INELASTIC MODELING OF REINFORCED CONCRETE ELEMENTS UNDER CYCLIC FLEXURAL AND SHEAR FORCES

휨 및 전단 반복하중을 받는 철근 콘크리트 부재의 비선형해석 모델

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요 약

반복하중하에서 휨과 전단거동을 구분할 수 있는 철근콘크리트의 비선형해석 모델이 개발되었다. 개발된 모델은 부재를 single component부재로 이상화시켰으며, 휨과 전단거동을 각각 표현할 수 있는 hysteretic rules의 결합형태로 이루어졌다. 개발된 모델을 이용한 반복하중을 받는 캔틸레버 보의 해석결과는 실험결과와 비교되었고, 그 결과는 만족스러운 값을 얻었다.

ABSTRACT

An inelastic element model capable of distinguishing between the shear and flexural hysteretic characteristics for reinforced concrete elements was developed. The developed model consists of a single component physical idealization of the element and hysteretic rules for flexural and shear force resistance mechanisms. The predictions using the developed inelastic model were compared with the results of a number of cyclic tests performed on reinforced concrete cantilever beams, and the comparisons between test and theory were satisfactory.

1. INTRODUCTION

Total lateral displacements in reinforced concrete beams result from the flexural and shear deformations. Each of these displacment components has distinct characteristic(Figure 1) [Refs. 2, 5]. The shear hysteresis is distingished from the flexural one by servere stiffness and strength deteriorations and lower energy dessipation. Shear deformations, due to their deteriorationg nature, tend to gradually dominate the lateral response of R/C beams to repeated inelastic load reversals.

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The available analytical models for predicting the hysteretic behavior of R/C beams are generally based on the assumption that flexure fully dominates the element behavior[8]. These models can not properly idealize the inelastic shear deformation, and this generally leads to poor comparisons between the analytical and experimental hystertic curves, especially after large inelastic cyclic deformations in elements subjected to relatively high shear stresses.

A practical model for predicting the hysteretic behavior of R/C beams has been developed in this paper. The model accounts for the distinct hysteretic chracteristics of shear and flexural deformations, and its predictions

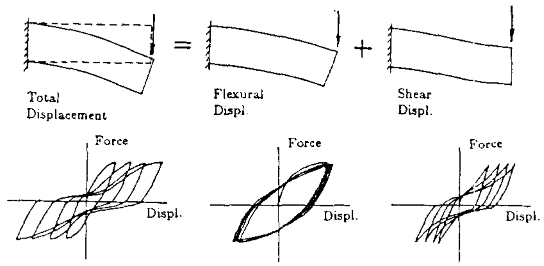


Fig. 1 Lateral Displ. Components of R/C Beams, and Their Hysteretic Characterestics

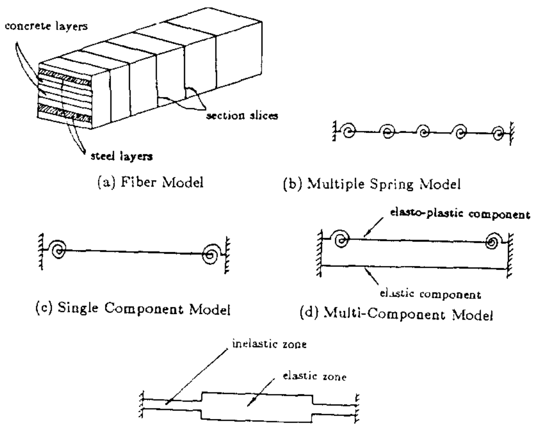


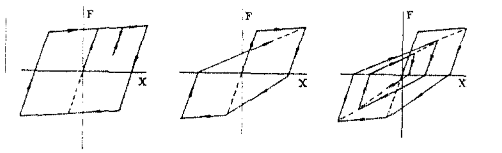
Fig. 2 Physical Models for Simulating the Flexural Behavior of R/C Beams

compare well with test results performed on R/C elements with a variety of geometric and material propeties.

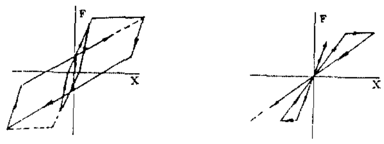
2. CURRENT ANALYTICAL MODELS

The available anlytical models for predicting the hysteretic behavior of R / C beams, usually consisting of a physical model(Figure 2) and some hysteretic rules(Figure 3), generally disregard the significant increase in shear deformations caused by repeated inelastic load reversals. A very popular model has been based on a single component physical idealization of flexural behavior(Figure 2c) and the Takeda hysteresis rules(Figure 3c). The shear deformations are accounted for in this model by assuming that the ratio of the shear to flexural deformations is constant in both the elastic and inelastic ranges. This ratio is, however, relatively small in the elastic range and it has been observed to increase (due to the more severe deteriorations of the shear resisting mechanisms compared with the flexural ones)under inelastic load reversals. Disregard for this increase in shear deformations is shown in Figure 4 to lead to major descrepancies between the analytical and experimental results. These discrepancies are observed to be more pronounced for beams with smaller shear span-to-depth ratios (and consequently higher shear stress levels).

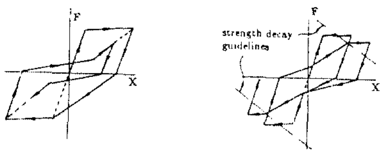
The analytical predictions of the inelasic seismic response chracteristics of R/C frames have generally been based on element models incapable of accurately predicting inelastic shear deformations. Large deviations from test results, similar to the one shown in Figure 4, are expected in the analytical predictions of these elements models. This shortcoming



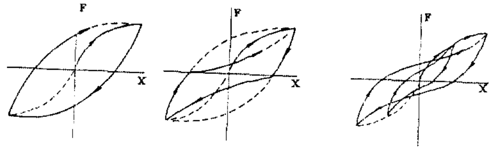
(a) Bilinear Model (b) Clough Model[4] (c) Takeda Model[14]



(d) D-Tri Model[13] (e) Origin-Oriented Model[13]

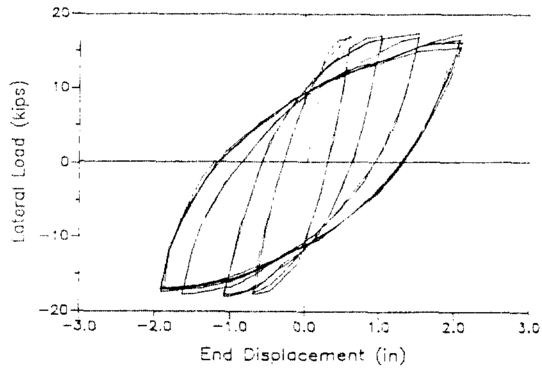


(f) Sinha Model[12] (g) Model with Strength Decay[8]

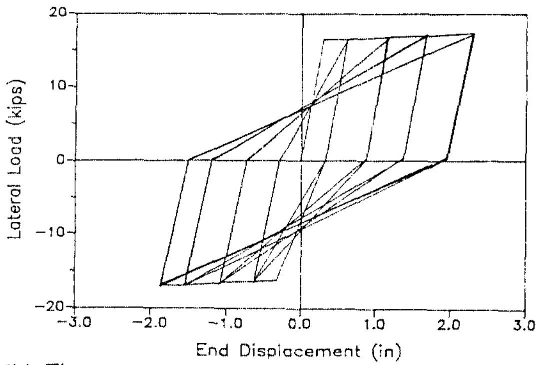


(h) Ramberg-Osgood Model[7] (i) Celebi Model[3] (J) Cubic Model[11]

Fig 3



(a) Experiment



(b) Theory

Fig 4

can produce large discrepancies in the predicted response of frame structures (especiallythosewith relatively deep elements) under severe seismic excitations. Hence, it is important to develop element models which account for the inelastic shear deformations in predicting the hystertic be-havior of R/C beams, in order to achieve more reliable predictions of nonlinear seismic response characteristic of R/C structures.

3. THE PROPOSED ELEMENT MODEL

The element model developed in this study, which consists of a physical idealization of the element and hysterestic rules for flexural and shear force resistance mechanisms, is described below.

3.1 Physical Idelization

Figure 5 presents the physical model of R/C beams used in this study. In this model, all the shear and flexural elasto-plastic deformations under seismic forces are assumed to be concentrated at dimensionless end springs. In order to differentiate between the hysteretic rules of shear and flexure, distinct springs representing each of these types of behavior are attached to the beam. The behavior of each spring is defined by a skeleton curve and a set of hysteretic rules.

The model shown in Figure 5 is a refined version of the single component model. It is practical (economical) because it involves limited number of degreesof freedom. A comprehensive review of the literatures also indicates that its accuracy in predicting the flexural response is comparable to that of the more complex ones like the multiple spring model(Figure 2d) and the single component model with distributed inelasticies(Figure 2e). The key advantage of the model proposed in Figure 5 is, of course, its capacity to account for the distinct hysteretic charac-teristics of shear and flexural deformations which leads to major improvements in its accuracy as the shear deformations tend to dominate the behavior in later inelastic load cycles.

3.2 Hystertic Rules of Flexure and Shear

In order to form the tangent stiffness matrix of the element model presented in Figure 5, the tangent stiffness of each of the spring needs to be drived as a function of the level and history of the loading. In the following, firstly a general set of hysteresis rules capable of simulating a variety of hysteresis characteristics(by proper selection of the variables involved)are introduced.

Then the hysteretic parameters for flexural and shear types of behavior, which have been drived empirically, are presented. In this presentation, the spring force(F) might be flexural or shear force, depending on the type of the spring, and the spring deformation (X) might be flexural rotation or shear deformation. The cyclic force−deformation model presented below consists of a skeleton curve and a set of hysteresis rules.

A bilinear curve(Figure 6) has been selected in this study, mainly due to the fact that the more complex triliner and curvilinear ones do not significantly improve the accuracy of the model[4, 8, 13]. The initial loading takes poace linearly with the stiffness K_i, up to the yield force F_y, at deformation X_y. Thereafter the stiffness drops to a strain hardening value of K_h.

Unloading before yielding takes place on the loading curve (with a stiffness eqaul to K_i). In the post−yield region, the unloading stiffness(K_u) will be decided using the parameter α(See Figure 6). This parameter should be derived empirically for each spring type.

The rule for reloading towards the skeleton curve in the opposite direction is illustrated in Figure 7. The point on the skeleton curve (R) towards which the reloading occurs is defined by a parameter β that can have any value greater than zero.

The basic reloading rules presented in Figure 7 have to be refined to account for the pinching effect which is dominant in shear behavior and the Bauschinger effect

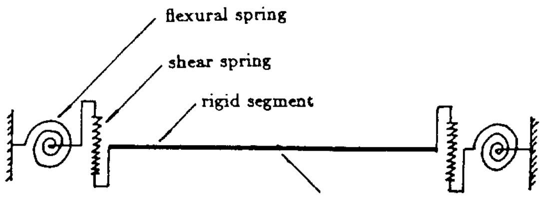


Fig 5 *Proposed Physical Idealization of Element*

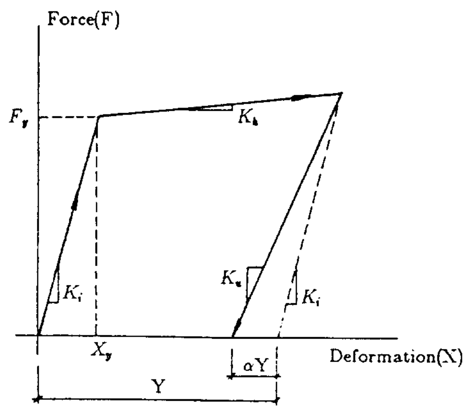
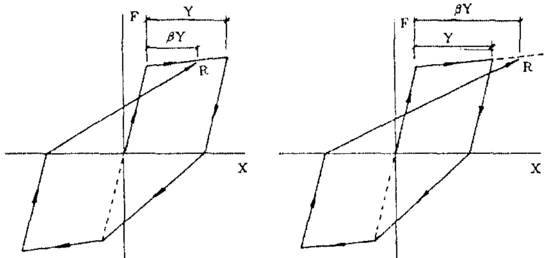


Fig 6 *Skeleton Curve and Unloading from The Skeleton Curve*

displayed especially by the flexural behavior. For this purpose, a line is specified which intersects the horizontal(deformation) axis at the same point as the strain hardening portion of the skeleton curve, and its slope is a fraction (ζ) of the skeleton curve hardening stiffness (see Figure 8). Another hysteretic parameter γ, that is between -1.0 and 1.0, defines the degree to which the actual reloading path deviates from a straight line. A positive value of γ (Figure 8a) is indicative of the Bauschinger effect, while a negative (Figure 8b) leads to the pinching of the reloading path.

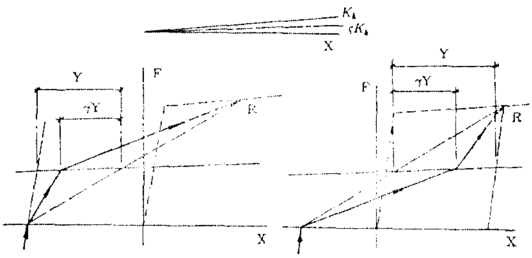
The majority of load cycles sustained by a structural element during a seismic event have deformation amplitudes within the bound of previous maximum and minimum deformations. Upon a small-amplitude load reversal, if the reloading takes place towards the skeleton curve(indicating a large amplitude reversal from skeleton curve in the previous half cycles), the path shown in Figure 9 will be followed. If reloading is in a direction with the previous load reversal being incomplete, the reloading would take place towards the peak point of the incomplete cycle. This reloading path will involve pinching and bowing if the incomplete cycle peak is above the line with slope ζ discussed earlier (Figure 10a). Otherwise, it will simply be a straight line from the horizontal axis to the peak of the previous incomplete cycle (Figure 10b). It should be noted that the small amplitude rules apply as far as the cyclic deformations are within the previous maximum and minimum values. If any of these limits are exceeded, the regular large amplitude rules will be effective again.

The suggested hysteretic rules are quite versatile and, edpending on the values of



(a) $0.0 \leq \beta \leq 1.0$ (b) $1.0 \leq \beta$

Fig 7 *Reloading towards The Skeleton Curve*



(a) $0.0 \leq \gamma \leq 1.0$ (b) $-1.0 \leq \gamma \leq 0.0$

Fig 8 *The Bauschinger and Pinching Effect on The Reloading Path*

thier parameters, they can represent a wide variety of hystertic characteristics as shown in Figure 11.

3.3 Selection of the Hystertic Parameters

In order to complete the hysteretic simulation of the flexural and shear springs in the proposed element model, the yield force, the initial and strain hardenings stiffnesses and the hystertic paremeters (α, β, γ and ζ) of these springs have to be determined. The empirical variahles these values of the analytical methods for calculating them are presented below.

Yielding of the flexural and shear springs in Figure 5 is assumed to occur simultaneously when the bending moment in the flexural spring reaches the flexural yield strength of the R/C cross section. The reported test data of R/C beams support this assumption [5, 6]. The yield moment in flexure can be derived

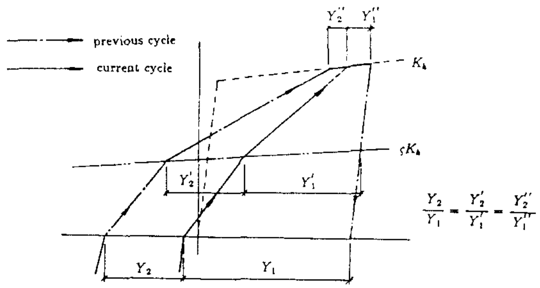


Fig 9 *Small-Amplitude Reloading towards The Skeleton Curve*

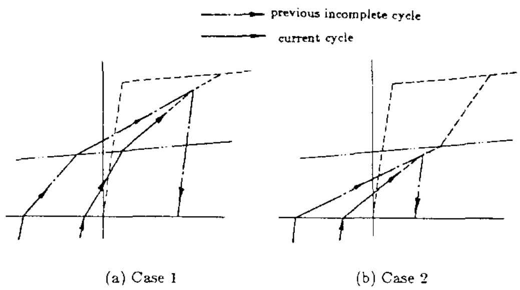


Fig 10 *Small-Amplitude Reloading in A Direction with Previous Incompete Cycle*

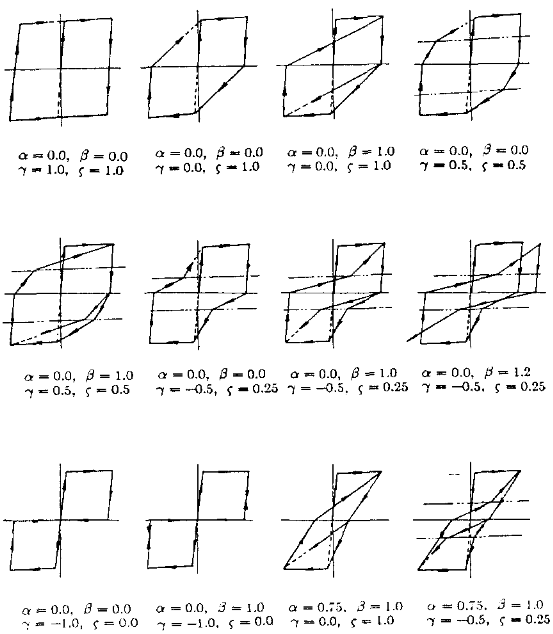


Fig 11 *Different Hysteresis Diagrams Formed By Varing Their Parameters of The Proposed Hysteretic Model*

using the simplifield strain and stress distributions presented in Figure 12. At the yield point, the concrete stress distribution across the section is assumed to be linear, and yielding of the tension steel is assumed to mark the yield point. The depth of neutral axis in Figure 12 can be obtained from the equilibrium of axial forces, and then yield moment of the cross section can be derived. The force in the shear spring at yielding of the flexural spring is assumed to be the yield shear force.

The initial flexural and shear stiffnesses of the springs are based on the moment of inertia and the area of the cracked transformed cross section using the conventional elastic formulations.

The strain hardening ratio(the strain hardening stiffness over the initial stiffness) and the hysteretic parameters of the flexural and shear springs were derived empirically using the test data presened in references 1, 2, 5, 6, 9 and 10. All these references have reported the results of cyclic tests on R/C cantilever beams with separte measurements of the flexural and shear deformations. The test techniques used in these refernces have eliminated the deformations associated with the fixed-end rotation. Table 1 presents the average values of these variables for the flexural and shear springs.

4. FORMULATION OF THE ELEMENT TANGENT STIFFNESS MATRIX

The hysteretic models of flexural and shear springs in proposed element model can be used to derive the tangent stiffness of these springs

Table 1 Empirical Values of The Strain Hardening Ratio and Hysteretic Parameters of The Flexural and Shear Springs

Spring Type	Strain Hardening Ratio	α	β	γ	δ
Flexural	0.021	0.16	0.97	0.42	0.75
Shear	0.043	0.11	1.22	-0.49	0.29

at any step in the loading history. These stiffness es can be used in constructing the overall tangent stiffness matrix of the element. In the following, the tangent stiffness matrix for cantil ever beam(commonly used in experimental studies) will be developed.

The cantilever beam(Figure 13) has only one degree of freedom, that is the lateral displacement (δ) at the free end of the element. This displacement results from the shear displacement of the shear spring and rotation of the flexural spring.

$$\begin{aligned} d\delta &= d\delta_s + \ell \cdot d\theta_f \\ &= \frac{dV}{K_s} + \frac{dV \cdot \ell}{K_f} \\ &= dV\left(\frac{1}{K_s} + \frac{\ell}{K_f}\right) \end{aligned}$$

where,

$d\delta$ = incermental displacement at free end ;
$d\delta_s$ = shear spring incremental displacement
$d\theta_f$ = flexural spring incremental rotation ;
dv = incremental lateral load at free end ;
ℓ = cantilver element length ;
K_s = shear spring tangent stiffness ;
K_f = flexural spring tangent stiffness.

Hence, the tnagent stiffness(K_t) of cantilever element can be derived from the following expression :

$$\begin{aligned} dV &= K_t \cdot d\delta \\ &= \left[\frac{1}{\frac{1}{K_s} + \frac{\ell}{K_f}}\right] d\delta \end{aligned}$$

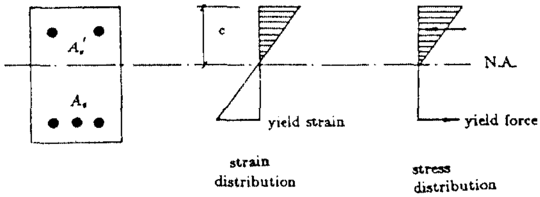


Fig 12 *Simplified Flexural Strain and Stress Distributions at Yielding*

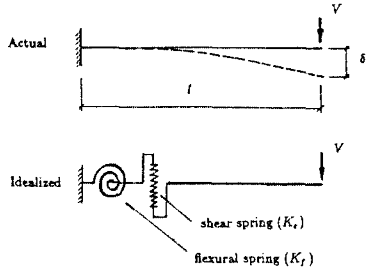


Fig 13 *Cantilever Element*

5. COMPARISON WITH TEST RESULTS

A large number of cyclic test data on R/C cantilever beams have been reported in the literatures[1, 2, 5, 6, 9 and 10]. This section compares the experimental cyclic load-deformation relationships with the predictions of the developed models. Six tested R/C beams with a variety of geometric and material charcteristics were selected for this comparative study. Some major properties of these elements are given in Table 2.

Figure 14(a) and (b) compares the cyclic force-deformation relationships obtained experimentally and theoretically, respectively, for the cantiver beam No. 1(see Table 2). The proposed model is observed to be capable of predicting the test results with a reasonable accuracy.

Figure 15 present the comparison between the experimental and theoretical total cyclic load displacement relationships for beam No. 4 of Table 2. In this beam, which have relatively low shear forces compares to beam Nos. 1 to 3, a resonable comparison can still

Table 2 Properties of R/C Beams Used Comparison Between The Tests and Theory.(in= 25.4㎜, 1psi= 6.89MPa)

Beam No.	ℓ(in)	ℓ/d	b	$\rho=\rho'$	ρ_s	f'_c(ksi)	f_y(ksi)
1	59	3.1	7.9	0.0178	0.0043	6.11	45.11
2	59	3.1	7.9	0.0178	0.0043	5.45	45.38
3	59	3.1	7.9	0.0178	0.0043	4.82	45.11
4	60	4.7	15.0	0.0103	0.011	4.75	54.4
5	60	4.7	15.0	0.0103	0.013	4.06	51.8
6	60	4.7	15.0	0.0103	0.020	4.25	51.8

ℓ = cantilever element length

ℓ/d = the shear span to depth ratio

b = the cross sectional width

ρ, ρ' = the ratios of bottom and top reinforcement, respectively

ρ_s = the volumetric ratio of transverse reinforcement at the critical location near the fixed end

f_c', f_y = the concerete compressive strength and steel yield strength, rspectively

be observed between the experimental and theoretical results.

6. SUMMARY AND CONCLUSIONS

A practical element model was developed in this study to predict the hysteretic behavior of R/C beams, accounting for the distinct hysteretic characteristics of shear and flexural deformations. All the elasto-plastic deformations were assumed to be concentrated in dimensionless serial shear and flexural springs at the element ends. The behavior of each spring was defined by a semi-empirical skeleton force-deformation curve and a set of empirical hysteretic rules which were derived from the available cyclic test results on R/C cantilever beam(in which the shear and flexural deformations were meansured

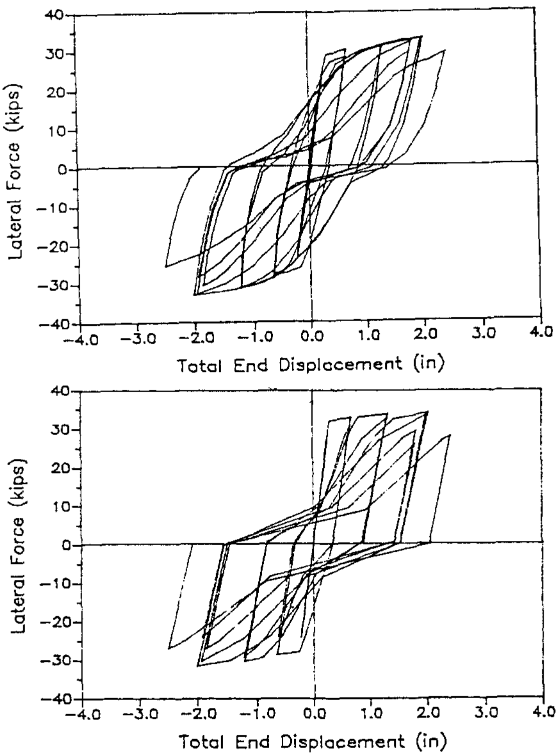


Fig 14 *Experimental and Theoretical Cyclic Load-Deformation Relationships of Beam No. 1* (1in=25.4 mm, 1 ksi=6.8957

separately).

Based on the developed physical model, tangent stiffness matrix was constructed for cantilever beams. The developed element model was checked against the results of a number of cyclic tests performed on R/C cantilever beams with wide ranges of geometric and material properties. The comparison between test and theory was satisfactory.

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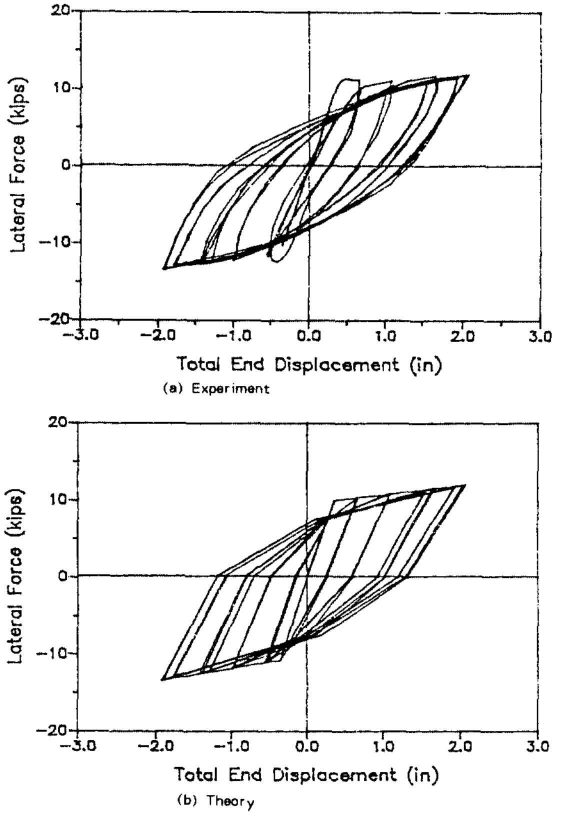

Fig 14 *Experimantal and Theoretical Total Cyclic Load-Displacement Relationships of Beam No. 4*

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