526 Journal of Ocean Engineering and Technology, Vol. 3, No. 2, pp. 526~534, December, 1989

⦿ Technical Paper

Dynamic Analysis of Cable with Intermediate Submerged Buoys for Offshore Applications+
— Frequency Domain Analysis —

H. Shin*
(*Received October* 7, 1989)

해양응용을 위한 중간잠수부표들이 부착된 케이블의 동적해석+

신 현 경*

Key Words : Cable-Buoy System(케이블-부표시스템), Self-Weight(자중), Mooring Line(계류선), Intermediate Submerged Buoys(중간잠수부표), Frequency Domain(주파수영역), Natural Frequency(고유진동수), Terminal Impedance(케이블 상단장력)

초 록

심해에서 사용되는 계류선의 큰 자중(self-weight)에 관련된 문제를 해결하기 위하여 케이블-부표 시스템을 이용하는 것은 매우 큰 도움이 된다. 계류선에 적절한 중간잠수부표(intermediate submerged buoys)를 붙임으로써 계류선에 발생하는 최대 장력을 줄일 수 있고, 따라서 케이블의 직경을 감소시킬 수 있다.

본 논문에서는 중간잠수부표가 부착된 케이블의 비선형 정적 방정식과 선형화된 운동방정식이 유도되고, 주파수 영역해석을 이용하여 케이블의 고유진동수와 케이블 상단에서의 장력을 구하고 그것들에 대한 중간잠수부표의 영향을 조사한다.

Nomenclature

- m : Mass per unit length of the cable
- m_a : Added mass per unit length of the cable
- E : Young's modulus of the cable
- a : Cross-sectional area of the cable
- D_0 : Unstretched diameter of the cable
- W_0 : Weight per unit length of the cable
- B_0 : Buoyancy force per unit length of the cable
- e : Total strain ($e=e_1+e_0$)
- e_0 : Static strain
- e_1 : Dynamic strain
- C_{dn} : Normal drag coefficient of circular cylinders
- C_{dt} : Tangential drag coefficient of circular cylinders
- F_{n0} : Normal static drag force per unit length of the cable
- F_{t0} : Tangential static drag force per unit length of the cable

---

+ Presented at the 1989 KCORE Spring Conference (June, 1989)
* Member, Naval Architecture & Ocean Engineering Dep't., University of Ulsan

F_{nd} : Normal dynamic drag force per unit length of the cable
F_{td} : Tangential dynamic drag force per unit length of the cable
T : Total tension of the cable ($T=T_0+T_1$)
T_0 : Static tension of the cable
T_1 : Dynamic tension of the cable
T_P : Pretension at the top of the cable
s : Lagrangian coordinate
p : Tangential displacement of the cable
q : Normal displacement of the cable
x : Horizontal axis of a Cartesian coordinate system
y : Vertical axis of a Cartesian coordinate system
ϕ : Total angle of the cable ($\phi=\phi_0+\phi_1$)
ϕ_0 : Static angle of the cable
ϕ_1 : Dynamic angle of the cable
U : Current velocity assumed to be paralled to the x-axis
V : Velocity of the buoy
ρ_w : Water density
m_b : Mass of the buoy
a_b : Added mass of the buoy
C_{sp} : Drag coefficient of a sphere
D_b : Diameter of the buoy
B_b : Buoyancy of the buoy
w_b : Weight of the buoy
D_T : Total drag force of the buoy
D_{xb} : x-component of the total drag force to the buoy
D_{yb} : y-component of the total drag force of the buoy
D_{xbs} : x-component of the static drag force of the buoy
D_{ybs} : y-component of the static drag force of the buoy
D_{xbd} : x-component of the dynamic drag force of the buoy
D_{ybd} : y-component of the dynamic drag force of the buoy
F_R : Resultant static force of the buoy
Ψ : Angle of the resultant static force F_R with the x-axis

1. Introduction

The recent move by the offshore industry toward deeper water, made mooring systems very important, and created an interest in studying such systems. Semi-submersibles, for example, are normally positioned with a multi-leg mooring system.

Researchers have studied the nonlinear dynamics of a mooring line in various coordinate systems. Bliek derived the cable dynamic equations by considering the kinematics and dynamics of a mooring line in three dimensions[7]. Shin has extended the dynamic equations derived by Bliek, using a coordinate system which is based on the moving configuration (dynamic reference) of a mooring line[2].

The main function of a mooring system is to provide a holding force. Mooring lines supported by intermediate buoys appear to be best suited for deep water applications, where the weight of the line itself is an important factor. By adding buoys to the mooring line, it is possible to reduce the maximum tensile force and therefore increase its capability for carrying external loads reducing the diameter[4, 9].

So far, most analyses are restricted to a cable without buoys and only a few papers refer to the mooring line with buoys[1, 3, 8, 9].

In this paper, the nonlinear static and linearized dynamic equations of a mooring line with buoys are derived[4]. The dynamic motions are assumed to be small oscillations around a mean position, which is the static configurations. The static equations are solved numerically using the Runge-Kutta method. A set of linearized partial differential equations are solved using the Finite Centered Difference method with linearized damping coefficients.

Terminal impedances, which are sometimes called transfer functions or complex frequency response functions, are calculated and natural frequencies of the cable-buoy system are obtained.

2. Statics

We assume that a buoy is connected to the cable by a hinge at the attachment point. At an attachment point, we should consider equilibrium of the forces due to the buoy and the forces from the adjacent cable elements (Fig. 1). Except at end points, the following static equations along the mooring line are derived from the static equilibrium condition of an infinitesimal segment of the cable (Fig. 2).

$$T_0 \frac{d\phi_0}{ds} = (w_0 - B_0)\cos\phi_0 + F_{n0}(1+\frac{e_0}{2})$$
$$\frac{dT_0}{ds} = (w_0 - B_0)\sin\phi_0 - F_{t0}(1+\frac{e_0}{2})$$
$$\frac{dx}{ds} = \cos\phi_0 \cdot (1+e_0)$$
$$\frac{dy}{ds} = \sin\phi_0 \cdot (1+e_0) \quad \cdots\cdots\cdots (1)$$

Also the following static continuity conditions at an attachment point are as follows (Fig. 1).

$$T_0^- \cos\phi_0^- = T_0^+ \cos\phi_0^+ + D_{xbs}$$
$$T_0^- \sin\phi_0^- = T_0^+ \sin\phi_0^+ + B_b - w_b + D_{ybs}$$
$$x^+ = x^-$$
$$y^+ = y^- \quad \cdots\cdots\cdots (2)$$

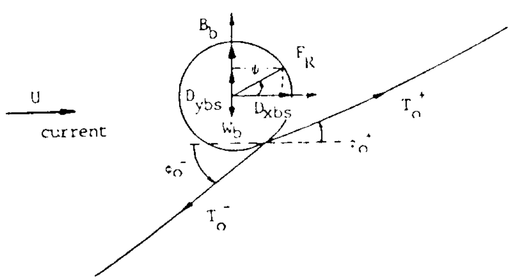

Fig. 1 Static forces at an attachment point

3. Dynamics

The 2-dimensional, linearized equations of motion with nonlinear drag force terms of the cable, whose static configuration is 2-dimensional, expressed along the local tangential and normal directions are obtained from considering the dynamic equilibrium of the cable segment[4),7)] : (see Figures 2 and 3).

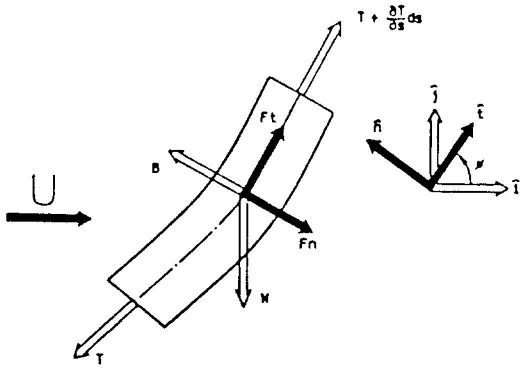

Fig. 2 Forces on a cable segement

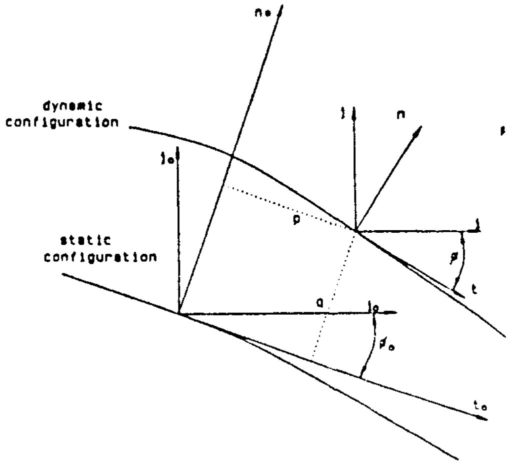

Fig. 3 Dynamic motion

$$m \frac{\partial^2 p}{\partial t^2} = \frac{\partial T_1}{\partial s} - T_0 \frac{d\phi_0}{ds}\phi_1 + F_{td}$$
$$M \frac{\partial^2 q}{\partial t^2} = \frac{dT_0}{ds}\phi_1 + T_0 \frac{\partial \phi_1}{\partial s} + T_1 \frac{d\phi_0}{ds} + F_{nd}$$

$$\frac{\partial p}{\partial s} - q\frac{d\phi_0}{ds} = \frac{T_1}{EA}$$
$$\frac{\partial q}{\partial s} + p\frac{d\phi_0}{ds} = \phi_1(1+e_0) \quad \cdots\cdots (3)$$

with

$$\phi = \phi_0 + \phi_1$$
$$V_n = -\frac{\partial p}{\partial t}\sin\phi_1 + \frac{\partial q}{\partial t}\cos\phi_1$$
$$V_t = \frac{\partial p}{\partial t}\cos\phi_1 + \frac{\partial q}{\partial t}\sin\phi_1$$
$$M = m + m_a \quad \cdots\cdots (4)$$

Boundary conditions :

$$p(0, t) = 0 \qquad q(0, t) = 0$$
$$p(l,t) = h(t)\cos(\phi_{otop} - \theta) \qquad q(l,t) = -h(t)\sin(\phi_{otop} - \theta) \quad \cdots\cdots (5)$$

Initial conditions ;

$$p(s,0) = f_1(s) \qquad \frac{\partial p}{\partial t}(s,0) = f_3(s)$$
$$q(s,0) = f_2(s) \qquad \frac{\partial q}{\partial t}(s,0) = f_4(s) \quad \cdots\cdots (6)$$

where $f_1(s)$, $f_2(s)$, $f_3(s)$ and $f_4(s)$ are arbitrary functions of the Lagrangian coordinates and h(t) is an excitation displacement imposed on the top of the cable (Fig. 4).

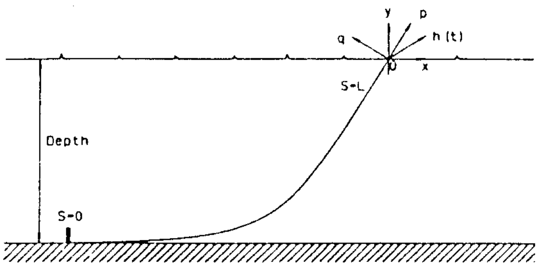

Fig. 4 Excitation and Largrangian coordinates of a cable with no attached buoys

3.1 Drag Forces on a Submerged Buoy

We assume that the submerged intermediate buoy is fixed on the cable, and we do not allow buoy rotation about the attachment point. The hydrodynamic forces on the buoy are described by a Morison type loading, based on the relative motions between the buoy and the surrounding fluid. In order to simplify the dynamic problem, the incident current velocity is assumed to be parallel to the x-axis (Then, $D_{ybs} = 0$).

The drag forces on a submerged buoy are described as follows ; (see Fig. 5)

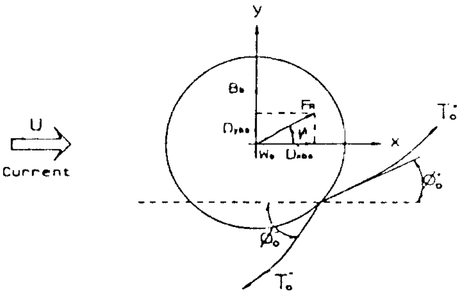

Fig. 5 Static forces at an attachment point

$$D_T = \frac{1}{2}\rho_w C_{sp}\frac{\pi}{4}D_b^2(\bar{U}-\bar{V})\,|\,\bar{U}-\bar{V}\,| \quad \cdots\cdots (7)$$

The total drag force is decomposed into two components ;

$$D_{xb} = D_T\cos\gamma \qquad D_{yb} = D_T\sin\gamma$$
$$\text{with } \bar{U} = (U,0) \qquad \gamma = \tan^{-1}\frac{-V_y}{U-V_x}$$
$$\bar{V} = (V_x, V_y) \quad \cdots\cdots (8)$$

Therefore the dynamic drag forces on a buoy are obtained from (8) as :

$$D_{xbd} = D_{xb} - D_{xbs}$$
$$D_{ybd} = D_{yb} \quad \cdots\cdots (9)$$

with

$$D_{xbs} = \frac{1}{2}\rho_w C_{sp}\frac{\pi}{4}D_b^2 U^2$$

3.2 Dynamic Equations of a Submerged Intermediate Buoy

From the dynamic equilibrium condition at an attachment point, we obtain the following equations :

$$m_b\ddot{x} = -a_b\ddot{x} + T^+\cos\phi^+ - T^-\cos\phi^- + D_{xb}$$
$$m_b\ddot{y} = -a_b\ddot{y} + T^+\sin\phi^+ - T^-\sin\phi^- + B_b - W_b + D_{yb}$$
$$x^+ = x^-$$
$$y^+ = y^- \quad \cdots\cdots (10)$$

The simplified dynamic equations with nonlinear drag force terms at an attachment point (Fig. 5) by substituting $T = T_o + T_d$ and $\phi = \phi_o + \phi_d$ in (10), subtracting the relations (2) and (9) and neglecting higher order terms ($T_d^+ d^+$ and $T_d^- d^-$, etc).

$$(m_b + a_b)\ddot{x} = T_d^+\cos\phi^+ - T_d^+\phi_d^+\sin\phi_o^+ - T_d^-\cos\phi_o^- + T_d^-\phi_d^-\sin\phi_o^- + D_{xbd}$$
$$(m_b + a_b)\ddot{y} = T_d^+\sin\phi^+ + T_d^+\phi_d^+\cos\phi_o^+ - T_d^-\sin\phi_o^- - T_d^-\phi_d^-\cos\phi_o^- + D_{ybd}$$
$$x^+ = x^-$$
$$y^+ = y^- \quad \cdots\cdots (11)$$

with

$$x = p\cos\phi_o - q\sin\phi_o$$
$$y = p\sin\phi_o + q\cos\phi_o \quad \cdots\cdots (12)$$

4. Frequency Domain Analysis

We assume that all dynamic quantities vary sinusoidally with time and we introduce the equivalent linearized damping coefficient in order to linearize nonlinear fluid drag forces acting on the the cable segment[1), 2)].

$$F_p = -b_p\frac{\partial p}{\partial t} : \quad b_p = \rho_w C_{Dt} D\frac{4}{3\pi}\omega p_a$$

(for small current)

$$F_q = -b_q\frac{\partial q}{\partial t} : \quad b_q = \rho_w C_{Dn} D\frac{4}{3\pi}\omega q_a \quad \cdots\cdots (13)$$

Therefore we can construct the following matrix equation from the cable dynamic equations (3) using the equivalent damping coefficient.
Boundary Conditions are obtained from (5).

$$\begin{bmatrix} \frac{dT_1}{ds} \\ \frac{d\phi_1}{ds} \\ \frac{dp}{ds} \\ \frac{dq}{ds} \end{bmatrix} = \begin{bmatrix} 0 & T_0 d\phi_0/ds & -m\omega^2 + \omega b_p i & 0 \\ -\frac{d\phi_0}{T_0 ds} & -\frac{dT_0/ds}{T_0} & 0 & \frac{1}{T_0}(-M\omega^2 + \omega b_q i) \\ \frac{1}{EA} & 0 & 0 & \frac{d\phi_0}{ds} \\ 0 & 1+e_0 & -\frac{d\phi_0}{ds} & 0 \end{bmatrix} \begin{bmatrix} T_1 \\ \phi_1 \\ p \\ q \end{bmatrix} \quad \cdots\cdots (14)$$

$$p(0) = 0 \qquad p(L) = h_a\cos(\phi_{o\,top} - \theta)$$
$$q(0) = 0 \qquad q(L) = -h_a\sin(\phi_{o\,top} - \theta) \quad \cdots\cdots (15)$$

Where h_a is the amplitude of h(t) in case of harmonic excitation.

5. Terminal Impedances.

A concise method of describing the frequency dependence of the amplitude and phase is to give the complex frequency response functions S(ω).

When the upper end of the cable is excited by an externally imposed harmonic motion the mooring line terminal impedances are defined in the following way (Fig. 6)[5)].

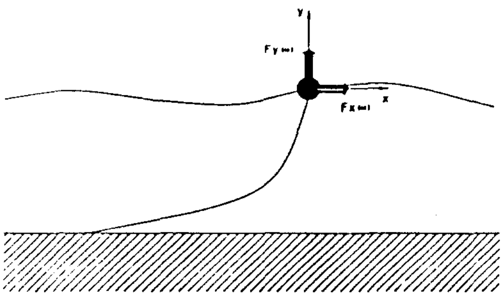

Fig. 6 Terminal impedances

$$\begin{bmatrix} S_{xx}(\omega) & S_{xy}(\omega) \\ S_{yx}(\omega) & S_{yy}(\omega) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \end{bmatrix} \quad \cdots (16)$$

where,

- x = Complex amplitude motion, horizontal direction
- y = Complex amplitude motion, vertical direction
- F_x = Complex amplitude force, horizontal direction
- F_y = Complex amplitude force, vertical direction.

At the top of the cable, sinusoidal motions in the x-direction and the y-direction are imposed. Then the dynamic forces at the top are obtained as :

$$S_{xx} = [T_{11} \cdot \cos\phi_0 - T_0 \cdot \sin\phi_0 \cdot \phi_{11}] / A_x$$
$$S_{yx} = [T_{11} \cdot \sin\phi_0 + T_0 \cdot \cos\phi_0 \cdot \phi_{11}] / A_x$$
$$S_{xy} = [T_{12} \cdot \cos\phi_0 - T_0 \cdot \sin\phi_0 \cdot \phi_{12}] / A_y$$
$$S_{yy} = [T_{12} \cdot \sin\phi_0 + T_0 \cdot \cos\phi_0 \cdot \phi_{12}] / A_y$$
$$\cdots\cdots\cdots \quad (17)$$

Where, $T_{11}(T_{12})$ and $\phi_{11}(\phi_{12})$ are the dynamic tension and the dynamic angle, respectively, caused by external motion in the x(y)-direction.

A_x and A_y are the external motion given at the top of the cable.

6. Numerical Applications

The static equations (1) are solved numerically using the Runge-Kutta method. A set of linearized partial differential equations, which consist of the dynamic equations (3) and appropriate boundary conditions (5), are solved using the Finite Centered Difference method with linearized damping coefficients. Natural frequencies are calculated in addition to the static configurations and terminal impedance matrices.

The principal parameters of the inclined cable are shown in Table 1. For the convenience of computations, we assume that the buoyancy of a buoy is much larger than its weight and the total buoyancy force of buoys is equal to the total weight of the cable in water. The excitation is imposed on the top of the cable in the horizontal direction.

Table 1 Principal parameters of the inclined cable

Length = 700*m*	Mass = 49.24*kgf/m*
Weight = 425.31 *N/m*	*Dia* = *0.09m*
Depth = 400*m*	U(current) = 1*m/s*
Pretension = 500000*N*	Added mass = 6.5*kgf/m* (*of cable*)

The static configuration of the inclined cable without buoys is shown in Fig. 7, and Figures 10 and 15 show the cable-buoy systems with one buoy and two buoys respectively.

Figures 8 and 9 show the real part and imaginary part respectively, of the terminal impedances (complex frequency responses) of the inclined cable without buoy, which denotes the top tensile force caused by the excitation with unit amplitude.

Terminal impedances of the cable with a buoy are shown in Figures 11 to 14. Also, terminal impedances of the cable with two buoys are shown in Figures 16 to 18.

The complex frequency response S_{xx} at $\omega = 0$, specially, denotes the holding force of the cable-buoy system and the calculated S_{xx} are shown in Table 2.

Figures 11, 12 and 16 show the natural frequencies of the cable-buoy system with a boy and two buoys, respectively and the results are summarized in Table 2.

Table 2 Natural frequencies and holding forces

Case \ N.F.	Without buoy	1 buoy	2 buoys
ω1	0.7379(*rad/s*)	0.4156	0.3822
ω2	1.0803	0.8669	0.9809
ω3	1.5127	1.6319	1.3633
S_{xx}, ω=0	12264.7(*N/m*)	27363.7	78760.6

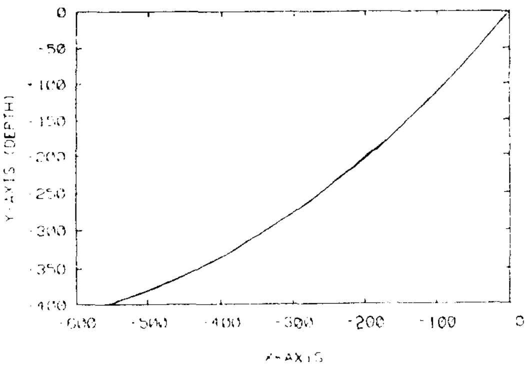

Fig. 7 Static configuration : without buoy, non-zero damping

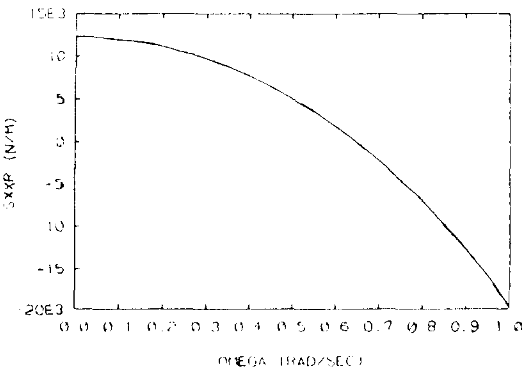

Fig. 8 Real part of S_{xx} : without buoy, non-zero damping

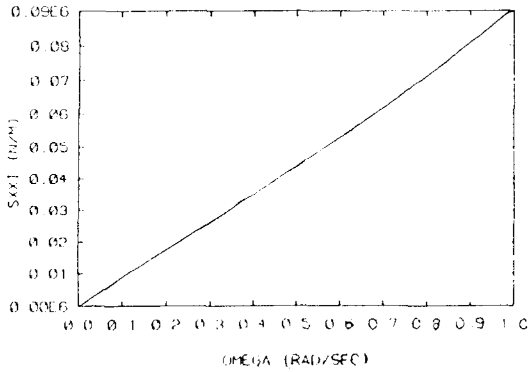

Fig. 9 Imaginary part of S_{xx} : without buoy, non-zero damping

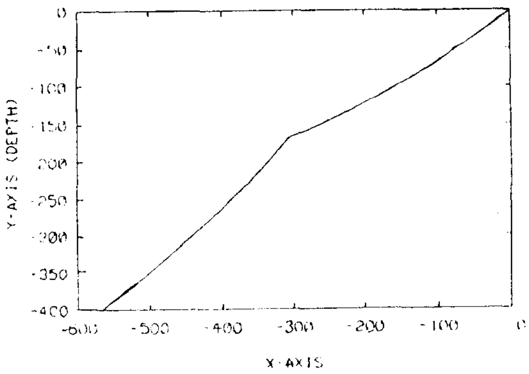

Fig. 10 Static configuration : 1 buoy, non-zero damping

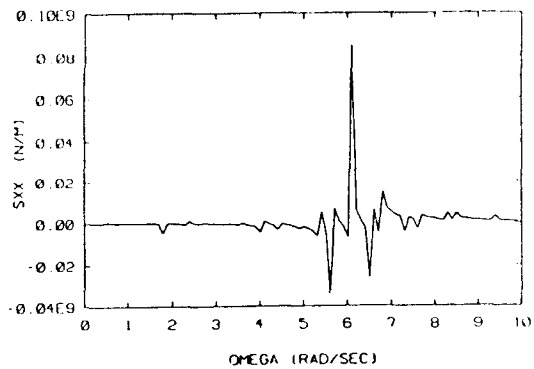

Fig. 11 S_{xx} : 1 buoy, no damping(natural frequencies)

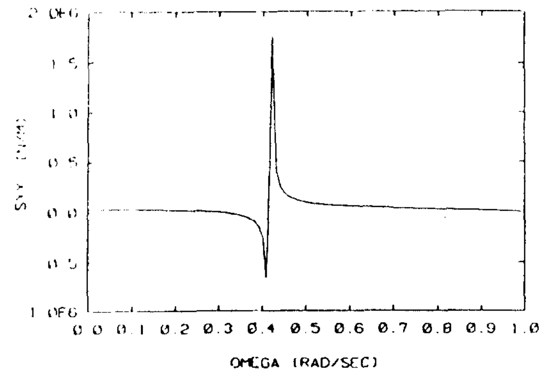

Fig. 12 (Repeated), S_{xx} : 1 bouy, no damping(natural frequency)

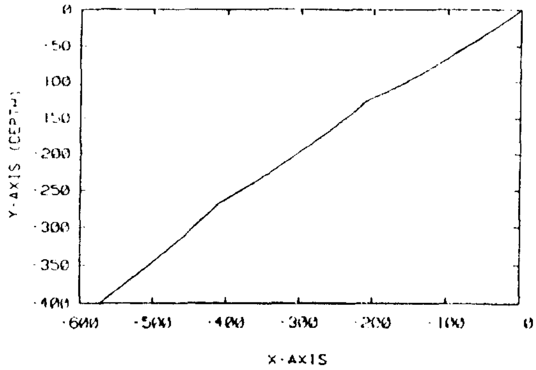

Fig. 15 Static configuration : 2 buoys, non-zero damping

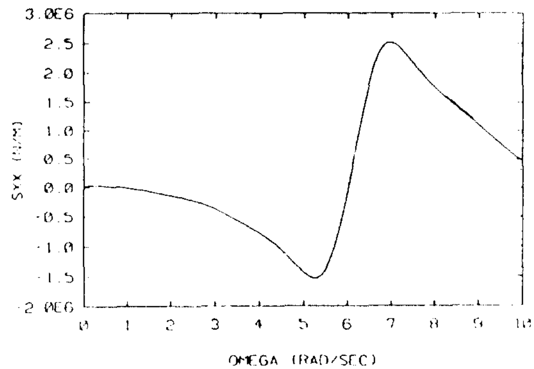

Fig. 13 Real part of S_{xx} : 1 buoy, non-zero damping

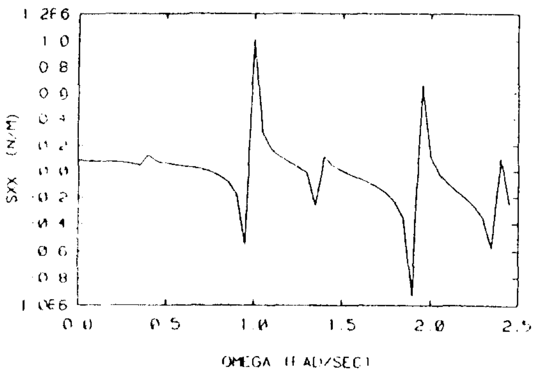

Fig. 16 S_{xx} : 2 buoys, no damping(natural frequencies)

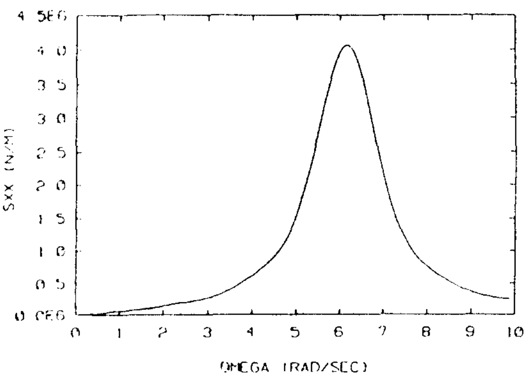

Fig. 14 Imaginary part of S_{xx} : 1 buoy non-zero damping

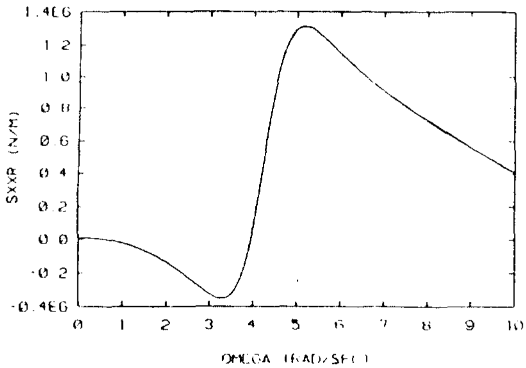

Fig. 17 Real part of S_{xx} : 2 buoys, non-zero damping

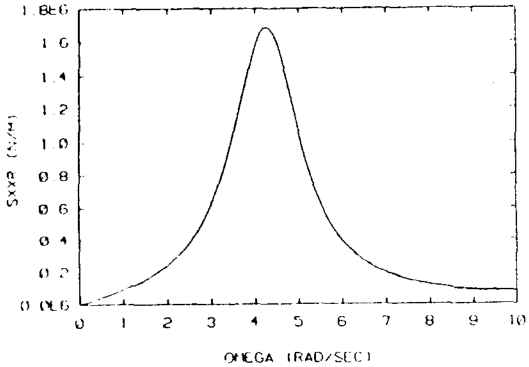

Fig. 18 Imaginary part of S_{xx} : 2 bouys, non-zero damping

7. Conclusions

As the number of buoys increases, the holding force (S_{xx}, $\omega=0$) in Table 2 increases and, therefore, it is possible to carry larger external loads.

The natural frequencies of a mooring line with buoys are smaller than those of a corresponding mooring line without buoys. Such a decrease in natural frequencies may be helpful or not in the dynamic behaviour of the mooring line, depending upon the surrounding ocean conditions.

Finally, we can see that the cable-buoy systems are very useful to solve the problems associated with the larger weight of the mooring line.

Acknowledgements

The research reported here is supported by nondirected research fund.

References

1) Shin, H, and M.S. Triantafyllou, "Dynamic Analysis of Cable with an Intermediate Submerged Buoy for Offshore Applications", Proceedings of the Eighth Joint International Conference on Offshore Mechanics and Polar Engineering, The Hague, The Netherlands, March. 1989
2) Shin, H., "Linearization of the Nonlinear Drag Forces Acting upon Cable", Journal of Ocean Engineering and Technology, Vol. 2, No. 2, pp. 221~226, December, 1989
3) Hanna, S., P. G. S. Dove, and G. B. H. Breese, "The Design, Procurement and Installation of Placid Oil Companys Green Canyon Floating Production Platform Mooring System", Proceedings of the Seventh International Conference of Offshore Mechanics and Arctic Engineering, Vol. 1, Houston, pp. 1~12, 1988
4) Shin, H., "Nonlinear Cable Dynamics" PhD thesis, MIT, 1987
5) Triantafyllou, M.S., A. Bliek, J. and H. Shin, "Mooring Dynamics for Offshore Applications, Part 1". MITSG 86-1, January. M. I. T., 1987
6) Triantafyllou, M. S., A. Bliek, and H. Shin, "Dynamic Analysis as a Tool for Open-Sea Mooring System Design", Transactions, The Society of Naval Arichitecture and Marine Engineering (12), November, 1985
7) Bliek, A., "Dynamic Analysis of Single Span Cables", PhD thesis, MIT, 1984.
8) Nakajima, T., S. Motora, and M. Fujino, "On the Dynamic Analysis of Multi-Component Mooring Lines", OTC 4309, the 14th Annual OTC in Houston, Texas, May. 1982
9) Berteaux, H. O., "Buoy Engineering", John Wiley & Sons, New York., 1976