499 0.054  σ <sub>ξ</sub> 0.0406 0.0405 0.0404 0.0402 0.0406 0.0470 0.0399 0.0492 0.0384 0.044  10 ε <sub>χMS</sub> 0.0863 0.0869 0.0820 0.0893 0.0792 0.0994 0.0744 0.0851 0.0754 0.084  σ <sub>ξ</sub> 0.0778 0.0781 0.0787 0.0795 0.0763 0.0956 0.0731 9.0841 0.0727 0.084  11 ε <sub>χMS</sub> 0.1781 0.1799 0.1831 0.1860 0.1785 0.2021 0.1593 0.1595 0.1553 0.155  σ <sub>ξ</sub> 0.1373 0.1379 0.1389 0.1397 0.1639 0.1725 0.1538 0.1586 0.1487 0.153  τ -0.0836 -0.0837 -0.0841 -0.0846 -0.0724 -0.0984 -0.0784 -0.0913 -0.0788 -0.091  12 ε <sub>χMS</sub> 0.1092 0.1093 0.1094 0.1096 0.1059 0.1189 0.1073 0.1146 0.1070 0.114  σ <sub>ξ</sub> 0.0702 0.0702 0.0699 0.0696 0.0772 0.0669 0.0733 0.0693 0.0724 0.0681  τ -0.0638 -0.0677 -0.0718 -0.0706 -0.0308 -0.1240 -0.0629 -0.0768 -0.0647 -0.0871  σ <sub>ξ</sub> 0.0703 0.0815 0.0833 0.0752 0.0992 0.0618 0.0990 0.0527 0.0809 0.074  ε <sub>χMS</sub> 0.1095 0.1097 0.1102 0.1015 0.1015 0.1260 0.1243 0.1351 0.1140 0.124			0.1212	0.1169	0.1077	0.1002	0.2197	0.1063	0.0893	0.0771	0.0955	0.0887
G <sub>E</sub> 0.0406         0.0405         0.0404         0.0402         0.0406         0.0470         0.0399         0.0492         0.0384         0.041           E         -0.0375         -0.0381         -0.0393         -0.0407         -0.0216         -0.0271         -0.0140         0.0128         -0.0199         0.006           G <sub>E</sub> 0.0778         0.0781         0.0787         0.0795         0.0763         0.0956         0.0731         0.0841         0.0727         0.084           I1         ε <sub>RMS</sub> 0.1781         0.1755         -0.1192         -0.1227         -0.0705         -0.1054         -0.0415         -0.0170         -0.0449         -0.022           I1         ε <sub>RMS</sub> 0.1781         0.1799         0.1831         0.1860         0.1785         0.2021         0.1593         0.1595         0.1553         0.155         0.1553         0.152         0.1533         0.1586         0.1487         0.153         0.158         0.1586         0.1487         0.153         0.158         0.1585         0.1586         0.1487         0.153         0.153         0.1586         0.1487         0.153         0.153         0.158         0.1586         0.1487         0.153         0.091			1		1	i .	1	1	-0.0295	-0.0221	-0.0318	-0.0245
T	9		i		i	1	1	i	i	1	!	0.0543
10   Z <sub>RMS</sub>   0.0863   0.0869   0.0890   0.0893   0.0792   0.0994   0.0744   0.0851   0.0754   0.084     σ <sub>E</sub>   0.0772   0.0781   0.0787   0.0795   0.0763   0.0956   0.0731   9.0841   0.0727   0.084     E   -0.1134   -0.1155   -0.1192   -0.1227   -0.0705   -0.1054   -0.0415   -0.0170   -0.0449   -0.022     11   Z <sub>RMS</sub>   0.1781   0.1799   0.1831   0.1860   0.1785   0.2021   0.1593   0.1595   0.1553   0.155     σ <sub>E</sub>   0.1373   0.1379   0.1389   0.1397   0.1639   0.1725   0.1538   0.1586   0.1487   0.153     12   Z <sub>RMS</sub>   0.1092   0.1093   0.1094   0.1096   0.1059   0.1189   0.1073   0.1146   0.1070   0.114     σ <sub>E</sub>   0.0702   0.0702   0.0699   0.0696   0.0772   0.0669   0.0733   0.0693   0.0724   0.081     13   Z <sub>RMS</sub>   0.1010   0.1059   0.1100   0.1031   0.0972   0.1385   0.1173   0.1128   0.1036   0.114     σ <sub>E</sub>   0.0783   0.0815   0.0833   0.0752   0.0922   0.0618   0.0990   0.0827   0.0809   0.074     14   Z <sub>RMS</sub>   0.1095   0.1097   0.1102   0.1120   0.1015   0.1026   0.1243   0.1251   0.1140   0.1240     14   Z <sub>RMS</sub>   0.1095   0.1097   0.1102   0.1120   0.1015   0.1260   0.1243   0.1251   0.1140   0.1240     14   Z <sub>RMS</sub>   0.1095   0.1097   0.1102   0.1120   0.1015   0.1260   0.1243   0.1251   0.1140   0.1240     15   0.0813   0.0915   0.1097   0.1100   0.1015   0.1260   0.1243   0.1251   0.1140   0.1240     15   0.0813   0.0915   0.1097   0.1100   0.1015   0.1260   0.1243   0.1251   0.1140   0.1240     15   0.0813   0.0815   0.0813   0.0815			0.0406	0.0405	0.0404	0.0402	0.0406	0.0470	0.0399	0.0492	0.0384	0.0484
σ <sub>E</sub> 0.0778         0.0781         0.0787         0.0795         0.0763         0.0956         0.0731         0.0841         0.0727         0.084           Ε         -0.1134         -0.1155         -0.1192         -0.1227         -0.0705         -0.1054         -0.0415         -0.0170         -0.0449         -0.024           11         ε <sub>RMS</sub> 0.1781         0.1799         0.1831         0.1860         0.1785         0.2021         0.1593         0.1595         0.1553         0.1553         0.1553         0.1553         0.1553         0.1553         0.1553         0.1586         0.1487         0.153         0.1586         0.1487         0.153         0.1586         0.1487         0.153         0.1586         0.1487         0.153         0.1586         0.1487         0.153         0.1586         0.1487         0.153         0.1586         0.1487         0.153         0.1586         0.1487         0.153         0.1586         0.1487         0.153         0.158         0.1586         0.1487         0.153         0.091         0.091         0.091         0.091         0.091         0.091         0.091         0.091         0.091         0.091         0.091         0.091         0.091         0.091	٠,		1	ŧ	ì	i	j	1	I.	0.0128	-0.0199	0.0068
E         -0.1134         -0.1155         -0.1192         -0.1227         -0.0705         -0.1054         -0.0415         -0.0170         -0.0449         -0.022           11         ε <sub>RMS</sub> 0.1781         0.1799         0.1831         0.1860         0.1785         0.2021         0.1593         0.1595         0.1553         0.1553         0.155           σ <sub>E</sub> 0.1373         0.1379         0.1389         0.1397         0.1639         0.1725         0.1538         0.1586         0.1487         0.153           12         ε <sub>RMS</sub> 0.1092         0.1093         0.1094         0.1096         0.1059         0.1189         0.1073         0.1146         0.1070         0.114           σ <sub>E</sub> 0.0702         0.0702         0.0699         0.0696         0.0772         0.0669         0.0733         0.0693         0.0724         0.081           13         ε <sub>RMS</sub> 0.1010         0.1059         0.1001         0.0972         0.1385         0.1173         0.1128         0.0647         -0.087           σ <sub>E</sub> 0.0783         0.0815         0.0833         0.0752         0.0922         0.0518         0.0990         0.0827         0.0667         -0.086	10			l .		ł	j	I	ì	ł	İ	0.0845
11 ε <sub>RMS</sub> 0.1781 0.1799 0.1831 0.1860 0.1785 0.2021 0.1593 0.1595 0.1553 0.155  σ <sub>ε</sub> 0.1373 0.1379 0.1389 0.1397 0.1639 0.1725 0.1538 0.1586 0.1487 0.153  τ -0.0836 -0.0837 -0.0841 -0.0846 -0.0724 -0.0984 -0.0784 -0.0913 -0.0788 -0.091  12 ε <sub>RMS</sub> 0.1092 0.1093 0.1094 0.1096 0.1059 0.1189 0.1073 0.1146 0.1070 0.114  σ <sub>ε</sub> 0.0702 0.0702 0.0669 0.06772 0.0669 0.0733 0.0693 0.0724 0.064  τ -0.0638 -0.0677 -0.0718 -0.0706 -0.0308 -0.1240 -0.0629 -0.0768 -0.0647 -0.087  13 ε <sub>RMS</sub> 0.1010 0.1059 0.1100 0.1031 0.0972 0.1385 0.1173 0.1128 0.1036 0.114  σ <sub>ε</sub> 0.0783 0.0815 0.0833 0.0752 0.0922 0.0618 0.0990 0.0827 0.0809 0.074  τ -0.0897 -0.0899 -0.0902 -0.0928 -0.0796 -0.1026 -0.0983 -0.1083 -0.0906 -0.1040 0.1241			<del> </del>	<del> </del>	<del> </del>	<del></del>	<del> </del>		0.0731	9.0841	0.0727	0.0842
σ <sub>E</sub> 0.1373         0.1379         0.1389         0.1397         0.1639         0.1725         0.1538         0.1586         0.1487         0.153           τ         -0.0836         -0.0837         -0.0846         -0.0724         -0.0984         -0.0784         -0.0913         -0.0788         -0.091           12         ε <sub>RMS</sub> 0.1092         0.1093         0.1094         0.1096         0.1059         0.1189         0.1073         0.1146         0.1070         0.114           σ <sub>E</sub> 0.0702         0.0702         0.0649         0.0696         0.0772         0.0669         0.0733         0.0693         0.0724         0.061           13         ε <sub>RMS</sub> 0.1010         0.1059         0.1100         0.1031         0.0972         0.1385         0.1173         0.1128         0.1036         0.114           σ <sub>E</sub> 0.0783         0.0815         0.0833         0.0752         0.0922         0.0618         0.0990         0.0827         0.0809         0.074           ε         -0.0897         -0.0899         -0.0902         -0.0796         -0.1026         -0.0983         -0.1083         -0.0908         -0.102           14         ε <sub>RMS</sub> 0.1095 <th>,,</th> <th>_</th> <th>1</th> <th></th> <th>1</th> <th>i</th> <th>1</th> <th>1</th> <th>i</th> <th></th> <th>1</th> <th>-0.0205</th>	,,	_	1		1	i	1	1	i		1	-0.0205
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12 ε <sub>RMS</sub> 0.1092 0.1093 0.1094 0.1096 0.1059 0.1189 0.1073 0.1146 0.1070 0.1146 σ <sub>E</sub> 0.0702 0.0702 0.0669 0.0772 0.0669 0.0733 0.0693 0.0724 0.0669 0.0733 0.0693 0.0724 0.0669 0.0733 0.0693 0.0724 0.0669 0.0733 0.0693 0.0724 0.0669 0.0733 0.0693 0.0724 0.0669 0.0733 0.0693 0.0724 0.0669 0.0733 0.0693 0.0724 0.0669 0.0724 0.0669 0.0733 0.1126 0.1146 0.1076 0.1146 0.1076 0.1146 0.1076 0.1146 0.1076 0.1146 0.1076 0.1146 0.1076 0.1146 0.1076 0.1146 0.1076 0.1146 0.1076 0.1146 0.1076 0.1146 0.1076 0.1146 0.1076 0.1146 0.1076 0.1146			<del> </del>	<del> </del>	<del> </del>	<del></del>	<del> </del>	<del> </del>	<del> </del>		<del></del>	
σ <sub>E</sub> 0.0702         0.0702         0.0699         0.0696         0.0772         0.0669         0.0733         0.0693         0.0724         0.061           E         -0.0638         -0.0577         -0.0718         -0.0706         -0.0308         -0.1240         -0.0629         -0.0768         -0.0647         -0.087           13         ε <sub>RMS</sub> 0.1010         0.1059         0.3100         0.1031         0.0972         0.1385         0.1173         0.1128         0.1036         0.114           σ <sub>E</sub> 0.0783         0.0815         0.0833         0.0752         0.0922         0.0618         0.0990         0.0827         0.0809         0.074           ξ         -0.0897         -0.0899         -0.0902         -0.0928         -0.0796         -0.1026         -0.0983         -0.1083         -0.0908         -0.100           14         ε <sub>RMS</sub> 0.1095         0.1097         0.1102         0.1120         0.1015         0.1260         0.1243         0.1351         0.1140         0.124	12		1		4		L	1	1		\$	-0.0917
E         -0.0632         -0.0677         -0.0718         -0.0706         -0.0308         -0.1240         -0.0629         -0.0768         -0.0647         -0.087           13         ε <sub>RMS</sub> 0.1010         0.1059         0.3100         0.1031         0.0972         0.1385         0.1173         0.1128         0.1036         0.114           σ <sub>E</sub> 0.0783         0.0815         0.0833         0.0752         0.0922         0.0618         0.0990         0.0827         0.0809         0.074           Ε         -0.0897         -0.0899         -0.0902         -0.0928         -0.0796         -0.1026         -0.0983         -0.1083         -0.0908         -0.104           14         ε <sub>RMS</sub> 0.1095         0.1097         0.3102         0.1120         0.1015         0.1260         0.1243         0.1251         0.1140         0.124			1		4	į.	1				1	1
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GE         0.0783         0.0815         0.0833         0.0752         0.0922         0.0618         0.0990         0.0827         0.0809         0.074           E         -0.0897         -0.0899         -0.0902         -0.0928         -0.0796         -0.1026         -0.0983         -0.1083         -0.0908         -0.100           14         ERMS         0.1095         0.1097         0.1102         0.1120         0.1015         0.1260         0.1243         0.1351         0.1140         0.1243	- 13		ı	t	i	1	ł .	ı	1	l .	1	1 1
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	14		i	ł	ł				1	1		0.1240
σ <sub>g</sub> 0.0629 0.0630 0.0633 0.0626 0.0631 0.0732 0.0761 0.0808 0.0690 0.072		σ <sub>ε</sub>	0.0629	0.0630	0.0633		0.0631	ł .	<b>!</b>	1		0.0722

Table V, on the other hand, shows the results of the evaluations of each correlations for 14 data subsets shown in Table II. Also, in Fig.2 the RMS error ( $\epsilon_{\rm RMS}$ ) ranges of existing correlations examined for each data subsets are shown in order of increasing RMS error along with the list of 4 best performing correlations for each data groups. From the Table V and Fig.2 it can be observed that the RMS error ranges of the correlation for

low quality region ( $x \le 0.1$ ) are considerably smaller than those for high quality region ( $x \ge 0.1$ ).

As can be seen from Figs. 3-5, comparisons of experimental and predicted total pressure drop using the Owens model [1], Martinelli-Nelson model [5], and Thom model [6] show that the experimental data fall within -28% and +23% spread of the correlation at a 95% probability level.

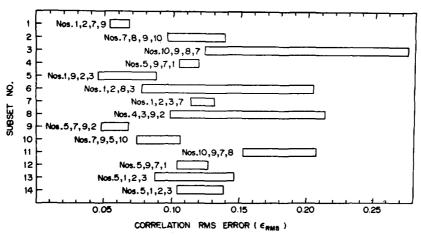


Fig 2. RMS Errors of All Correlations for Each Data Subsets.

# **4.Outlines of a New Method and Its Assessment**Outlines of the New Method Proposed

The basic concept of the new method proposed here is to calculate the total two-phase pressure drop using the best performing correlation for a specific data range and then apply a correction factor to compensate for the expected error of the selected correlation.

To summarize, the step-by-step calculation performed by the computer code developed here (i.e., the 'K-TWOPD' code) is as follows:

- (1) Select the best performing correlation that has the smallest RMS error out of the 10 methods shown in Table I for a given specific property and flow conditions with the aid of Table V and Fig.2.
- (2) Calculate the total two-phase pressure drop

- $(\Delta P)_{\text{opt}}$  using the selected correlation.
- (3) Obtain a correction factor  $\Psi$  to compensate the expected error of the selected correlation by

$$\Psi = B_{1X} + B_{2}(\frac{q^{*}}{G}) + B_{3}L^{2} + B_{4}log_{10}(\frac{q^{*}}{G}) + B_{5}log_{10}(\frac{L}{D_{e}}) + C_{0}$$
(19)

where

$$B_1 = 0.26739$$
,  $B_2 = -0.36968$ ,  $B_3 = 0.00024$ ,  $B_4 = -0.07543$ ,  $B_5 = -0.13777$ ,  $C_0 = 1.35224$ ,

x is the exit quality, q\* is the heat flux in Btu/hr-ft\*, G is the mass velocity in lbm/hr-ft\*, L is the pipe length in ft, and De is the equivalent diameter in ft.

(4) Finally, the predicted value  $(\Delta P)_{new}$  is

obtained by substituting  $(\Delta \, P)_{\mbox{\tiny opt}}$  and  $\Psi$  into the the following expression :

$$(\Delta P)_{\text{new}} = \frac{(\Delta P)_{\text{opt}}}{\Psi} \tag{20}$$

# Procedures Used to Obtain the Correction Factor $\underline{\Psi}$

If some input variables to evaluate  $(\Delta P)_{corr}$  are very influential on the error €, those input variables or their combinations require a close study in composing a correction factor that can reduce the error vectors. The stepwise regression technique (SRT) [13] is used to select these sensitive parameters, thereby to build a regression equation composed of not undue member of input parameters while the constructed regression equation reveals the input-output relationship. This procedure enables to select or remove the most important variables sequentially. At each step, to decide the adequacy of the constructed regression model composed of the selected input parameters, the analysis of variance (ANOVA) calculation has been performed.

The procedure used to derive the correction factor  $\Psi$  is as follows:

- Select the best performing correlation for each data subset with the aid of Table V and Fig.2, and collect the selected correlations.
- (2) Calculate the error of those correlations collected in step 1.
- (3) Select the most important parameters sensitive to produce errors obtained in step 2 by the stepwise regression technique.
- (4) Introduce new varibables by the combination of selected parameters, taking its logarithm or powers.
- (5) The stepwise regression technique is used again to select or remove the introduced new variables in step 4.
- (6) Build up the error response, €, which is a function of the selected variables.
- (7) Finally, from the definition of the error, Eq.(15), the correction factor  $\Psi$  to compen-

sate the expected error  $\epsilon$  in the  $(\Delta P)_{\text{opt}}$  obtained by the best performing correlation is derived as

$$\Psi = 1 + \epsilon \tag{21}$$

where  $\epsilon$  is the predicted error regressed with the important parameters as shown in Eq. (19).

### Evaluation of the New Method

As described in the previous section, once the best performing correlation that has the least RMS error is selected for a specific range of pressure, mass velocity, and exit quality, the two-phase pressure drop calculation with a correction factor  $\Psi$  to compensate the expected error of the selected best performing correlation can be carried out by the computer code 'K-TWOPD': This code calculates the exit quality internally by the Bowring model [14]. To examine the goodness of fit and the accuracy of this new procedure the following statistical analysis has been performed.

(1) Stepwise Regression Results: the importance of variables and regression of the correction factor  $\Psi$ , Eq. (19), are investigated by stepwise regression procedure [13]. In addition, the analysis of variance (ANOVA) has been performed to see the difference between  $\epsilon$  and  $\hat{\epsilon}$ . The analysis of variance table obtained from the final stage of the stepwise regression is shown in Table VI. There are basically three factors that describe how well the model actually fits the observed data [13], (1) the mean square due to residual variation (MSE), (2) the coefficient of determination R2, and (3) the calculated F value. The smaller the MSE value, the better is the regression. Notice that the MSE value is 0.00388 as shown in Table VI. Also one should be pleased if the sum of squares (SS) due to regression is much greater than the SS due to the residuals, or what amount to the same thing if the ratio R2 is not too far from unity. The F value serves to test how well the regression

model fits the data. If the calculated F value exceeds the 'critical-F' value obtained from a statistic table for F(k, n-k-1;  $\alpha$ ) the regression is significant at a level of 100 (1 –  $\alpha$ )%. It can be observed that all the values shown in Table VI pass the above tests for the precision of the estimated regression and the F-test for significance of regression.

Table VI. The Analysis of Variance Table for Final Stage of Stepwise Regression of Error

Degrees of Freedom	208
Variable Entering	5
Multiple R	.72977
R Square	.53256
Adjusted R Square	.52105
Standard Error	.06229

### AVOVA

Source	D.F	s.s	N.S	F	Critical-F
Regression	5	. 89725	.17945	46.25625	2.25
Residual	203	. 78753	.00388		
Total	208	1.68478			

D.F : Degrees of Freedom S.S : Sum of Squares M.S : Mean Square

S: M.S. Due to Regression/M.S due to Residual Critical-F: Critical F Value in 5% significance level

(2) Histograms of Errors: to examine the frequency distribution of errors from the new method two histograms of errors are compared as shown in Fig.6: top figure (Fig.6a) shows the distribution of errors obtained by using the best performing correlations for each 14 subsets (shown in Table II) with the

- aid of Fig.2, whereas lower figure (Fig.6b) shows the results obtained from the new procedure with a correction factor  $\Psi$ . From these figures two facts can be observed: first, the distribution of error is nearly symmetrical for both cases, with the highest frequency occuring in the middle where the error is zero. Secondly, the magnitude of all errors, such as  $\epsilon$ ,  $\epsilon$  RMS, and  $\sigma_{\epsilon}$  of the new procedure is appreciably smaller than those of the best performing correlations.
- (3) Comparison of Experimental and Predicted Total Pressure Drop Using a New Procedure: Fig.7 shows the comparison between between experimental data and predicted total two-phase pressure drop using the present new procedure with a correction factor. As can be seen from this figure, all the experimental data fall within ±11% spread of the new procedure at a 95% confidence level, whereas the same data has approximately ±25% spread for the existing correlations examined in Figs. 3–5.

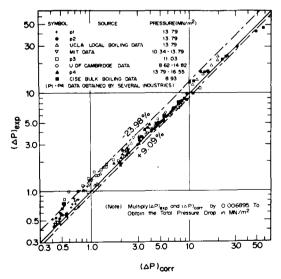


Fig.3 Comparison of Experimental and Predicted Local and Bulk Boiling Total Pressure Drop Using Owens Homogeneous Model.

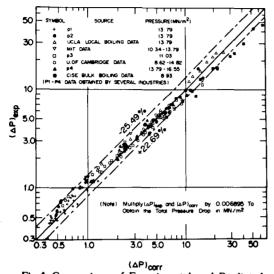


Fig.4 Comparison of Experimental and Predicted
Local and Bulk Boiling Total Pressure Drop
Using Martinelli-Nelson Model.

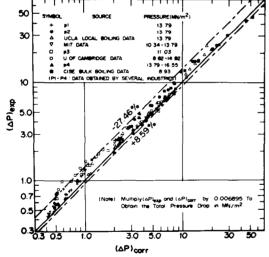


Fig.5 Comparison of Experimental and Predicted Local and Bulk Boiling Total Pressure Drop Using Thom Model.

### Applicability of the New Method to PWR Analysis

In previous sections 10 different approaches of existing correlations have been compared against measured data for conditions representing transient as well as steady state PWR operation. The data subsets investigated that are pertinent to the normal operation of the PWR, in particular, are those representing the following properties:

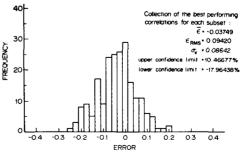
 $Pressure: 15.1{-}15.8~MN/m^2 (2200{-}2300~psia)~;$ 

Mass Velocity :  $3254.4-3525.6 \text{ Kg/m}^2-\text{s}(2.4-2.6 \text{ M1b/ft}^2-\text{hr})$ 

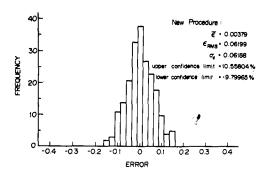
Quality: 0-1.

As can be seen in Fig.2, the correlation which had the least overall RMS error for the above conditions (equivalent to the subsets No.14 in Table II) is the Martinelli-Nelson correlation (ID No.5 in Table I).

In the event of a nuclear reactor transient such as the small and large LOCAs, the quality can be higher, whereas the mass velocity and the pressure in the core can be lower. Under these circumstances, conditions above may be changed and the best performing existing correlation for the changed conditions can be found from Fig.2. However, the new procedure outlined in previous sections is recommended for the more accurate analysis of PWR pressure drop at steady and transient core conditions. For the above property ranges representing the normal operation of the PWR, the RMS error of the new approach (for the subset No.14) is  $\epsilon_{RMS} = 0.0569$  whereas that of the best performing correlation (ID No.5) is  $\epsilon_{RMS}$ = 0.1015. This indicates that the new approach is superior to the best performing correlation for the subset No.14. In general, this is true for all the subsets as can be deduced from Fig.6 and the derivation of the correction factor  $\Psi$ .



(a) Distribution of errors of the best performing correlations



(b) Distribution of errors of the new procedure.

Fig.6 Comparison of Two Histograms of Errors:

The Best Performing Correlation VS. the New Procedure.

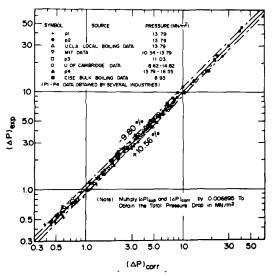


Fig.7 Comparison of Experimental and Predicted Total Two-Phase Pressure Drop Using a New Procedure with a Correction Factor  $\Psi$ .

#### 5. Conclusions

The performance of the ten two-phase pressure drop prediction methods were evaluated for their accuracy and applicability to PWR conditions. Specifically, the best performing correlations were identified for each data subsets representing specific ranges of pressure, mass velocity, and quality and data sets representing PWR conditions. The root-mean-square error between model prediction and data was used as the criteria to evaluate mod-

el performance.

- (1) Considering the total data bank, the existing models exhibiting minimum error were (a) Chisholm model(ID No.9 and 10), (b) Baroczy model(ID No.7 and 8), and (c) the homogeneous model with the two-phase viscosity term based on all-liquid flow (ID No.1).
- (2) The best performing models or methods for each data range can be found with the aid of Fig.2 or Table V.
- (3) The new method proposed here is to calculate the total two-phase pressure drop using the best performing correlation for a specific data range and apply a correction factor  $\Psi$ , defined by Eq. (19), to compensate the expected error of the selected correlation: that is, use Eq. (20). This new approach fits all the pressure drop data collected in the present work within  $\pm 11\%$  at a 95% confidence level compared with  $\pm 25\%$  for the existing correlations.

#### Nomenclature

Coefficient in Eq. (13)

В

De	Hydraulic equivalent diameter
$f_{\text{fo}}$	Friction factor based on total fow
	assumed liquid
$f_{\text{fo.iso}}$	Isothermal friction factor based on total
	flow assumed liquid
fтР	Two-phase friction factor
G	Mass velocity
g	Acceleration due to gravity
L	Length of channel
n	Exponent in Blasius' relation for fric-
	tion factor
P	Static pressure
q´	Surface heat flux
$V_{\rm f}$	Specific volume of liquid
$V_g$	Specific volume of vapor or gas
$V_{\rm fg}$	Difference in specific volumes of satu-

A Two-Pha	se Pressure Drop ··· M.H. Chun and J.G. C
	rated liquid and vapor
x	Mass vapor quality
z	Axial coordinate
α	Void fraction
γ	Dimensionless slip factor used in
	Eq.(11)
۳	Physical property coefficient defined by
	Eq. (14)
€	Fractional error defined by Eq. (15)
$\epsilon$	Pipe roughness
ξ	Mean error
€ <sub>RMS</sub>	Root-mean-square error
ê	Predicted error used in Eq. (21)
$\theta$	Angle to horizontal plane
$\mu_{_{\mathrm{f}}}$	Liquid viscosity
$\mu_{g}$	Vapor viscosity
$\mu$	Mean viscosity of homogeneous fluid
$ ho_{{}_{f}}$	Liquid density
ho g	Vapor density
$\rho$	Average density of homogeneous fluid
σ.	Standard deviation of the error from
	the mean
$\phi$ $^2$ fo	Two-phase frictional multiplier based
	on pressure gradient for total flow
	assumed liquid
Ψ	Correction factor defined by Eq. (19)
$(\Delta P)_{corr}$	Total two-phase pressure drop calcu-
	lated by correlations
$(\Delta P)_{\text{exp}}$	Total two-phase pressure drop
	obtained by experiment
$(\Delta P)_{new}$	Total two-phase pressure drop calcu-
	lated by the new method proposed,
	Eq. (20)
$(\Delta P)_{\mathrm{opt}}$	Total two-phase pressure drop calcu-
	lated by the best performing correla-
	tion
$(\frac{\mathrm{dp}}{\mathrm{dz}})$	Total pressure gradient
$(\frac{dp}{dz}F)$	Pressure gradient due to friction
dp 、	The state of the s

Pressure gradient due to acceleration

 $(\frac{dp}{dz}z)$  Pressure gradient due to static head

#### Subscripts

- fo Assuming total flow to be liquid
- f Liquid
- g Gas or vapor

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