# Develpoment of an Equipment Replacement Decision System considering Technological Changes

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## 1. Introduction

The study of equipment replacement is one of the oldest and most important subjects in the field of engineering economy. In the economy where profit is the motive, equipment replacement should be based on the economy of future operations. The main considerations leading to replacement of existing equipment may be classified as inadequacy, obsolescence, increasing maintenance, and declining efficiency.

1. Inadequacy. A physical asset that is inadequate in its capacity to perform its required services is a logical candidate for replacement.

2. Obsolescence. The current machine is obsolescent as new models can perform the same function more economically. There are a number of factors that can enter into the obsolescence of the existing machine. It may result from a change in customer tastes which through the dynamic forces of the market render the existing machine less desires. Technological change could permit the performance of the same function with lower operating cost, lower acquisition cost, or a combination of both lower acquisition cost and lower operating cost.

3. Increasing maintenance. Experience has proven that it is economical to repair many types of assets in order to maintain and extend thier usefulness. Before and expenditure for major repairs is made to extend the service life of a machine, analysis should be made to determine if the needed service might be more economically provided by other alternatives.

4. Declining efficiency. Equipment operates at peak efficiency initially and suffers a loss of efficiency with usage and age. When the loss of efficiency is due to the malfunctioning of only a few parts of a whole machine, it may be economical to replace them periodically and maintain a high level of efficiency over a long time periods. However, if it is not economical to restore efficiency by maintenance, the system should be replaced at intervals on the basis of economy.

Based on these considerations, theorists in the field of engineering economy have attempted to give an answer to the question : When should the existing equipment be replaced? For the analysis in this paper, equipment shall be divided into three main categories :

1. Presently-owned equipment(Current Defender)
2. Best currently available equipment(Current Challenger)
3. New equipment that may be available in the future(Future Challenger)

The equipment replacement decision problem involves a required service that is to be performed for a specified period of time. The decision-maker is faced with a choice between keeping existing equipment, commonly referred to as the current defender, or replacing existing equipment immediately with new equipment, referred to as the current challenger. If the existing equipment is not replaced at the present time, the decision-maker must take into account the succession of future challengers that may be more efficient.

In a world of technological progress, it might be that there is future equipment worth waiting for. Therefore, the comparison is not only between what you have now and the best available challenger, but also between the best current challenger and the future challengers.

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In discussing equipment repacement, it must be noted that two of the types of equipment listed above may suffer from obsolescence, either through improved equipment designs or a radical change in the product calling for the services of the equipment. Since these two cases are largely unpredictable, a certain element of chance enters into the decision to replace any equipment. It must also be noted that inadequacy is, in the majority of cases, predictable.

In the study of the classical approaches to the problem of equipment replacement, it seems that the models presented are limited in their applicability in two respects. First, the models deal exclusively with replacement of equipment of the first category. Second, the treatment of equipment is inadequate from both theoretical as well as practical points of view because of the changes in technology and the demand for service, both of which are very difficult to predict.

The first part of this paper discusses the current approaches based on the classical method. The second part deals with the development of a "futuristic approach" and a computerized equipment replacement decision system based on the futuristic approach using actual data.

In summary the objectives of the study are :

i ) To develop an approach for replacing equipment considering not only current challengers but also future challengers.

ii ) To develop a computerized system, Equipment Replacement Decision System, based on the developed approach.

iii ) To illustrate the use of the system based on actual data of the computing machinery.

## 2. Review of The Literature

The traditional treatment of the equipment replacement problem assumes that any future challengers would be identical to the current challenger. This assumption not only ignores the effects of inflation and technological improvements, but it also greatly simplifies the replacement economy analysis. Terborgh[3], in 1949, and Alchian[1], in 1952, presented the earliest models that partially relaxed the assumption of repeatability. They permitted the receipts and disbursements of the future challengers to vary linearly with time from their predecessors but required that the economic service lives of the current and future challegers remain constant. Oakford generalized these models and permitted the receipts and disbursements as well as the first costs and salvage values to vary linearly, geometrically, or bounded-geometrically with time. The constant economic service lives of the current and future challengers remained unchanged.

### 1) the MAPI Formulation

The MAPI formulation is clearly a step forward in the engineering way of thinking about equipment replacement. Its value lies in the following two conditions :

1. It exposed the underlying assumptions in the classical engineering economy liturature concerning equipment replacement. It also formulated new standard assumptions on which its own theory was built. "And the least be said of these assumptions, they are clear and definite. In using them --- or rather the replacement formula they yield --- the analyst knows exactly where he stands. If he thinks the assumptions inapplicable in some respects to the case in hand, he can modify the results of the formula as his judgment indicates, but he has at least a benchmark or point of departure to work from."[4]

2. It introduced the concept that the replacement problem is not : With what shall I replace now? Rather, it is : Shall I replace new or defer replacement to future time? The emergence of the current challenger and its descendent was a natural result of such an approach. At no time is the always indirect via the descendents of the current challenger.[4]

The concepts introduced by Terborgh can be summarized in Figure 1. Line A represents the increasing yearly cost of operating the existing machine. Line B shows the increasing(but at a different rate) yearly cost of the current challenger. Line C represents the declining yearly operating cost of the continuously improved line of descendents. Both the defender and the current challenger are compared to the descen-

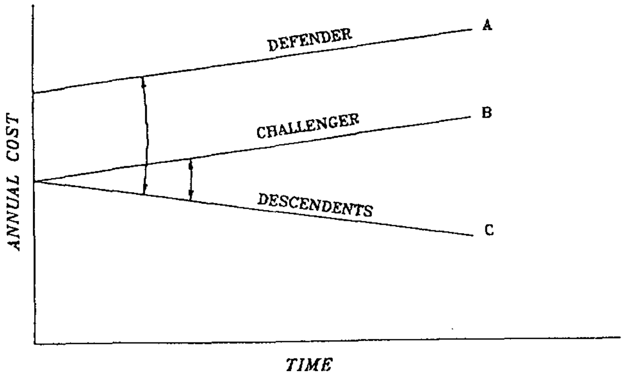



Figure 1

dents, and the two adverse minima are then compared with each other. This explains even more forcibly the dynamic nature of replacement studies, since the base on which both defender and challenger are compared is changing with time. Credit must be given to Orensteen for casting Terborgh's formulation in the classical language of minimum cost, as well as giving a simplified graphical representation of the theory. [12]

### 2) Comparison of the MAPI With Other Methods

Because of Orensteen's finding, the classicists were not unware of obsolescence as a factor in determining the replacement decision, and they did try to take it into consideration in more than one way. Orensteen observes that semantic difficulties appreciably interfere with an understanding of the conceptual basis of the MAPI formula. By showing a development of the MAPI formula through the orthodox minimum cost formula, he provides a more straight forward approach to the problem than most other engineering economists. Orensteen's paper contains a demonstration of how obsolescence may be taken into account in the rate of return, the capital recovery periods, or the salvage values to produce the same results as the MAPI formula. His answer appears to have been achieved more through a demonstration of the fundamental soundness of the MAPI concepts. It is proposed that a conceptual framework be set up in which both the MAPI proponents, and the critics can be clearly defined and the merit of each position be assessed.

The development of the most successful analytical models has been characterized by a large share of intuition in the initial phases. There have been instances in the history of engineering economy where techniques have envolved through the addition of new assumptions to compensate for the effects of bad initial assumptions. The most unrealistic standard assumption is the supposition that each succeeding improved machine has an operating cost lower than that of a new replica of the previous year's machine. It is a logical consequence of this assumption that, as the machines are improved year, eventually there will be a developed machine which has a negative operating cost. Actually, nobody has constructed and analytical model of the costs of operations.

However, the most serious deficiency of the MAPI formula as an analytical model on which to base dynamic equipment policy is its implicit assumption as to the nature of obsolescence. Insofar as the MAPI formula portrays it, obsolescence consists in the development of new machines which are improved but functionally identical to the old. A machine may become effectively obsolete because the entire process in which it is employed becomes obsolete. Obsolescence of a process may come about not merely because of the development of an improved process, but may also occur when a necessary change in product design dictates the use of a new process. The impact of product design changes on management of the equipment investment is potentially great in any highly competitive consumer good industry. Because this concept of the nature of obsolescence is far more difficult to modify in the MAPI formula than the assumption of uniformly decreasing cost of improved machines and the assumption of constant acquisition cost for future machines, it appears to be the most serious limitation on the potential usefulness of the MAPI approach as a general analytical model. The ability of the MAPI formula to produce satisfactory answers as a forecasting function comes into consideration as an independent question. Orensteen's demonstration of the equivalence of the MAPI formula to older methods certainly shows that the answers obtainable by use of the MAPI formula can be as good as those obtained by these other methods. Moreover, it would appear that, as a forecasting function, the MAPI formula is not limited to applications in which obsolescence conforms to comparatively narrow concepts adopted in the development of the formula.

The distinction suggested is between a general category of forecasting functions which includes all quantitative prediction methods and a more restrictive sub-class of the category, called analytical models. When this distinction is made it appears that most fundamental criticisms of the MAPI formula have to do with its validity as an analytical model. Orensteen describes that he dismisses the question of analytical model validity and pleads his case entirely on the validity of the formula as a forecasting function.

Dr. Terborgh is most outspoken on this point : "We have no desire to claim too much for this procedure... No standard method, whatever its merits, can encompass the infinite variety and complexity encountered in practice, nor can it be a substitute for sound judgement.[3]

Nordin presented the replacement analysis method called the total discounted expenditure method. It is applied only to simple conditions, but ways of dealing with complications are outlined. The problem may be stated as follows : Given a machine installed at time 0, find the chain of replacements that will minimize the present worth of total expenditures over a selected period.

Terborgh doesn't begin his analysis with a statement about either management objectives or the period over which the objective are to be pursued. He states that correct equipment policy is that which minimizes the time adjusted sum or combined average of capital costs and operating inferiority. Time adjusted sum means the sum of discounted values. Capital cost is cost of buying replacement. The rate of operating inferiority includes both deterioration and obsolescence. There is an adverse minimum for the defender and an adverse minimum for the challenger. The challenger replaces the defender if and only if the challenger's adverse minimum is lower than the defender's adverse minimum. To select an optimal policy, he must consider both management objective and anticipated factual conditions over the whole planning period. Terborgh assumes a chain of replacements in calculating the adverse minimum of challenger and defender, but his calculations do not extend far enough into the future. Suppose that, a short time after the time after the period for which the challenger's adverse minimum has been calculated, new machines become very expensive or very poorly made. Management ought to arrange its affairs so that no replacement will be needed during this period. But the MAPI formula will not facilitate such an arrangemtnt. The MAPI formula provides a guidepost or point of reference. Since it is based on certain standard assumptions as to the future, it gives a result correct only when these are valid. The analyst may wish to shade the results one way or the other in accordance with his judgement as to the appropriateness of the assumptions to the to case in hand.

Terborgh concludes that "it is appropriate...to emphasize that the problem of shading the formula results is in no way peculiar to the MAPI prescriptions." It seems that all of the existing formulations are equally valid, once it is shown that they produce results which prove to be correct. In other words, if accepting the different formulations for what they really are aids judgement, arguments without statistical data are pointless. There is no conceptual difference between the MAPI formulation on the one hand and classical formulations on the other in dealing with notations such as obsolescence, deterioration,s economic life or salvage value.

## 3. Futuristic Approach To Replacement Decision Problems

### 1) Definition

A replacement decision problem is a situation in which a service is to be performed for some period of time, and the asset that currently performs this service will have to be replaced one or more times during the period of time the service is required. This type of problem can be found in most typical manufacturing operations where the machinery and equipment producing goods must be replaced from time to time.

A replacment decision involves more than just a decision about the asset currently performing the service. The decision must also take into account the sequence of future replacements that will continue to perform the service once the current asset is retired. The productivity of the future replacements influences the replacement time of the current asset. The problem assumes the decision-maker is faced with a replacement decision involving the sequence of future replacements.

Technological improvement decreases the operating cost of a future facility continuously over time at a certain rate. Technological improvement is reflected not only in the form of decreases in the operating cost and/or decrease in the price of the future facility.

The defender is the existing equipment and the challenger is the best available replacement equipment. Replacement analysis may have as its endproduct a recommendation that some particular equipment be replaced and that money for the replacement be included in the capital expenditures budget.If there is not a recommendation to replace the equipment now, this recommendation may be next year or some subsequent year. The question is : Shall I replace the defender now, or shall I keep it for one or more additional years ? Thus the question is not whether or not I am going to remove the defender equipment. The equipment may be removed when the task performed by the equipment is no longer needed or when the task can be better performed by different equipment.

This study evaluates future challengers when the current challenger is considered as the best available alternative at the present time. If a future challenger will be better than the current challenger, what impact will it have on an analysis now? The prospect of better future challengers makes it more desirable to retain the defender and to reject the current challenger. If the defender is kept for now, it may be replaced by a better future challenger. For this reason, the approach using for replacement decision in this study is a "futuristic approach."

### 2) Inferiority Level vs. Operating Cost

It is insufficient to limit the comparison to the current defender and the current challenger. The current defender and challenger will accumulate obsolescence, deterioration and declining efficiency. The costs of equipment can be categorized in two general ways : acquisition costs and operating costs. The acquisition costs are lump-sum costs at the time of purchase. The operating costs are incurred at points in time during the life of the equipment. In both the theory and practice of replacement, operating costs are extremely important. Acquisition costs are related to present transactions, whereas the operating costs are related to problems of the future. The nature of operating costs motivates equipment analysis and replacement. It is assumed that operating costs will increase over the life of the equipment and that there are operating costs which may be incurred even when the equipment is not operating. Certain maintenance costs may fall into this category.

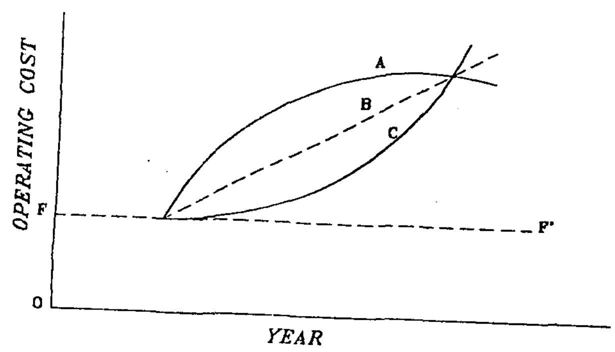



Figure 2

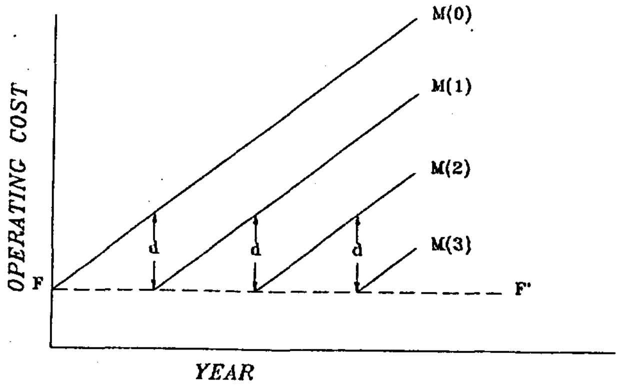



Figure 3

Figure 2 illustrates some operating cost patterns : FF' represents the fixed portion of operating costs;A represents a higher rate of increase in the early life of the equipment;B represents a constant amount rate of increase throughout the life of the equipment;and C represents a lower rate of increase in the early life of the equipment. The first year operating cost of the identical equipment is assumed to be a fixed amount, OF. The excess of total operating costs of the current equipment over that of new equipment in a particular year is due to the deterioration. In the following discussion, M(n) represents the equipment purchased at years n. In Figure 3, the line M(0) represents the operating costs of new equipment purchased today, M(1) that of the equipment purchased after one year, M(2) after two years, etc. The costs themselves would remain unchanged. They begin at a point in time when the equipment is new. The cost lines of an annual series of the new equipment can be represented by parallel shifts to the right of the new equipment can be represented by parallel shifts to the right of the operating cost lines of preceding year's equipment. This task measures the deterioration of M(0) every year throughout its life. The deterioration of M(0) the first year will be d, the difference between M(0) and M(1) for first year. The deterioration of M(0) the second year will be 2d, the difference between M(0) and M(2). It will be convenient to construct a horizontal reference line FF' as shown in Figure 3. The amount of deterioration in a particular year can be measured by the vertical distance between the line FF' and M(n) line. As time passes, the inital equipment becomes more and more inferior to the conceivably always available alternative, the then new equipment. This is inferiority due to age...deterioration.

In Figure 4, the operating cost lines M(0) and M(1) and the reference line FF' are represented. The graph assumes that one year hence there will be machine available capable of performing the same function as M(0), but superior to the one year old M(0). Such a machine could be a new replica, which is superior to orginal machine. A new machine must be superior to the new replica : it must have lower operating costs by reason of a lower and parallel shift in the operating cost line. This is analogous to the shifts in the operating cost line by reason of deterioration. However, the fall in the operating cost line must be greater than of the new replica in order for the improved machine to be superior to the new replica. The operating cost line of the new replica one year hence would be M(1), drawn through the line FF' and parallel to M(0). Thus, for an improved machine to be superior to a new replica M(1) and exhibit the desired line shift, the cost line of the improved machine must likewise be parallel to M(0), but it must go through a point lower than A. The improved machine of M(1) is designated M'(1). The amount by which a new replica is inferior to the improved machine M'(1) is due to obsolescence and is the vertical distance AB which is denoted by 0 in Figure 4. The amount by which M(0) was inferior to its new replica M(1) is d. Thus, the total amount of inferiority of M(0), in relation to the available improved machine M'(1), is d plus o, denoted by symbol I in the Figure 4.

As a result, the present equipment declines in efficiency as time passes. Considering technological improvement, it might be that there is future equipment worth waiting for. Therefore, the comparison is not

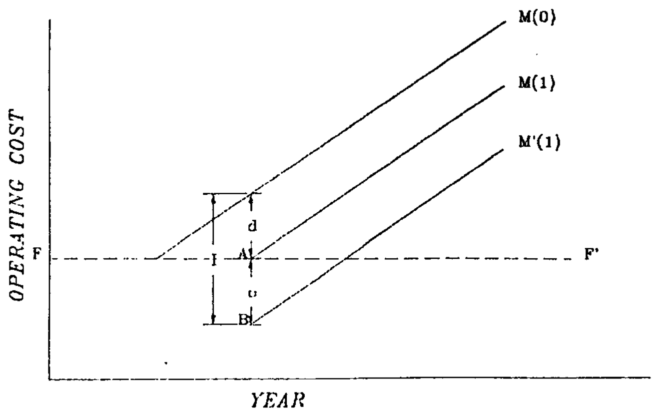



Figure 4. Operating Costs for Improved Machine

only between the current defender and the current challenger, but also between the current challenger and the future challengers.

### 3) Salvage Value

The salvage value is the price that can be obtained from the sale of the used equipment. Equipment that has been well-maintained and is in good condition will obviously be of greater value than that which has been neglected and would require considerable repairs. It is customary and convenient to assume a particular pattern of salvage values over the life of equipment. For example, one might assume a higher than usual rate of decline in value in the early years of equipment when obsolescence is expected to be high. Certainly the salvage value of the equipment is related to the existence of improved equipment. Therefore, the salvage value of defender is affected by the efficiency of the challengers and its remaining service life. The salvage value of the challenger is considerably negligible when service life is considered infinite.

### 4) Effects of Technological Changes

Although technological improvement in a future productive facility affects a capital investment decision in various ways, this study examines its effects on the operating cost and the price of such a facility.

A) Change in the operating cost

The reduction of the operating cost per unit of products often is attributed to technological improvement in a productive facility. This reduction may be realized in one or any combination of the following ways :

a. A decrease in the labor cost due to eased requirements for the skill of labor.

b. A decrease in the labor and other non-material costs due to an increased rate of production.

c. A decrease in the material cost due to either a reduced material wastage or the use of cheaper materials.

Let the current input for a facility available at time 0 be denoted by $X[y(t), z]$ where X is the input function, y(t) the amount of production at t, and z the size of the facility. If it is assumed that this input requirement for a future facility decreases continuously over time at a constant rate, h, because of technological improvement, the operating cost of the facility purchased at time t can be written

$$C(1)=c(t)xX[y(t), z]e^{-ht} \qquad t>=0 \qquad (1)$$

where c(t) is the operating cost per unit input. Since different degrees of technological improvement are expected for facilities of different sizes, the improvement factor denoted by h might be better represented by h(z), a function of the size of the facility.

B) Change in the price of a *facility*

Rarely is the price of a facility unaffected by technological improvement. On the other hand, the price of an existing facility tends to decrease as a newer, improved version of that type of facility becomes available. On the other hand, the price of a future facility may decrease because of the availability of a more efficient method of producing the facility, or it may go up as technological improvement demands increasing complexity in the design of the facility.

It is assumed that the price of a facility available at time 0 is a function of size z, denoted by K(z), and that the price of a future facility either increases or decreases from K(z) continuously over time at a constant rate k because of technological improvement. The price of a facility purchased at time t would be as follows :

$$P = K(z)e^{-kt} \qquad (2)$$

Such a price change is expected to be different for facilities of different sizes, and the factor denoted by k might be better represented by K(z), a function of the size of the facility. In the above expression, parameter k represents this technological improvement factor for facilities of all sizes. If the purchase of an identical model is under consideration, parameter h in Eq.[1] and parameter k in Eq.[2] should be 0, denoting no improvement. If it is assumed that an improved model is under consideration, then both parameter h and k are positive.

### 5) Decision analysis

It is assumed that the salvage value and operating costs are to be a function of the age of machine. Difficulties in cash flow comparison arise when the assets have different service lives. Standard textbook approaches to the resolution of these difficulties are use of a *minimum common multiple-life year or conversion* to an equal lives problem. The minimum common multiple-life year approach computes the present value of the cost of a series of identical assets for each alternative over a minimum common multiple-life year of their service lives. An equivalent procedure is to compute a uniform annual equivalent cost for each alternative over its full service life. This method makes the comparison as if assets are replaced at the end of their service lives by successors having identical cost characteristics until the end of the planning horzon. An alternative is to convert the problem to an equal lives problem by confining the study to a horizon determined by the life of the shortest-lived alternative. This method requires the estimation of a salvage value at the end of the study period for one or more of the alternatives. The comparison is made as if all assets will be disposed of at the end of the planning horizon. The decision process can be divided into two general steps. The first step of the decision is the choice between the current defender and the current challenger. If the current challenger is chosen, then it is necessary to consider the future challengers. The second step of the decision is a choice between the current challenger and the future challengers. However, if the future challenger is chosen instead of the current challenger, the current defender is to be used until the future challenger arrives.

The futuristic approach described in this study assumes that there are a current defender and a series of challengers...the current challenger and the future challengers. Thus, a choice from the model would be one of the following alternatives :

A. Keep the current defender.

B. Replace the current defender with the current challenger.

C. Keep the current defender for n years, n=1, 2, 3, ..., and then replace it with a future challenger. Since the economic life of the defender and the challengers will differ from each other, the cash flows of the defender and each of the challengers are assumed to continue according to the function of the respective equipment. Therefore, annual equivalent amounts are used as comparison decision bases. However, the pattern of the cash flow is assumed to continue until the end of planning horizon, finite or infinite, or until the end of the optimum replacement cycle, the economic life of the asset. The annual equivalent amounts are based on the economic life.

a) Economic life decision rule

To calculate the optimal economic life of the challengers, it is assumed that the salvage value Fj is given, the interest rate i is not zero, and the operating and maintenance costs are nondecreasing with the age of the machines. Let TC(n) stand for the present worth of all future costs associated with an indefinite sequence of identical machines, each of which is replaced after n years, where n is the economic life for these machines.

$$TC(n)=F_0+\sum_{j=1}^{n}\frac{C_j}{(1+i)^j}-\frac{F_n}{(1+i)^n}+\frac{F_0}{(1+i)^n}+\sum_{j=1}^{n}\frac{C_j}{(1+i)^{n+j}}-\frac{F_n}{(1+i)^{2n}}+\frac{F_o}{(1+i)^{2n}}$$
$$+\sum_{j=1}^{n}\frac{C_j}{(1+i)^{2n+j}}-\frac{F_n}{(1+i)^{3n}}+\cdots\cdots$$
$$TC(n)=[F_0+\sum_{j=1}^{n}\frac{C_j}{(1+i)^j}-\frac{F_n}{(1+i)^n}]\ [1+\frac{1}{(1+i)^n}+\frac{1}{(1+i)^{2n}}+\cdots]$$

$F_o$=*initial investment*

$C_j$=operating cost for the $j^{th}$ year

This is a geometric series of the form $HR^m$ where

$$H=F_o+\sum_{j=1}^{n}\frac{C_j}{(1+i)^j}-\frac{F_n}{(1+i)^n}$$

$$R=\frac{1}{(1+i)^n}$$

$m=1,\ 2,\ 3,\ \cdots$

and

$$sum=\frac{H}{1-R}$$

thus

$$TC(n)=\frac{F_o+\sum_{j=1}^{n}\frac{C^{C}j}{(1+i)^j}-\frac{F_n}{\cdot(1+i)^n}}{\frac{(1+i)^n-1}{(1+i)^n}}$$

Notice this has the following interpretation. Let

$$P=F_o+\sum_{j=1}^{n}\frac{C_j}{(1+i)^j}-\frac{F_n}{(1+i)^n}$$

P, then, is the present worth of all the expenses associated with the first asset. Since

$P(1+i)^n=F$=future worth of P after n periods

and $(1+i)^n-1$=effective interest rate for n periods

then $TC(n)=\frac{F}{(1+i)^n-1}=\frac{P(1+i)^n}{(1+i)^n-1}$

*or* $F=TC(n)[(1+i)^n-1]$

It can be seen that F is in the form of $A=P'i^*$ where

$A'=F$, $P'=TC(n)$ and $i^*=(1+i)^n-1$.

thus TC(n), is the capitalized equivalent amount. That is, it is the amount invested now which is capitalized equivalent to an indefinite sequence of payments F at the end of every n periods.

If N=optimal service life, then

$TC(N+1)-TC(N)>0$

$TC(N+1)-TC(N)>0$

Let

$$q=\frac{1}{1+i}$$

$$TC(N+1)=[F_o+\sum_{j=1}^{N+1} C_jq^j-F_{N+1}q^{N+1}]\ [\frac{1}{1-q^{N+1}}]$$

$$[(TC(N))(1-q^N)+C_{N+1}q^{N+1}+F_Nq^N-F_{N+1}q^{N+1}]\ [\frac{1}{1-w^{N+1}}]$$

$$TC(N+1)-TC(N)=[\frac{1-q^N}{1-q^{N+1}}-1]\ [TC(N)]+\ [C_{N+1}\ q^{N+1}+F_Nq^N-F_{N+1}q^{N+1}]\ [\frac{1}{1-q^{N+1}}]>=0$$

$$TC(N+1)-TC(N)=[q^{N+1}-q^N]\ [TC(N)]+C_{N+1}q^{N+1}+F_Nq^N-F_{N+1}q^{N+1}>=0$$

$$=(q-1)TC(N)+C_{n+1}q+F_N-F_{N+1}q>=0$$

For convenience these may be written

$$C_{N+1}q+F_N-F_{N+1}q>=\frac{F_o+\sum_{j=1}^{N} C_jq^j-F_Nq^N}{1+q+q^2+\cdots\ \cdots+q^{N+1}}$$

$$C_Nq+F_{N-1}-F_Nq<=\frac{F_o+\sum_{j=1}^{N-1} C_jq^j-F_{N+1}q^{N+1}}{1+q+q^2+\cdots\ \cdots+q^{N+1}}$$

since the comparison basis generally used for the unequal service life situation is the annual equivalent amount, the capitalized equivalent amount discussed above will be converted to annual equivalent cost as follow. Let AC(n) stand for the annual equivalent cost, given n years of replacement intervals. Then

$AC(N)=TC(n)i$

it can be easily shown that the foregoing considerations for n may be written as follows :

$AC(n)<=C_{n+1}+F_n(1+i)-F_{n+1}$

and

$AC(n-1)>=C_n+F_{n+1}(1+i)-F_n$

the expressions can be interpreted as follows. The sign Fj indicates that the value of Fj is the opportunity cost the salvage value. That is, if it is positive, then that is the opportunity cost due to not replacing in that year. On the other hand, if Fj is negative, it is the salvage value. So long as the annual equivalent cost is greater than the marginal cost of extending the life of the asset by one additional year, do not replace the asset. As soon as the marginal cost of one additional year's service exceeds the annual equivalent cost, the asset should be replaced. That value of the life of the asset when replaced is the economic life of the asset, n.

b) Annual equivalent amount decision rule

In order to discuss annual equivalent amounts, the following symbols are used :

1. (CDAE)=Annual equivalent amount of the current defender
2. (CCAE)=Annual equivalent amount of the current challenger
3. (FCAE)=Annual equivalent amount of the future challenger
4. (CDFCAE)=Annual equivalent amount of the combination of the defender and the future challengers

Assume that technological improvement will continue and each asset is more effective than its predecessor. If the present equipment is compared with the new equipment each year, the present equipment becomes less favorable than the new equipment as time passes. The present equipment itself is growing older, and thus its operating costs are increasing more rapidly than the new equipment. The new equipment may be getting better each year, and its operating costs will be lower than that of the previous year's equipment. The improvement in the operating costs of future challengers is the result of the technological improvement. The operating costs of future challengers are assumed to be lower than that of the current challenger.

This relationship can be shown as follows :

$C_{0j} > C_{1j} > C_{2j}$.

Based on the operating costs, annual equivalent amounts of the current defender, the current challenger and the future challengers are computed using the initial investment, operating and maintenance costs, and the salvage value at the end of the economic life. The mathematical expressions for the annual equivalent amounts can be shown as follows :

$$CDAE = (F_0 + \sum_{j=1}^{n0} \frac{C_{0j}}{(1+i)^j} - \frac{F_{0n^0}}{(1+i)^{n0}})(A/P,\ i,\ n_0)$$

$$CCAE = (F_1 \sum_{j=1}^{n1} \frac{C_{1j}}{(1+i)^j} - \frac{F_{1n^1}}{(1+i)^{n1}})(A/P,\ i,\ n_1)$$

Where

$F_0$=opportunity cost of not replacing the present equipment

$C_{0j}$=operating cost for the present equipment during the $j^{th}$ year

$F_{0j}$=salvage value of the present equipment at the end of the $j^{th}$ year

$F_1$=initial investment in the new equipment

$C_{1j}$=operating cost for the new equipment during the $j^{th}$ year

$F_{1j}$=salvage value of the new equipment at the end of the $j^{th}$ year

$n_0$=economic life of the present equipment

$n_1$=decoomic life of the new equipment

$$FCAE = (F_2 + \sum_{j=1}^{n2} \frac{C_{2j}}{(1+i)^j} - \frac{F_{2n^2}}{(1+i)^{n2}})(A/P,\ i,\ n_2)$$

where

$F_2$=initial investment in the improved equipment

$C_{2j}$=operating cost for the improved equipment during the $j^{th}$ year from the use of the future challenger

$F_{2j}$=salvage value for the improved equipment during the $j^{th}$ year from the use of the future challenger

$n_2$=economic life of the improved equipment

$$CDFCAE = [(CDAE)(P/A,\ i,\ n_0) + (FCAE)(P/A,\ i,\ n_2)(P/F,\ i,\ n_2)]\ [A/P,\ i,\ n_0 + n_2]$$

where

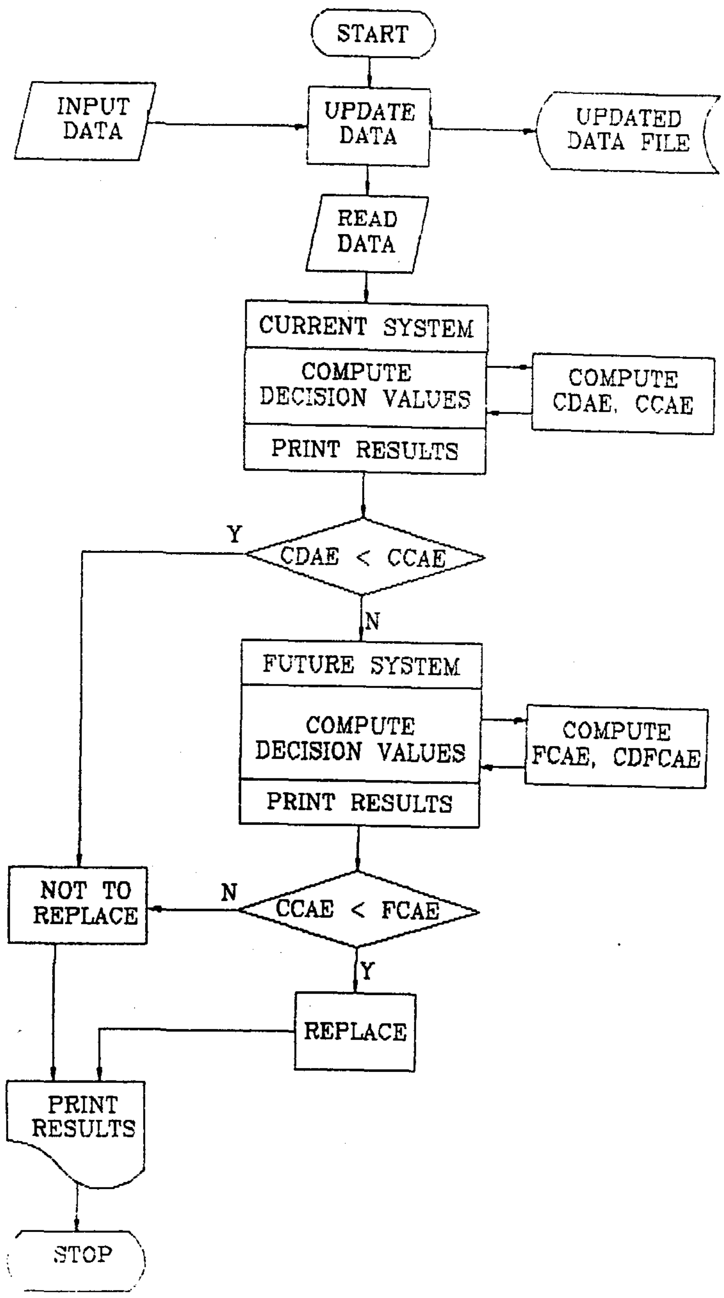
$n_0$=economic life of the current defender

$n_2$=economic life of the future challengers

## 4. The Replacement Decision System

### 1) Overall system

The overall system for making a replacement decision is shown in Figure 5. Fundamentally the replacement decision has been concerned with the problem of selecting equipment for a given task. The economic analysis will start considering the existing equipment. The question should be then whether to retain or replace the existing equipment. The replacement decision system requires input data, such as : interest rate, planning horizon, initial investment, salvage value, inflation rate, and operating cost for the defender and each of the challengers. The first step is to compare the option of replacing the current defender now nad that of maintaing it without replacing and to determine which is economically better. If keeping the current defender is better, then it will be not replaced but if replacing the defender with the current challenger is better, then there is a second step in the decision process. That is, the current callenger will be analyed further considering the future challenger. The first step is called the "Current System." The major decision criterion is the minimum annual equivalent amount. The second step is called the "Future System." The decision criterion of the future system is again the annual equivalent amount. In this system, the current cr is compared with future challengers based on the criterion. If the future challenger is better than the current challenger, then since the future challenger is better, it is more desirable to reject the current challenger and to retain the defender until the future challenger actually will be available.



2) **Subsystem**

a) The current system

The current system reads input data, such as interest rate, planning horizon, initial investment, salvage value, inflation rate, and the operating costs for the first year for each of the challengers and defender;updates the date file using new data;and writes them onto a new file which can be used in the subroutine. Using the input data values, this current system computes the decision bases for comparing the annual equivalent amount of the current defender and that of the current challenger. The main decision is to choose between the current defender and the current challenger. If the current challenger is chosen in the current system, this process continues to the next process which is called the future system.

b) The future system

This future system is used when there is a choice between replacing the current defender with the challenger now and replacing the current defender with a future challenger at some future time. This future system computes the decision bases of the current challenger according to the current system, and those of the combination of the defender and the future challenger. The decision basis values and the results are displayed on the screen and printed if desired for the final decision.

## 5. Conclusions

This paper has presented a method which utilizes the annual equivalent amounts in determining a minimum cost of the current defender, the current challenger and the future challengers. Typically such models presume explicit knowledge of :

...initial investments

...operating costs

...salvage value

Development of such model in the light of changing technological improvement and market conditions is difficult. In this replacement decision system annual equivalent amounts for the current defender and challenger and the future challengers are used for the futuristic approach. The decision system is formulated in a cost minimization context. This paper has developed the mathematical expressions for the annual equivalent amounts and described the futuristic approach. For example, if the current challengers is chosen, then it is necessary to consider the future challengers. The futuristic approach considers that there are a current defender and a series of challengers...the current challenger and the future challengers. In order for the model to be more useful to decision-makers, a futher research is suggested on computerizations of the models developed.

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