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A Stable Model Reference Adaptive Control With a Generalized Adaptive Law

(一般化된 適應法則을 사용한 安定한 기준모델 適應制御)

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要 約

本 論文에서는 媒介 變數 調整을 위하여 有理函數 형태의 演算子를 사용하는 一般化된 형태의 適應法則을 提案하였다. 媒介 變數를 適應시키는 블럭에서의 受動性(passivity) 條件을 滿足시키기 위하여 演算子외에도 常數 歸還 利得을 導入하였다. 이 適應法則을 相對次數가 1이고 最小位相이며 線形 時不變인 連續時間 시스템의 기준모델 適應制御에 適用하여, 出力 誤差가 漸近的으로 安定됨을 超安定度(hyperstability) 方法에 의해 보였다. 이 適應法則을 適用하면, 기존의 單純 傾斜(gradient) 適應法則을 適用할 때보다 向上된 出力誤差 過渡應答을 얻을 수 있는 경우가 있음을 디지탈 컴퓨터 시뮬레이션을 통하여 보였다. 또한 어떤 종류의 非모델化 운동이 존재하는 경우에서의 出力誤差에 대해 提案된 適應法則의 여러 형태를 시뮬레이션을 통하여 살펴보았다.

Abstract

In this paper, a generalized adaptive law is proposed which uses a rational function type operator for parameter adjustment. To satisfy the passivity condition of the adaptation block, we introduce a constant feedback gain into the adaptation block. This adaptation scheme is applied to the model reference adaptive control of a continuous-time, linear time-invariant, minimum-phase system whose relative degree is 1. We prove the asymptotic stability of the output error of this adaptive system by hyperstability method. It is shown that by digital computer simulations this law can give a better output error transient response in some cases than the conventional gradient adaptive law. And the output error responses for the several types of the proposed adaptation law are examined in the presence of a kind of unmodeled dynamics.

I. Introduction

Adaptive control systems can be considered as a special class of nonlinear time-varying systems. Since the most critical problem arising in adaptive systems is to assure the global stability of them, there have been many results of stability for various structures of adaptive controllers and adaptive algorithms in both continuous and discrete time versions. In the approaches of system

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stability analysis, the Lyapunov approach is well known and has been applied to most adaptive system[8] [9] [12]. This approach was sometimes used for the design of adaptive controllers to guarantee the stability of adaptive systems[11].

When the Lyapunov's method is employed, the stability of a set of error differential equations is investigated by a suitable choice of Lyapunov function candidate. However, the Lyapunov function cannot be easily found for most cases except some simple or special cases. Hence most model reference adaptive control problems have been solved by rather a simple and special adaptive law.

The difficulty of finding appropriate Lyapunov functions for adaptive systems has led some researchers to the hyperstability approach which gives a greater flexibility in choosing adaptive laws. In this point of view, Narendra and Valavani [10] showed that the hyperstability could be successfully applied to adaptive systems and that two approaches gave the same results when a simple gradient adaptation algorithm was used with an additional feedback.

In adaptive control systems, the strictly positive realness plays an important role in both stability analysis methods. Since the error model resulting from given adaptive control structure mainly determines the evolution of the error between plant and model, it is critical to the stability whether or not the error transfer function is strictly positive real. In general, in the model reference adaptive control, model cannot have an arbitrary transfer function irrespective of the plant transfer function because the transfer function of the plant together with a controller must match that of the model asymptotically. Thus we should take into account the notion of relative degree which is denoted by $n^* \triangleq n\text{-}m$ (the plant has 'n' poles and 'm' zeros).

For relative degreen $n^* = 1$, however, since the model transfer function may be chosen strictly positive real, a stable adaptive control objective can be gained with a typical adaptive control structure. Thus we only consider this case because there is no need to introduce a stable filter into the control structure to make the resulting transfer function strictly positive real. In this case, the error transfer function can be made strictly positive real and hence one can readily apply the hyperstability method to design stable adaptive laws.

The design of adaptive laws based on hyperstability was tried for a class of discrete-time linear time-invariant systems [14] and applied to a robust adaptive control in a bounded external disturbance environment [15]. But was only considered in [14] the case where a second-order operator was used for adaptation algorithm. Moreover the boundedness of auxiliary signals were not explicitly discussed and consequently the proof of the asymptotic stability was incomplete.

Motivated by these hyperstability approaches, we suggest a generalized adaptive law for continuous-time linear time-invariant systems in the sense that this employs a wider class of linear operators in it. And we prove the asymptotic stability of adaptive systems having this type of adaptive laws by the hyperstability method.

This paper consists of four sections. In section Ⅱ some definitions and theorems of hyperstability are introduced and a preliminary lemma is established. Section Ⅲ deals a basic adaptive controller with a simple illustrative proposed adaptation algorithm. The main results based on hyperstability are established in this section. Section IV shows some digital computer simulation results.

Ⅱ. Hyperstability in Adaptive Systems

Hyperstability approach to the model reference adaptive control has been used as an alternative to Lyapunov's method by authors such as Landau [3]. Literatures have also been published [1] [3] [4] [5] about the interpretation of the original Popov's theory for the application of it to adaptive systems. To proceed with the hyperstability in adaptive systems, consider a continuous-time, linear time-invariant, completely controllable and observable system B1 with single input u(t) and single output y(t) described by

$$\dot{x}(t) = Ax(t) + bu(t)$$
$$y(t) = c^T x(t) + du(t) \qquad (1)$$

where $x(t) \in R^m$ is a state vector and $A \in R^{m \times m}$, $b \in R^m$, $c \in R^m$, and d is a scalar. And the block B1 under consideration is to be connected in a negative feedback configuration, as shown in Fig. 1, by a nonlinear time-verying block B2 which has an input y(t) and an output -u(t) with the inequality (this corresponds to the passivity condition)

$$\int_0^T u(t)y(t)dt \le \delta^2 \tag{2}$$

where δ is an arbitrary constant independent of T for all $T \ge 0$.

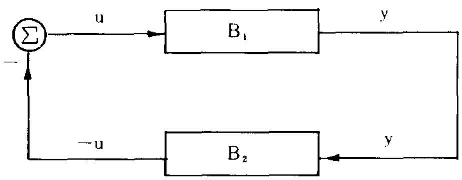

Fig.1. Negative feedback connection of two blocks

Then the hyperstability of Bl is defined by the property which requires that the state x(t) be bounded for the input u(t) satisfying the inequality (2).

Definition 1 [5]

The feedforward block B1 defined by equation (1) is hyperstable if there exists a positive constant γ such that all the solution x(t) of system (1) satisfies

$$\| x(t) \| \le \gamma [\| x(0) \| + \delta] \text{ for all } t \ge 0 \tag{3}$$

for any block B2 satisfying the inequality (2).

Definition 2 [5]

The feedforward block B1 is asymptotically hyperstable if it is hyperstable and satisfies

$$\lim_{t \to \infty} x(t) = 0 \tag{4}$$

for any block B2 which has a bounded u(t) satisfying the inequality (2).

Two important hyperstability theorems are as follows:

Theorem 1 [10]

The system B1 (1) is hyperstable if and only if the transfer function of the block B1

$$W(s) = d + c^T (sI - A)^{-1} b \tag{5}$$

is positive real for the input u(t) satisfying the inequality (2).

Theorem 2

Consider a nonlinear time-varying block B2 connected as in Fig. 1. Suppose that the block B2 can be described by the input-output relation for a piecewise continuous function vector $\zeta(t) \in R^m$ for all $t \geqslant 0$

$$u(t) = -\zeta^T(t) H(s) I[\zeta(t) y(t)] \tag{6}$$

where the operator H(s) is a positive real operator having a simple pole at s=0. (With an abuse of notation, we denote H(s) as a transfer function or as a linear operator in s $\triangleq$ d/dt according to the context.) And suppose that u(t) can also be expressed in terms of a measurable state variable $\pi \in R^m$ as

$$u(t) = \zeta^T(t) \pi(t) - d_1 \zeta^T(t) \zeta(t) y(t)$$
$$\pi(t) = -H_1(s) I[\zeta(t) y(t)] \tag{7}$$

where $H(s) = d_1 + c_1^T (sI - A_1)^{-1} b_1 = d_1 + H_1(s)$ for appropriate dimensional matrix A_1, vectors b_1, c_1, and a scalar d_1. (Again, I represents a unity matrix or an identity transfer matrix of appropriate dimension.)

Then the closed loop system is hyperstable if the transfer function of the block B1 is strictly positive real. Furthermore if $\zeta(t)$ is bounded, then the block B1 is asymptotically hyperstable.

(Proof)

Since the operator H(s) is positive real, there exists a time-invariant square matrix kernel $K(t, \tau) = K(t-\tau)$ such that $K(t-\tau)$ has the Laplace transform matrix H(s) I and is positive definite. Hence the Popov inequality (2) is given from equation (6) by

$$\int_0^T u(t)y(t)dt = -\int_0^T [\zeta(t)y(t)]^T H(s) I [\zeta(t)y(t)] dt$$
$$= \int_0^T [\zeta(t)y(t)]^T \cdot$$
$$\left| \int_0^t K(t-\tau)\zeta(\tau)y(\tau)d\tau \right| dt \le \delta_1^2 \tag{8}$$

for an arbitrary positive constant δ_1, independent of T. This with Theorem 1 implies the hyperstability of block Bl. In a similar way, from the strictly positive realness of block B1, it can be shown [5] that for an arbitrary constant δ_2

$$\int_0^T -u(t)y(t)dt = -[4(x(t))]_0^T - \int_0^T -\rho(x(t),u(t))dt \leq \zeta_2^2 \quad (9)$$

with a quadratic positive definite function $\psi(x(t))$ for all $x(t) \in R^m$ and a function $\rho(x(t), u(t)) \geqslant 0$ for all $x(t) \in R^m$, $u(t) \in R$, and $T \geqslant 0$. And with equation (7) one can show that the condition (3) with π replaced for x can be satisfied by using the property of positive realness of H(s) by deriving the same form of relation (9). Therefore one can see that B2 is also hyperstable. From the lemma of Popov [13] it follows that the closed loop system is hyperstable. If $\zeta(t)$ is bounded we can find a uniformly continuous positive definite function $\sigma(x)$ such that

$$\int_0^T \sigma(x)dt \leq \int_0^T u(t)y(t)dt, \quad T \geq 0. \quad (10)$$

Hence, from Barbalat's lemma, condition (4) holds and it follows that B1 is asymptotically hyperstable. □

Since the hyperstability merely requires a passive operator in the feedback path for the linear time-invariant system B1, we can readily devise a more flexible adaptive law as long as it governs the block B2 with satisfying passivity conditions. To introduce a general form of positive operator into the adaptive laws in the feedback path, we need a lemma.

Lemma 1

Let h(s) be a rational function of the complex variable "s" as

$$h(s) = \frac{1}{s \prod_{i=1}^{n-1} (s+\beta_i)} \quad (11)$$

where βi is a positive real constant for i=1,2, ..., n-1 ($n \geqslant 3$). And assume that $\beta i > \beta j$ if $i > j$ for i, j=1,2, ..., n-1.

Then there exists a positive constant μ such that a composite function $h_1(s) = \mu + h(s)$ is positive real and μ should be given by the inequality as

$$\mu > \sum_{k=0}^{n/2-1} \frac{1}{\beta_{2k+1}^2 \prod_{i=1, i \neq 2k+1} (\beta_i - \beta_{2k+1})}, \quad n: \text{even}$$

or

$$\mu > \sum_{k=0}^{(n-1)/2-1} \frac{1}{\beta_{2k+1}^2 \prod_{i=1, i \neq 2k+1}^{n-1} (\beta_i - \beta_{2k+1})}, \quad n: \text{odd}. \quad (12)$$

In particular, if n=2, μ should not be smaller than $1/\beta_1^{\ 2}$ for $h_1(s)$ to be positive real.

(Proof)

Since h(s) is real for real s and the residue of the simple pole at s=0 is given by

$$\frac{1}{\prod_{i=1}^{n-1} \beta_i} > 0, \quad n \geq 3. \quad (13)$$

So, from the definition [2] of positive realaness of transfer function, it is sufficient of check the satisfaction of condition that $Re[h_1(jw)]$ should be nonnegative for any $w \in (-\infty, \infty)$.

Nothing that h(s) can be expanded in a partial fraction form as

$$h(s) = \frac{a_0}{s} + \frac{a_1}{s+\beta_1} + \frac{a_2}{s+\beta_2} + \cdots + \frac{a_{n-1}}{s+\beta_{n-1}} \quad (14)$$

where a_i (i=0,1,2,...,n-1) is a real constant, one can see that by equating equation (11) to equation (14)

$$\sum_{i=0}^{n-1} a_i = 0. \quad (15)$$

This implies that some negative a_i's are present.

From the assumption, after some observations one can see that for $n \geqslant 3$ it is true that $a_{2k} > 0$ and $a_{2k+1} < 0$ for k=0,1,2,..., n/2-1 (when n is even) or for k=0,1,2, ..., (n-1)/2-1 (when n is odd). In either case a_{2k+1} is negative and hence there exists a positive constant μ such that for any values of $w \in (-\infty, \infty)$.

$$\mu + \sum_{k=0}^{n/2-1} \frac{a_{2k+1}\beta_{2k+1}}{\beta_{2k+1}^2 + w^2} > 0, \quad n: \text{even}$$

or

$$\mu + \sum_{k=0}^{(n-1)/2-1} \frac{a_{2k+1}\beta_{2k+1}}{\beta_{2k+1}^2 + w^2} > 0, \quad n: \text{odd} \quad (16)$$

Since for k=0,1,..., n-1, a_{2k+1} may be represented by in either case

$$a_{2k+1}=\frac{1}{-\beta_{2k+1}\prod_{\substack{i=1,\\ i\neq 2k+1}}^{n-1}(\beta_i-\beta_{2k+1})} \tag{17}$$

it follows that it is sufficient for $Re[h_1(jw)]$ to be positive if μ is chosen to satisfy the inequality (12).

For n=2 we can readily see that $h_1(s)$ becomes positive real if $\mu > 1/\beta_1{}^2$ □

The Lemma 1 is shown to be useful to design by the hyperstability method an adaptive law updating the parameter vector.

Ⅲ. Model Reference Adaptive Control Based on Hyperstability

A single-input single-output continuous linear time-invariant minimum-phase plant may be represented by a transfer function n

$$W_P(s)=c_P^T(SI-A_P)^{-1}b_P=k_P\frac{N_P(s)}{D_P(s)} \tag{18}$$

where $W_p(s)$ is strictly proper with monic polynomials $N_p(s)$ and $D_p(s)$ of degrees $m(<n)$ and n respectively with a constant gain parameter k_p. Assume that m,n, and the sign of k_p are known. Thus the sign of k_p can be assumed to be positive.

A model that represents the desired behavior which the controlled plant is to follow is supposed to be described by the transfer function

$$W_m(s)=c_m^T(sI-A_m)^{-1}b_m=k_m\frac{N_m(s)}{D_m(s)} \tag{19}$$

where $N_m(s)$ and $D_m(s)$ are monic Hurwitz polynomials whose degrees are m (< n) and n respectively with a constant gain k_m.

Then the adaptive control problem is to design a controller for the plant having the properties asymptotically

$$\lim_{t\to\infty} e_1(t) \triangleq \lim_{t\to\infty}(y_P(t)-y_m(t))=0 \tag{20}$$

where $y_p(t)$ is the output of the plant and $y_m(t)$ is that of the model.

Following[9], let the adaptive controller have the following structure as shown in Fig.2. Two auxiliary signal generators S_1, S_2 are employed to generate filtered signals. S_1 contains an (n-1) dimensional parameter vector c(t) and S_2 contains a parameter $d_o(t)$ and an (n-1) dimensional parameter vector $d(t)$. S_1 and S_2 are described by the differential equations

$$\dot{s}_1(t)=F\,s_1(t)+gu(t)$$

$$\omega_1(t)=c^T(t)\,s_1(t)$$

$$\dot{s}_2(t)=F\,s_2(t)+gy_P(t)$$

$$\omega_2(t)=d_o(t)y_P(t)+d^T(t)s_2(t) \tag{21}$$

where F is an (n-1)x(n-1) stable matrix and g is an (n-1) dimensional vector. It is realized so that (F,g) is a phase-variable canonical pair. Let us modify slightly the control structure so that the control input to the plant is given by

$$u(t)=\theta^T(t)\zeta_c(t)-\mu\zeta_c^T(t)\,\Gamma\,\zeta_c(t)e_1(t) \tag{22}$$

where $e_1(t)$ is the output error, $\Gamma=\Gamma^T$ is a positive definite matrix. Here, μ is a constant satisfying the condition of Lemma 1 and the 2n-dimensional parameter vector $\theta(t)$ is defined as

$$\theta^T(t)\triangleq[c_o(t),\ c^T(t),\ d_o(t),\ d^T(t)] \tag{23}$$

and composite filtered signal vectors are defined as:

$$\zeta_c^T(t)\triangleq[r(t),\ \zeta^T(t)] \tag{24}$$

$$\zeta^T(t)\triangleq[s_1^T(t),\ y_P(t),\ s_2^T(t)] \tag{25}$$

which have dimensions 2n and 2n-1 respectively.

It was shown that a constant control parameter vector θ^* exists such that if $\theta(t)\equiv\theta^*$ the transfer function of the plant together with a controller matches that of the model exactly. For this θ^* the state equation may be written as a (3n-2) dimensional vector equation

$$\dot{x}(t)=A_c\,x(t)+b_c[c_o{}^*r(t)]+ b_c[\phi^T(t)\zeta_c(t)-\mu\zeta_c^T(t)\,\Gamma\zeta_c(t)e_1(t)] \tag{26}$$

where $x^T(t)\triangleq[x_p^T(t),\ s_1{}^T(t), s_2{}^T(t)]$, $x_p(t)$ is the state of the plant, $\theta(t)\triangleq\phi(t)+\theta^*$, and $c_o{}^*=k_m/k_p$. The matrix A_c is a stable (3n-2)x(3n-2) matrix and b_c is a (3n-2) dimensional vector. These are determind by the values of θ^*.

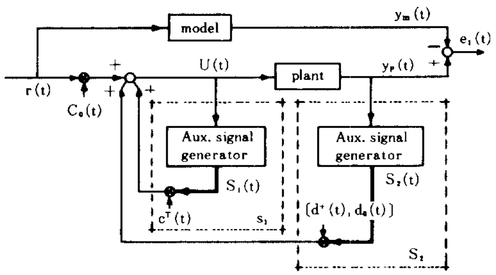

Fig.2. A typical structure of adaptive controller.

Since when matched (or $\phi(t) \equiv 0$) the resulting state differential equation describes a nonminimal representation of the model, we have

$$\dot{x}_f(t) = A_c\, x_f(t) + b_c\, c_o^*\, r(t) \tag{27}$$

where $x_f^T(t) \triangleq [x_m^T(t), s_{1m}^T(t), s_{2m}^T(t)]$, $x_m(t)$ is the state of the model, and s_{1m}, s_{2m} are the resulting signal vectors of s_1, s_2 respectively when matched. If $e(t) \triangleq x(t)-x_f(t)$ represents the state error between model and plant, the error differential equation may be given by

$$\dot{e}(t) = A_c\, e(t) + b_c[\phi^T(t)\zeta_c(t) - \mu e_1(t)\zeta_c^T(t)\Gamma\zeta_c(t)] \tag{28}$$

$$e_1(t) = h_c^T e(t)$$

where the transfer function of $e_1(t)$

$$W_f(s) \triangleq h_c^T(sI - A_c)^{-1} b_c = \frac{k_p}{k_m} W_m(s) \tag{29}$$

is strictly positive real and $hc^T = [\ 1\ 0\ 0\ \dots\ 0]$.

For most cases a simple gradient type algorithm has been used to update the parameter vector in the adaptive law. It was shown [7] that this leads to a stable adaptive control with the boundedness of output error and parameter vector. From Theorem 2, we can choose a generalized adaptive law which includes higher order integrators for $\theta(t)$ instead of a mere pure integrator.

To get the continuous-time counterpart of [14] we first consider a second-order adaptive law represented by

$$\theta(t) = \frac{-1}{s(s+b_1)}[\Gamma\,\zeta_c(t)e_1(t)] \tag{30}$$

Or equivalently

$$\ddot{\phi}(t) = -b_1\,\dot{\phi}(t) - \Gamma\,\zeta_c(t)\,e_1(t) \tag{31}$$

where b_1 is a positive real constant. The error model for equations (28) and (30) is shown in Fig. 3. Note that the parameter vector $\theta(t)$ can be decomposed into $\theta_1(t)$ and $\theta_2(t)$ as

$$\theta(t) = \theta_1(t) + \theta_2(t) \tag{32}$$

where

$$\theta_1(t) = \frac{1}{b_1} - \Gamma\,\zeta_c(t)e_1(t)$$

$$\theta_2(t) = -b_1\,\theta_2(t) + \frac{1}{b_2}\Gamma\,\zeta_c(t)\,e_1(t). \tag{33}$$

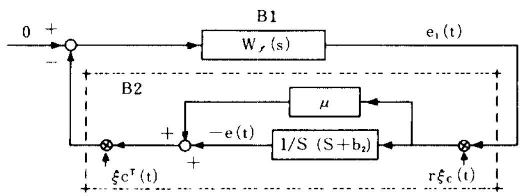

Fig.3. The error model with a second-order parameter adaptation.

Theorem 3

Consider the error system (28) with the adaptive law (31). For $\mu > 1/b_1$, the state error e(t) and the parameter error $\phi(t)$ are bounded. Furthermore e(t) and $e_1(t)$ tend to zero as time goes to infinity.

(Proof)

The hyperstability of this system can be directly obtained by applying the Theorem 2. But to give an insight for Theorem 2 and to prove the asymptotic hyperstability, we show the procedure in some detail. If we define $w(t) \triangleq \zeta_c^T(t) \cdot [\phi(t) - \mu\,\Gamma\,\zeta_c(t)e_1(t)]$, for all $0 \leq t \leq T$ with finite T, we obtain followings after some manipulations

$$\int_0^T w(t)e_1(t)dt = -(\mu b_1^2-1)\int_0^T \dot{\phi}(t)\Gamma^{-1}\dot{\phi}(t)dt$$
$$-\mu\int_0^T \ddot{\phi}(t)\Gamma^{-1}\ddot{\phi}(t)dt$$
$$-\mu b_1\Big[\,[\dot{\phi}(t)+\frac{1}{2\mu b_1}\phi(t)]^T\Gamma^{-1}$$
$$[\dot{\phi}(t)+\frac{1}{2\mu b_1}\phi(t)]\,\Big]_0^T$$
$$-\left(\frac{b_1}{2}-\frac{1}{4\mu b_1}\right)[\phi(t)^T\Gamma^{-1}\phi(t)]_0^T, \quad (34)$$

it follows that, for finite $\phi(0)$ and $\dot{\phi}(0)$,

$$\int_0^T w(t)e_1(t)dt \le \lambda^2 \quad (35)$$

where

$$\lambda^2 = \frac{b_1}{2}\left[\phi(0)+\frac{1}{b_1}\dot{\phi}(0)\right]^T\Gamma^{-1}\left[\phi(0)+\frac{1}{b_1}\dot{\phi}(0)\right]$$
$$+\left(\mu b_1-\frac{1}{2b_1}\right)\left[\dot{\phi}^T(0)\Gamma^{-1}\dot{\phi}(0)\right] \quad (36)$$

Since the transfer function of B1 is strictly positive real, it can be shown that by the Kalman-Yacubovich lemma for some $Q=Q^T>0$

$$\int_0^T w(t)e_1(t)dt \ge \frac{1}{2}[e^T(T)Q_e(T)-e^T(0)Q\,e(0)]. \quad (37)$$

This implies that the block B1 is hyperstable and hence e(t) is bounded. In the similar way, from equation (34) and (37), considering that the transfer function of B2 is positive real, $\phi(t)$ can be shown to be bounded. Since e(t) is bounded and r(t) is a uniformly bounded continuous, we have the boundedness of the plant output $y_p(t)$. As is observed in equation (21), since the auxiliary signal generators have eigenvalues with negative real parts and since the transfer function of the plant $W_p(s)$ has zeros in the open left half plane, one can show that ζ_c=(t) and u(t) are bounded. Consequently ζ_c = (t) e_1 (t) are bounded and w(t) is also bounded. From the fact that $W_f(s)$ is strictly positive real it follows that $e(t)\to 0$ and $e_1(t)\to 0$ as $t\to\infty$. □

Remark 1

The additional feedback term may be regarded as a proportional feedback term as in [5][14]. In [8], for the case where relative degree was greater than 2, an additional feedpack positive gain μ was introduced for the purpose of the signal-boundedness analysis and was stressed as essential. This argument can be interpreted in this adaptation algorithm that it should be introduced to satisfy the condition of positive realness of adaptation block regardless of relative degree for the (asymptotic) hyperstability of the system.

Remark 2

As pointed out by Popov [13], a Lyapunov function for the system (28) with adaptive law (31) can be found with the help of equation (34). This can be seen by the fact that B1 is strictly positive real and strictly proper and that the adaptive law has the passivity property. If a Lyapunov-like function $V_1(e(t))$ is chosen as for some $P=P^T>0$

$$V_1(e(t)) = \frac{1}{2}e^T(t)P\,e(t), \quad (38)$$

then we can show that by the Kalman-Yacubovich lemma

$$\int_0^t w(\tau)e_1(\tau)d\tau = \frac{1}{2}\int_0^t e^T(\tau)Q\,e(\tau)d\tau +$$
$$\frac{1}{2}[e^T(\tau)P\,e(\tau)]_0^t \quad (39)$$

for some Q such that $A^TP+PA^T=-Q<0$.

As stated early, from the property of the linear operator in adaptive law, one can find operators $g_1: R^{m1}\to R^+$ and $g_2: R^{m2}\to R^+$, as in equation (34), such that for all $\phi(t)\in R^{m2}$

$$\int_0^t w(\tau)e_1(\tau)d\tau = -\Big[g_1(\phi(\tau))\Big]_0^t$$
$$-\int_0^t g_2(\phi(\tau))d\tau. \quad (40)$$

Hence if a Lyapunov function candidate $V(e(t), \phi(t))$ is chosen as

$$V(e(t),\phi(t)) = V_1(e(t))+g_1(\phi(t)), \quad (41)$$

it can be readily shown that if $\mu>1/b_1$

$$\frac{dV(t)}{dt} = -\frac{1}{2}e^T(t)\,Q\,e(t) - g_2(\phi(t)) \le 0. \quad (42)$$

This leads to the same conclusion as Theorem 3.

Now we are going to generalize the adaptive law used in Theorem 3. Let us choose a monic Hurwitz polynomial P(s) of degree r (>1) such that the equation P(s)=0 has positive real distinct roots (this condition seems not so restricitive). If we choose the adaptive law as

$$[P(s) - P(0)]\,I\,\theta(t) = -\Gamma\,\zeta_c(t)\,e_1(t) \quad (43)$$

and choose the control input u(t) as equation (22), we still have the same error equations (28). Note that [P(s)-P(0)] contains a pure integrator which enables us to obtain the derivatives of $\theta(t)$ by known signals. Hence we have an error model which gives an output error $e_1(t)$ as

$$e_1(t) = W_r(s)\,[\phi^T(t)\,\zeta_c(t) - \mu\,e_1(t)\cdot \zeta_c^T(t)\,\Gamma\,\zeta_c(t)]. \quad (44)$$

This error model can be thought as an equivalent system that uses an adaptive law

$$\theta_c(t) = \left[\mu + \frac{1}{P(s)-P(0)}\right] I\,[-\Gamma\,\zeta_c(t)\,e_1(t)\,] = \theta(t) - \mu\,\Gamma\,\zeta_c(t)\,e_1(t) \quad (45)$$

and generates the plant input u(t) as

$$u(t) = \theta_c^T(t)\,\zeta_c(t). \quad (46)$$

This equivalent error model is shown in Fig. 4.

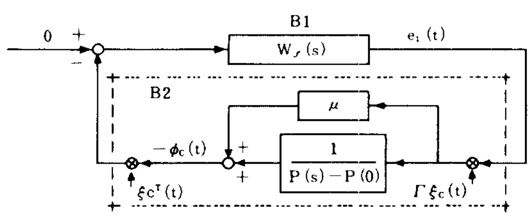

Fig.4. The equivalent error model with a positive real operator in the parameter adaptation.

Remark 3

Note that the adaptation law of Narendra [9] is a special form of this algorithm if P(s) is chosen as "s+a" where a is a positive constant. Since a choice of P(s) does not alter the control structure one can select P(s) according to the design objectives. If k_p is known (or equivalently if $k_p=k_m$) c_0 can be fixed as 1. Thus the parameter vector may be of dimension 2n-1.

Now we state the main result for a generalized adaptive laws giving the error model (44).

Theorem 4

Consider the error system described by equations(28),(29) with the adaptive law (43). For μ satisfying the condition of Lemma 1 for given P(s), the state error e(t) and the parameter error $\phi(t)$ are bounded. Furthermore, all the signals in the system are bounded and e(t) and $e_1(t)$ tend to zero as $t \to \infty$.

(Proof)

As shown in Fig. 4., the error model (44) represents two blocks which are connected in the form of negative feedback with zero external input. Since the transfer function of block B1 is strictly positive real and the feedback path block B2 has a positive real operator for the chosen μ, the block B1 is hyperstable and therefore the overall system is hyperstable from Theorem 2. This implies that e(t) and $\phi(t)$ are bounded, and $e_1(t)$ is bounded. The reaminings are followed as in the proof of Theorem 3.

IV. Computer Simulation Results

In this section we present some simulation results for a second order continuous-time linear time-invariant system for four cases. The plant was assumed minimum phase and of relative degree 1. The simulations were done on the IBM personal computer and the integral operation was simulated by the Runge-Kutta algorithm. The plant and the model transfer functions for the four cases are chosen as shown in Table 1.

In the cases (a) and (b), unstable plants were used but in other cases stable plants wer employed. In all cases the model was fixed to be strictly positive real. When k_p was assumed known it was set 1 so that $k_p = k_m = 1$. When it was supposed to be unknown it was assumed 2. Four illustrative adaptive laws are employed for

Table 1. The transfer functions of the plant and the model.

Case	Plant $W_P(s)$	Model $W_m(s)$
(a)	$\frac{s+0.5}{s^2-2s+1.25}$	$\frac{s+0.56}{s^2+1.3s+0.4}$
(b)	$\frac{2(s+0.5)}{s^2-2s+1.25}$	
(c)	$\frac{s+0.5}{s^2+2.5s+1.5}$	
(d)	$\frac{2(s+0.5)}{s^2+2.5s+1.5}$	

Table 2. Operators and corresponding μ' for each type.

Type	operator H(s)	μ
0	1/s	0
1	1/s	0.12
2	1/s(s+3)	0.12
3	1/s(s+3)(s+4)	0.2

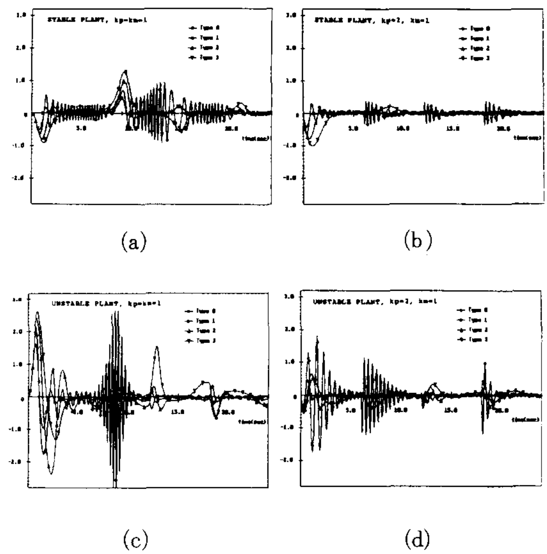

Fig.5. The output error for the four cases.

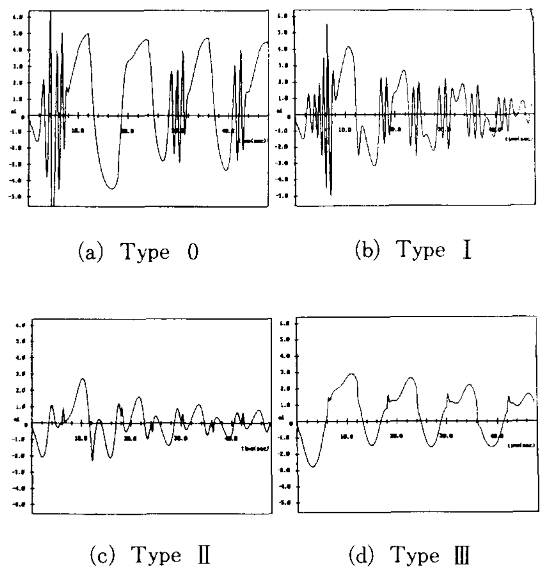

Fig.6. The output error for the case 1 when unmodeled dynamics are present.

simulations. Type 0 represents the conventional simple gradient adaptive law. For Types 1, 2, and 3, the operators H(s)=1/[P(s)-P(0)] of the form a rational function of "s" and corresponding μ's are tabulated in Table 2.

For each case, four types of adaptive laws were applied. The output errors are drawn together in Fig. 5. Since Wm(s) has a stable zero at s=-0.56, the auxiliary signal generators were chosen to have a factor (s+0.56) in the characteristic polynomial. The reference input r(t) in each case was set to a square wave with an amplitude of 4 units and frequency 1/12 Hz. The constant parameter adaptation gain matrix Γ was chosen a unity matrix.

The simulation results for the proposed adaptive law show that this law gives a bounded output error and a bounded parameter vector. And it is observed that Type 0 shows a severe oscillatory transient response but the proposed law doesn't. In the case where no unmodeled dynamics are present, Type 1 exhibit the best output error responses. A slower convergence rate can be observed as the degree of P(s) increases. In general, a higher order integrator causes a slower convergence rate. But the convergence rate can be recovered to a competitive one by raising the magnitude of the adaptation gain matrix.

And it is observed in this simulation that a still better output error transient response may be obtained if a 2n-dimensional parameter adaptation is used. This implies that, in adddition to the filtering of high frequency components of the reference input, a gradual increase of the control input can give a better transient response. Extended time simulations exhibit that the output error $e_1(t)$ goes to 0 as time elapses in all types of adaptive law used.

On the other hand, if unmodeled dynamics are present, a higher order integrator may function as a sharper lowpass filter and gives a better output response and/or a robustness of adaptive control. A simulation result for this situation is shown in Fig. 6. The transfer function of plant is given by $104\ (s+0.5)/[(s+1)\ (s+1.5)\ (s^2+20s+104)]$. Type 2 shows the best results in the sense of a fast convergence and a smaller output error magnitude. Thus it should be considered in selecting P(s) that there is a compromise between the convergence rate of output error and the magnitude of output error (or robustness).

V. Conclusions

In this paper, a generalized adaptive law is proposed which uses a rational function type operator for parameter adjustment. To satisfy the passivity condition of the adaptation block, we introduce a constant feedback gain into the adaptation block. This adaptation scheme is applied to the model reference adaptive control of a continuous-time, linear time-invariant, minimum-phase system whose relative degree is 1. We prove the asymptotic stability of the output error of this adaptive system by byperstability method. It is shown that by digital computer simulations this law can give a better output error transient response in some cases than the conventional gradient adaptive law. And the output error responses for the several types of the proposed adaptation law are examined in the presence of a kind of unmodeled dynamics. An extension of this algorithm is expected to arbitrary relative-degree systems.

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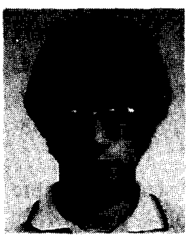

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