ALGORITHMS FOR AGGREGATE INTERLOCK CONSTITUTIVE BEHAVIOR AT R/C CRACKS

철근콘크리트 균열에서의 골재맞물림 거동에 대한 해석연구

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요 약

이 논문에서는 철근콘크리트 균열에서의 활동전단변형에 대해 골재맞물림에 따라 수반되는 저항응력을 예측할 수 있는 해석방법들이 개발되어졌다. 이러한 방법들은 거의 정확하게 실험결과들을 예측하였으며 초기 균열폭, 콘크리트 압축강도, 최대 골재크기, 그리고 균열틈에 따라 수반되는 구속력에 의해 영향을 받는 골재맞물림 거동에 대한 해석연구를 위해 사용되어졌다.

ABSTRACT

Algorithms have been developed for predicting the resistance provided by aggregate interlock against sliding shear deformations at reinforced concrete cracks. These methods are shown to be capable of Perdicting test results with a reasonable accuracy, and they are used for an analytical study on the aggregate interlock behavior as influenced by the initial crack width, concrete compressive strength, maximum aggregate size, and the restraint provided against crack opening.

INTRODUCTION

Sliding shear deformations at the open cracks in reinforced concrete structures are resisted by the dowel action of steel bars crossing the cracks, and also by the aggregate interlock resulting from roughness of the crack faces(Fig. 1). The aggregate interlock resistance is partially provided by the bearing of the adjacent pieces of aggregate and mortar at the two crack faces against each other. Sliding also results in overriding of the roughnesses of crack faces and thus opening of the crack. The crack opening is resisted by the tension in the steel bars crossing the crack, which in turn induce pressure on the crack faces. The frictional forces resulting from this action also contribute to the aggregate interlock resistance.

Sliding across the cracks is a major cause of shear deformations in reinforced concrete elements and connections.[1, 2, 3] Limited analytical and experimental studies have been per-

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formed on the sliding shear resisting mechanisms(aggregate interlock and dowel action). This investigation was conducted to develop some analytical techniques needed in refined analysis of the shear behavior of reinforced concrete elements and structures.

BACKGROUND

In the reported experimental studies on aggregate interlock, [4, 5, 6, 7] a shear force has generally been applied across a crack with a preset initial width. Further crack opening with the increase in shear force has been either partially restrained by some external reinforcement[Fig. 2(a)], or fully prevented by a transverse pressure normal to the crack[Fig. 2(b)]. A typical shear stress−sliding shear deformation diagram obtained from such aggregate interlock tests is shown in Fig. 3. Initially, the aggregate interlock stiffness is low, but it increases with sliding as the contacts between crack roughnesses increase. Thereafter, the stiffness stays constant and then starts to decrease with further sliding up to the peak resistance. The post−peak region of the shear stress−sliding defection diagram is almost flat.

The aggregate interlock stiffness and strength have been oabserved to increase as the restraining stiffness(or pressure)and concrete compressive strength increase and the initial crack width decreases. There is also a slight increase in stiffness and strength with increasing maximum aggregate size, provided that good quality cement paste and aggregates are used.

The following empirical expressions have been developed by different investigatior for predicting the initial aggregate interlocking stiffness(K_a, MPa/㎜)in terms of the initial crack width(C,㎜)and the compressive strength of concrete(f'_c, MPa) :

Based on tests with a constant crack width (restraining pressure) :

Ref.8 : $K_a = (3.21/C - 2.28)(0.307\sqrt{f'_c} - 0.016C)$ ································ (1)

for tests with $0.06 \leq C \leq 0.4$㎜
$18.5 \leq f'_c \leq 55.5$MPa

Ref. 9 : $K_a = 0.98(1/C)^{1.5}(f'_c/34.5)^{0.5}$ ··(2)

for tests with $0.05 \leq C \leq 0.5$㎜
$16.5 \leq f'_c \leq 50.5$MPa

Ref. 5 : $K_a = (3.34/C - 2.25)$ ········· (3)

for tests with $0.08 \leq C \leq 0.5$㎜
$f'_c = 34.5$MPa.

Based on tests with a variable crack width in which a restraining stiffness(K_r, MPa/㎜)is provided by steel bars :

$$\text{Ref. 7} : K_a = \frac{1}{566.42(C-0.05) - 8.487\,K_r/C + 135.42} \quad \cdots\cdots (4)$$

for tests with $0.127 < C \leq 0.76$㎜
$19.5 \leq f'_c \leq 26.5$MPa

A more comprehensive model for predicting the constitutive behavior of aggregate interlock has been developed in Ref. 10. In this model, which is developed on the basis of both test data and the physics of aggregate interlock behavior, the shear and normal stresses(δ_t and δ_n, respectively, MPa)are expressed in terms of the sliding deformation and opening of the crack(δ_t and δ_n, respectively, ㎜) :

$$\sigma_t = 0.245f'_c \cdot \frac{0.01D^2}{0.01D^2 + \delta^2_n} \cdot \frac{\delta_t}{\delta_n} \cdot \frac{10/f'_c + 2.44(1 - 16.31/f'_c)\,|\delta_t/\delta_n|^3}{1 + 2.44(1 - 16.31/f'_c)(\delta_t/\delta_n)^4} \quad (5)$$

$$\sigma_m = \frac{0.00053}{\delta_n} \cdot [\,|145\sigma_t|\,]^p \quad (6)$$

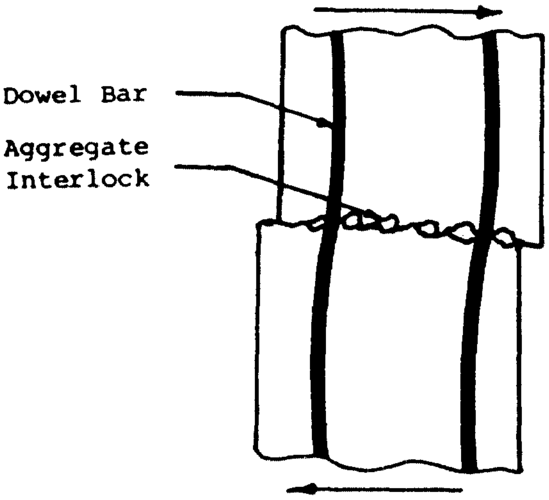


Fig. 1. Mechanisms Resisting Sliding Shear at Cracks

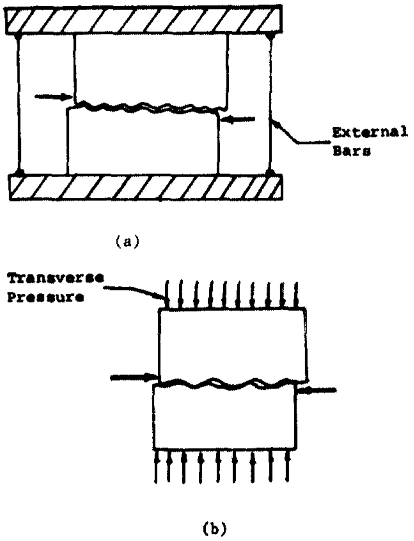


Fig. 2. Agregate Interiock Test Techniques : (a) With Restraining Reinforcement and Variable Crack Width : (b) With Restraining Pressure and Constant Crack Width.

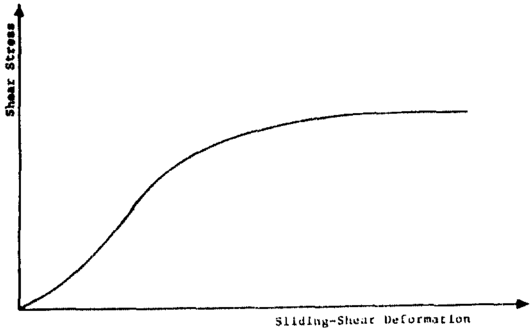


Fig 3. General Shape of the Shear Stress—Sliding Deflection Obtained from Aggregate Interlock Test.

where : $p = 1.30\left(1 - \frac{0.231}{1+0.185\delta_n+5.639\delta_n^2}\right)$; and

D = maximum aggregate size(mm)

In the above model, sliding can occur only after some finite opening of the crack.

ANALYTICAL MODELING

In this study, two algorithms were developed on the basis of Eqns.(5) and (6) for predicting the agtreagate interlock constitutive behavior in both cases with variable and constant crack widths.

For the case with a variable crack width having a restraing stiffness of K_r(before yielding of the restraining steel)and an initial crack width of C :

(1) given the value of sliding shear deformation, δ_t, which is an increment over its previous value, assume a value for the shear stress(σ_t) ;

(2) find the values of normal stress(σ_n)and crack opening(δ_n)by solving equation(6), noting that $\sigma_n = K_r(\delta_n - C) \leq f_y \cdot \rho_S$, where f_y is the restraining steel yield strength and ρ_S is the ratio of the re-

straining steel area to that of the crack surface ;

(3) find the value of sliding shear deformation(estimated δ_t) from Eqn. (5) ;
(4) if the estimated δ_t is not close enough to the input value of δ_t, modify σ_t and repeat from step 2 until convergence is achieved(with the final values of the variables being the solutions) ; and
(5) repeat steps(1) through(4) until loading is completed.

For the case with a constant crack width(C) in which further crack opening is prevented by a restraing pressure :

(1) set the crack opening(δ_n)equal to C ;
(2) assume a sliding shear deformation(δ_t) which is an increment over its previous value ;
(3) find the value of shear stress(δ_t)from Eqn. (5) ;
(4) find the value of normal stress(δ_n)from Eqn. (6) ; and
(5) repeat steps(2)through(4)until loading is completed.

COMPARISON WITH TEST RESULTS

The algorithms introduced above were used to predict the constitutive behavior of aggregate interlock in some tests reported in the literature. Figs. 4(a) through 4(d) compare the experimental results obtained in tests with variable crack widths(constant restraining stiffnesses), with the theoretical results obtained by the proposed algorithm and also by Eqn. (4). The proposed algorithm is observed to predict test results with a reasonable accuracy, while Eqn.(4)can only approximate the intial

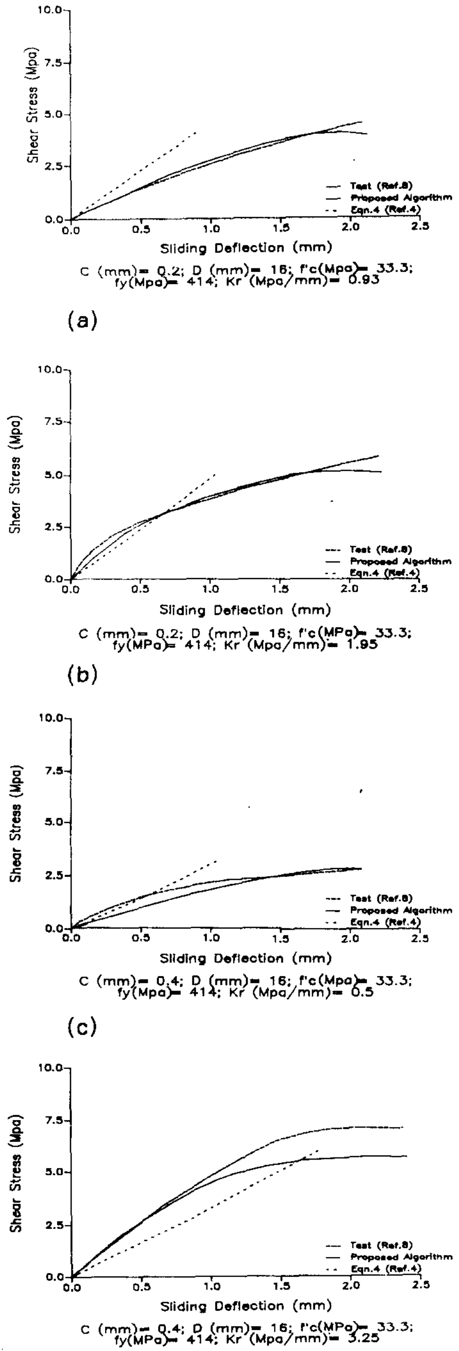
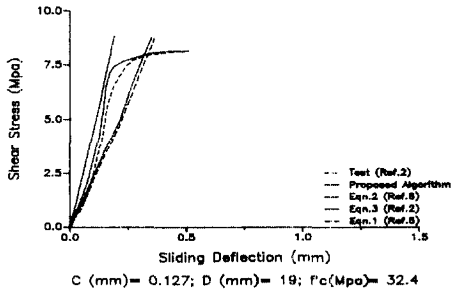
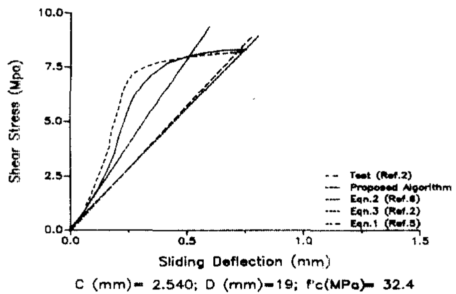


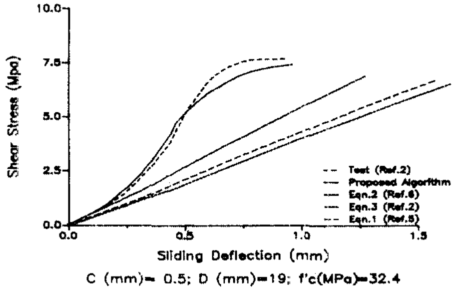
Fig. 4 (d)Theoretical Predictions vs. Test Results(Ref. 8) Perrormed with Constant Restraining Stiffness.(1 in.=25.4㎜ ; 1 ksi=6.895 MPa)



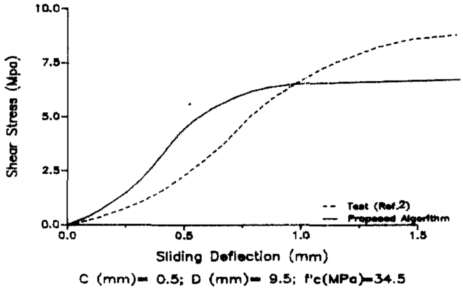
(a)Ref. 2 for c=0.005 in.



(b)Ref. 2 for c=0.01 in.

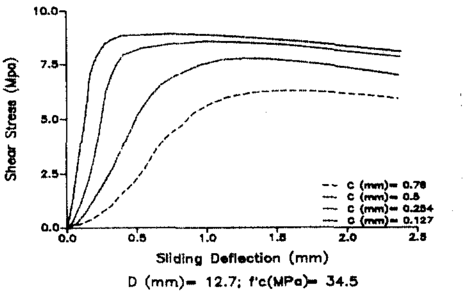


(c)Ref. 2 for c=0.02 in.

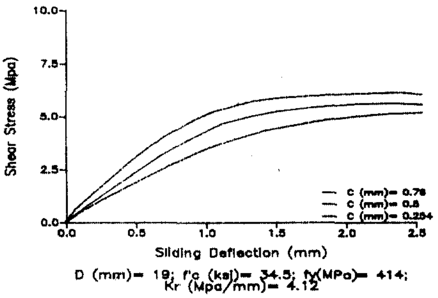


(d)Ref. 2 for c=0.02 in.

Fig.5. Theoretical Predictions vs. Test Results Performed with a Constant Crack Width :



(a)Constant Crack Width



(b)Constant Restraining Stiffness

Fig. 6. Effect of the Initial Crack Width on Aggregate interlock Behavior

stiiffness of aggregate interlock.

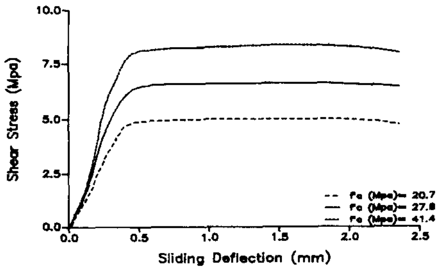
results of tests performed with a constant crack width are compared in Figs. 5(a)through 5(d)with the predictions of the proposed algorithm as well as those of Eqns. (1), (2) and (3). In this case also, the proposed algorithm predicts test results satisfactorily. Eqns. (1), (2) or (3), however, can only give a rough measure of the initial aggregate interlock stiffness.

PARAMETRIC STUDIES

The proposed algorithms were used to study the effects of the initial crack width, concrete compressive strength, maximum aggregate size, and the magnitude of the restraining stiffness on the constitutive behavior of aggregate interlock. The initial crack width is observed in Figs.

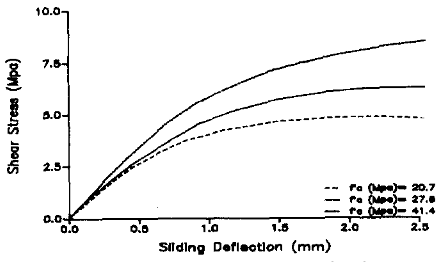
6(a) and 6(b) to have detrimental effects on the aggregate interlock behavior in cases with constant and variable crack widths, respectively. The stiffness and ultimate strength of aggregate interlock in both cases increase significantly with decreasing the initial crack width.

Increasing the concrete compressive strength is also shown in Figs. 7(a) and 7(b) for cases with constant and variable crack widths, respectively, to considerably increase the aggregate interlock stiffness and ultimate strength. Fig. 8(a) and 8(b) show the effect of maximum aggregate size on the aggregate interlock behavior for relatively small and large constant crack widths, respectively. The maximum aggregate size effect seems to be important only in conditions with large intitial crack width, For the case with a variable crack width. however, as shown in Fig. 8(c) and 8(d), the effect of maximum aggregate size seems to be important irrespective of the intial crack width.



C (mm)= 0.254; D (mm)= 38

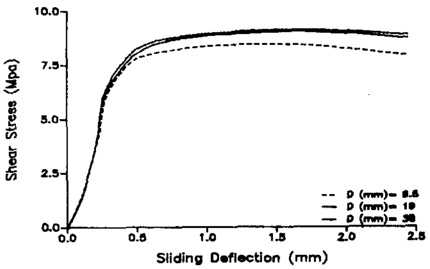
(a)Constant Crack Width



C (mm)= 0.254; D (mm)= 38; fy (Mpa)= 414; Kr (Mpa/mm)= 4.12

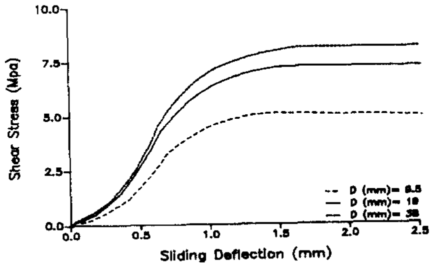
(b)Constant Restraining Stiffness

Fig. 7. Effect of the Concrete Compressive Strength on Aggregate Interlock Behaior



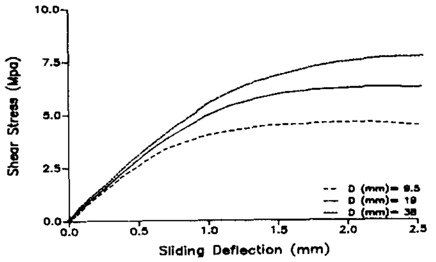
C (mm)= 0.254; f'c (Mpa)= 34.5

(a)Relatively Small Constant Crack Width



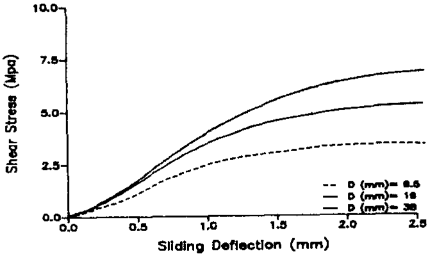
C (mm)= 0.76; f'c (Mpa)= 34.5

(b)Relatively Large Constant Crack Width



C (mm)= 0.254; f'c (Mpa)= 34.5; fy (Mpa)= 414; Kr (Mpa/mm)= 4.12

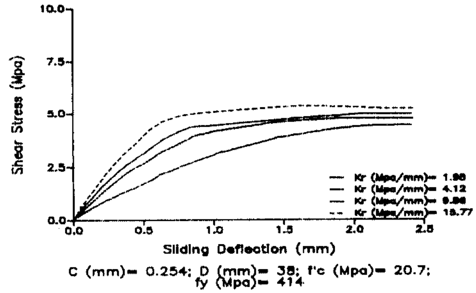
(c)Constant Restraining Stiffness with Relatively Small Initial Crack Width



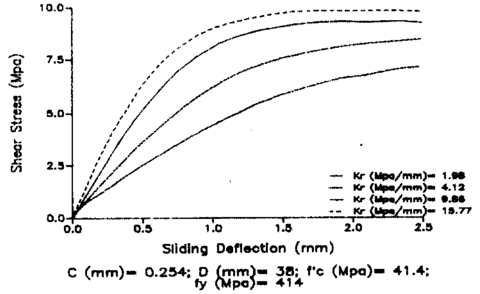
C (mm)=0.76; f'c (Mpa)= 34.5; fy (Mpa)= 414; Kr (Mpa/mm)= 4.12

(d)Constant Restraining Stiffness with Relatively Large Initial Crack Width

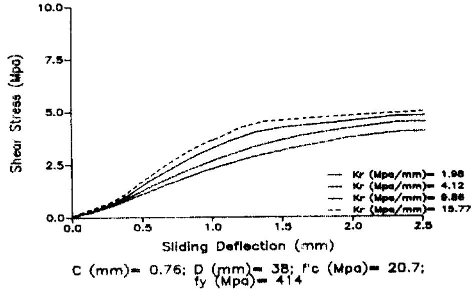
Fig. 8. Effect of the Maximum Aggregate Size on Aggregate Interlock Behavior.



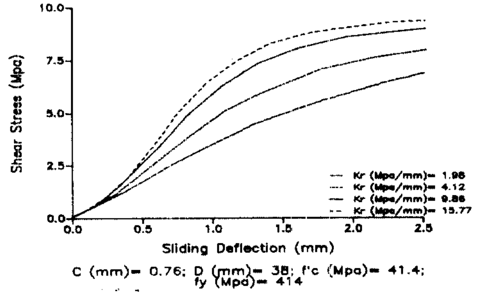
(a) Relatively Low Initial Crack Width and Concrete Compressive Strength



(b) Relatively Low Initial Crack Width and high Concrete Compressive Strength



(c) Relatively High Initial Crack Width and Low Concrete Compressive Strength



(d) Relatively High Initial Crack Width and Concrete Compressive Strength

Fig. 9. Effect of the Magnitude of Restraining Stiffness on Aggregate Interlock Behavior

In the case with a variable crack width, the magnitude of the restraining stiffness is observed in Figs. 9(a)through 9(d) to be another variable significantly influencing the constitutive behavior of aggregate interlock. This influence shows similar tendencies irrespective of the value of initial crack width and concrete compressive strength.

SUMMARY AND CONCLUSIONS

Algorithms were developed for predicting the aggregate interlock constitutive begavior using some empirical formulations. The analytical results obtained from these algorithms compared well with test results performed with either constant or variable crack widths. A numerical study with****** the developed algorithms indicated that the sliding shear stiffness and strength provided by aggregate interlocking increase significantly with decreasing crack width and increasing concrete compressive strength and restraing stiffness, and also to some extent with increasing maximum aggregate size.

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