A New Grillage Method for Analyzing Orthogonally Stiffened Plated Structures

직교 이방성으로 보강된 평판 구조물 해석을 위한 새로운 방법 연구

Cho, Kyu Nam*
조 규 남

요 약

직교 이방성으로 보강된 평판 구조물 해석을 위한 방법제시가 본 논문의 주요 내용이다. 이 방법에서는 변형된 정적 압축법을 이용하여 직교 이방성으로 보강된 평판 구조물과 동등한 강성을 가지는 2차원 그릴리지 구조물을 생성하여 해석에 응용하고 있다. 대표적인 구조물을 선택하여 이론을 적용시켜 해석을 한 결과 직교 이방성으로 보강된 평판 구조물 해석에는 본 논문의 방법이 매우 효과적임이 입증되었다.

Abstract

Development of a procedure for improved modeling of orthogonally stiffened plated grillages is the primary subject of this paper. In the method developed here a modified static condensation procedure is used to get a complete 2-dimensional grillage which represents the stiffness of the original orthogonally stiffened plated structure. The theory and numerical model are applied to a typical structure and the method has been demonstrated to work well for the analysis of orthogonally stiffened plate structures.

Section 1

INTRODUCTION

More than half of elements of the ship's structure are rectangular stiffened plates. Such plates are commonly stiffened in two directions : the system is composed of two sets of stiffeners intersecting each other orthogonally. Generally this structure consists of the plating, transverse frames, and longitudinal stringers of different sizes and with varying spacing. The analysis of bending, and

* 정회원, 現代重工業 綜合研究所 海洋研究室長, 工學博士

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associate twisting, of these stiffened plate structures contains many complexities which arise largely from the intricate interactions of the intersection parts.

The most common procedures for analyzing orthogonally stiffened plate are : 1) orthotropic plate theory [1-4] and, 2) 2-dimensional finite element grillage modeling [5]. These are two limiting physical idealizations. Orthotropic plate theory lumps the stiffeners with the plating to form an equivalent homogeneous plate of uniform thickness, but with different rigidity properties in the orthogonal directions corresponding to the stiffener directions. Two-dimensional grillage modeling, on the other hand, lumps the plating with the stiffeners in forming a planar gridwork of orthogonally interconnecting beams.

The orthotropic plate modeling technique is based on the assumption that plate-stiffener combinations can be accurately replaced by equivalent homogeneous orthotropic plates. But the application of orthotropic plate theory to plates with orthogonally attached stiffeners has inherent difficulties. Orthotropic plate theory is approximately applicable to stiffened plates provided that the ratios of stiffener spacing to plate boundary dimensions are sufficiently small, [1]. Furthermore, for approximate homogeneity, the stiffeners should be identical and equally spaced. Nonuniformity of the stiffeners and their variable and sparse spacing make it difficult to replace the orthogonally stiffened plate by an orthotropic plate which has equivalent elastic constants. One special case of orthotropic plate modeling of a plate with nonuniformly disposed stiffeners was discussed by Schade, [4].

In the 2-dimensional finite element grillage approach to analysing orthogonally stiffened plate, the system is converted into an equivalent gridwork of co-planar beams, i.e., the plating is lumped with the stiffeners in achieving the grillage. This method is the converse of orthotropic plate theory where the stiffeners are lumped with the plate. However, the widths of the effective flanges of the grillage beam elements, which totally represent the plating cover, appear to be nearly as uncertain as the equivalent rigidity constants in the alternative orthotropic plate model.

The simplest approach to lumping the plating with the stiffeners in the grillage model is just to assume a width of plating as the effective flange of each beam element equal to the spacing of the respective stiffeners. This approach however, ignores both shear lag effects and the nonlinearities associated with the coupling of plating in-plane and normal deformations. Both of these effects produce an attenuation of the in-plane plating stresses between the stiffeners. Allowance for this stress non-uniformity requires the concepts of "effective widths" and "effective breadths" of plating for estimating the stiffener flanges : these effective flanges, in reflecting the plating stress non-uniformity, are necessarily smaller in width than the local stiffener spacing [6].

Actually, the terms "effective breadth" and "effective width" are each used to distinguish the two different physical mechanisms leading to the need for "effective" flange considerations. "Effective breadth" allows for shear lag effects associated with bending moment gradients along the stiffeners : most of the work on effective breadth due to shear lag in ship structures was done by Schade, [7, 8]. "Effective width", on the other hand, allows for the loss of plating in-plane rigidity due to coupling with normal displacements, such normal displacements being due to normal loading, fabrication distortions, buckling, and other extraneous sources of plating deformations.

The established procedures for estimating effective widths are not so rational as their effective breadth counterparts. The "60t" or "40t" rules for effective width have existed in ship codes for many years, [7, 9]. These rules require an effective width of flange of "60 times the plating thickness" or "40 times the plating thickness", respectively. The origin of and basis for these rules are uncertain.

In practical 2-dimensional finite element grillage analysis, the effective breadth theory is seldom used. The 40t and 60t rules for effective width are widely accepted as rules of thumb. However, neither of these procedures can be expected to give consistent representation of the actual structure. Especially, the torsional properties of the actual stiffened plate elements, which comes from the continuity of the plate, is not specifically considered with these traditional approaches.

In spite of the shortcomings of the two established procedures for analysing orthogonally stiffened plates, as discussed above, any other relatively simple alternative with better accuracy potential is not obvious. Fully 3-dimensional finite element modeling does arise as a possibility. However, even though 3-D analysis is possible, an improved simplified modeling approach is needed in view of the inherent computational complexities of the 3-D approach. Development of a procedure for improved approximate modeling of orthogonally stiffened plated grillages is the primary subject of this paper. The 2-dimensional grillage modeling method, which is consistent with modern numerical anlaysis and compatible with the 3-D basis modeling, is chosen as the simplified approach to be improved. Improvement of this simplified modeling approach is based on rational reduction from the accurate 3-dimensional modeling characteristics.

In the simplified method assembled, the global stiffeness matrix of a 3-dimensional orthogonally stiffened plate model is first generated. For this purpose, the 3-dimension method outlined in Cho and Vorus, [10], has been employed. The 3-D matrix is then condensed to the end nodal points of an equivalent grillage element of interest using a modified static condensation procedure. This is performed for each grillage element, in turn. A complete 2-dimensional grillage which represents the stiffeness of the original orthogonally stiffened plated grillage structure is so obtained.

The theory and numerical model are applied to a typical orthogonally stiffened plated grillage structure. Section properties of the grillage beam elements and displacements of the structure are calculated by various established methods. Displacements are compared both with those of a relatively exact 3-D solution and with the grillage model assembled by the subject new procedure. The following sections describe the general theory employed, define the numerical models, and present the results of the analysis of the sample structure.

Section 2

EQUIVALENT 2-D GRILLAGE MODELING

Equivalent Stiffness Concept

In the 2-dimensional grillage approach to analyzing an orthogonally stiffened plate system, the plating is lumped with the stiffeners in achieving the equivalent grillage. This grillage is composed of beams representing the set of discrete plate--stiffener combinations, which have equivalent section properties I and J. Here, I and J denote the bending moment of inertia and torsional rigidity constant, respectively, of an equivalent grillage beam.

The elements of a general 6-degree of freedom beam stiffness matrix are in terms of the section properties of the beam. Therefore, if the stiffness matrix of a stiffener element, including the effects of the plating cover, can be obtained, the equivalent section properties of the 2-D grillage element can be identified by comparing the elements of the subject matrix with those of the general beam element stiffness matrix.

In the method developed here, the global stiffeness matrix of the 3-dimensional orthogonally stiffened plate system is first constructed by the method described in the proceding section. This fully coupled matrix is then condensed to the nodal points of the stiffener element of interest to provide the stiffness matrix of the equivalent beam element. This is performed for each grillage stiffener, in turn. Then, the equivalent section properties 2-D grillage beam elements can be identified by the comparison mentioned above. A complete 2-dimensional grillage which represents the stiffness of the orig inal orthogonally stiffened plated grillage structure is so obtained.

General Modeling Procedure

Details of this procedure for the derivation of the equivalent grillage beam section properties for the orthogonally stiffened plate system is as follows.

1) The global stiffness matrix of the complete structural model is first assembled using the 3-D procedures described previously. After applying the boundary conditions, the matrix is written in partitioned form as,

$$\begin{bmatrix} K_{nn} & K_{nm} & K_{nb} \\ K_{mn} & K_{mm} & K_{mb} \\ K_{bn} & K_{bm} & K_{bb} \end{bmatrix} \begin{Bmatrix} X_n \\ X_m \\ X_b \end{Bmatrix} = \begin{Bmatrix} F_n \\ F_m \\ F_b \end{Bmatrix} \quad (1)$$

Where, X_m represents the degrees of freedom to be condensed to the selected siffener nodal points, X_n represents the residual degrees of freedom of the stiffener nodal points. X_b represents the nonassigned(in-plane) degrees of freedom corresponding to the nodal points on the model boundary.

2) The applied loads F in the equation (1) are set to zero. Equivalent stiffness is sought, which is independent of the applied loading. Note the difference here with Schade's effective breadth theory [7, 8], for example, which is based on equivalent load carrying capacity, versus equivalent stiffness. Schade's effective breadth, as associated with shear lag effects, is a function of the loading distribution.

3) The stiffener element of interest is next removed from the model in proceeding to determine the effective plating flange. At this point the mechanics of the interaction between the plate and stiffener elements in the 3-dimensional analysis procedure described above is utilized. Flexure of the stiffener elements loads the plate elements at the stiffener nodal points. Therefore the plating can be viewed as brought into participation with the stiffener by displacements imposed by flexure at the stiffener nodal points. The effective breadth of plating participating with a respective stiffener is therefore independent of the stiffener properties. In fact, it can be determined independently of the stiffener by fixing the plating at one stiffener nodal point and applying unit displacements on the plating at the other in the direction of interest, with the stiffener element removed. Formally, this is achieved by simply statically condensing the structure to the two stiffener nodal points, with the stiffener element removed (and with one additional step to be explained below). The third step in the procedure therefore requires removing the stiffener element of interest from the model in determining the associated effective breadth of plating.

4) The procedure of condensing X_m into X_n is then executed as follows. The first 2 sub-matrix equations in (1) with $\{F\}=0$, are:

$$K_{nn}X_n + K_{nm}X_m = -K_{nb}X_b \quad (2)$$

$$K_{mn}X_n + K_{mm}X_m = -K_{mb}X_b \quad (3)$$

Inverting (3) for X_m yields,

$$X_m = -K_{mm}^{-1}(K_{mn}X_n + K_{mb}X_b) \quad (4)$$

Subustiution of (4) into (2) yields,

$$(K_{nn} - K_{nm}K_{mm}^{-1}K_{mn})X_n = (K_{nm}K_{mm}^{-1}K_{mb} - K_{nb})X_b \quad (5)$$

The left−hand−side stiffness matrix in (5) indicates the modification of the equivalent beam element stiffness due to the condensed m degress of freedon. This stiffeness matrix contains the stiffness of an equivalent beam, representing the stiffener and its associated plating plus "end springs", as depicted on Figure 1. The end springs are the stiffness of the surrounding structure excluding the stiffener element and its associated plating.

The (1, 1) element of the condensed matrix $K_{nn} - K_{nm} K_{mm}^{-1} K_{mn}$ in the equation (5), for example, represents the axial stiffness of the plating associated with stiffener vertical bending, plus the end spring in axial direction. The (2, 2) element of the same condensed matrix in the (5) represents the horizontal bending stiffness of an equivalent flange, plus the end spring in that direction. In−plane horizontal plate bending is active in warping restrained torsion of the equivalent beam, as will be shown.

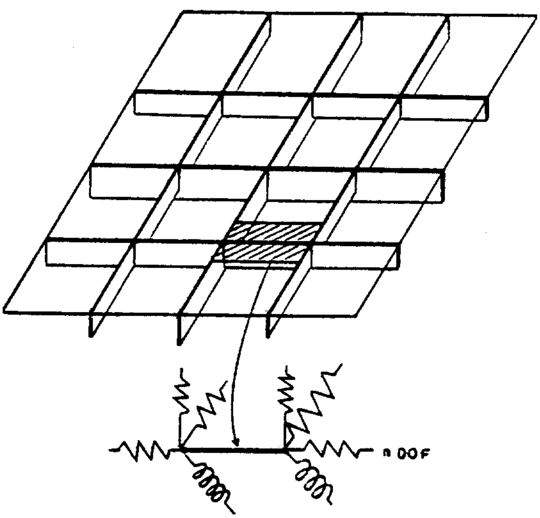

Figure 1 Extracted Equivalent Beam with End Springs

5) Elimination of the end springs from the condensed stiffness matrix elements referred to above will provide the matrix from which the net equivalent beam section properities of the grillage element of interest can be identified. Elimination of the end spring stiffness from the (1, 1) element, for example, of the condensed matrix is carried out by subtracting the end spring stiffness in the axial direction from element (1, 1). The end spring stiffness is obtained from the right hand side stiffness matrix, $K_{nm} K_{mm}^{-1} K_{mb} - K_{nb}$, in equation (5). For example, the end spring stiffness in the axial direction is the (1, 1) element of this stiffness matrix.

$$(K_{nm} K_{mm}^{-1} K_{mb} - K_{nb}) \tag{6}$$

where $K_{nm} K_{mm}^{-1} K_{mb} - K_{nb}$ is the stiffness matrix of the righthand side of the equation (5).

The physical interpretation of this is as follows.

Consider a plate with clamped boundary as shown in Figure 2, for example. X_n denotes the in−plane axial displacement at point n. The (1, 1) element of the condensed matrix gives the equivalent flange section axial stiffeness plus the end spring at point n, as mentioned above. It is now necessary to remove the end spring to leave only the equivalent flange stiffness. The end spring loads alone can be imposed by fixing n and imposing $-X_n$ on boundary as shown in Figure 3. Since both equivalent beam nodal points are now fixed, there is no participation of the equivalent beam flange : the loads at point must therefore be the end spring loads. Now, subtract the nodal loads of the system II in Figure 3, from system I in Figure 2, for which unit X_n gives the stiffness of the equivalent beam flange. Therefore, the end spring load at node n in the axial direction is the first element of the right hand side load vector in the equation (5) with X_b being unity. Physical superposition of the two problems is shown in Figure 4, for which the equivalent flange stiffness is the (1, 1) element of the matrix below,

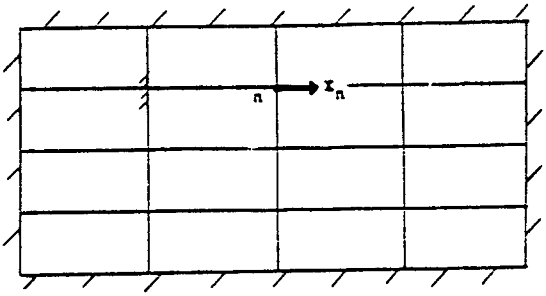

Figure 2 A Plate with Fixed Boundary, System Ⅰ

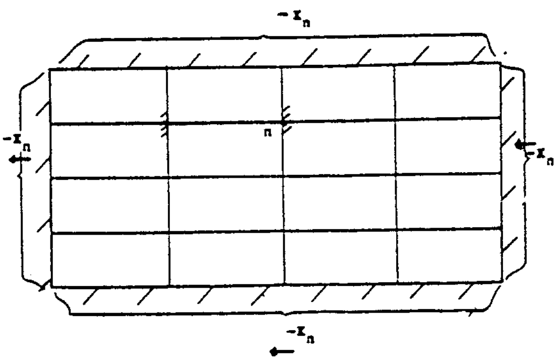

Figure 3 A Plate with Fixed Boundary, System Ⅱ

$$[K_{nn} - K_{nm} K_{mm}^{-1} K_{mn}] - [K_{nm} K_{mm}^{-1} K_{mb} - K_{nb}] \quad (7)$$

The cross sectional area of the effective breadth of plating associated with bending of the stiffener is determined from the (1, 1) element of (7). Comparison of the (1, 1) element of this matrix with the (1, 1) element of a general beam element stiffness matrix gives the effective breadth of the frange of the 2-D grillage beam for vertical bending. Note that this effective breadth is definitely different from that defined by Schade. Schade's "effective breadth" is derived on the basis of shear lag and equivalent load carrying capacity. The effective breadth defined here is that for equivalent stiffness.

6) For the calculation of the effective flange for torsion of the equivalent beam, the same procedure is applied except that the boundary displacements are set to unity in the direction orthogonal to the stiffener as shown in Figure 5. Setting the boundary displacement to unity in the orthogonal direction and subtracting the right-hand-side matrix of (5) similarly removes the end spring loads in that direction.

The equivalent effective flange which horizontally bends as a cantilever as the stiffener twists is obtained by comparing the (2, 2) element of the resulting 12x 12 condensed matrix with the (2, 2)

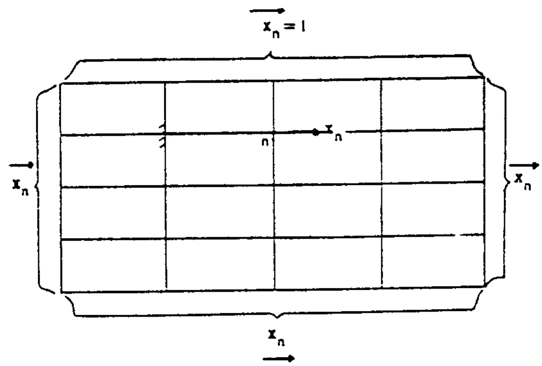

Figure 4 Physical Superposition of Two Systems Ⅰ, Ⅱ

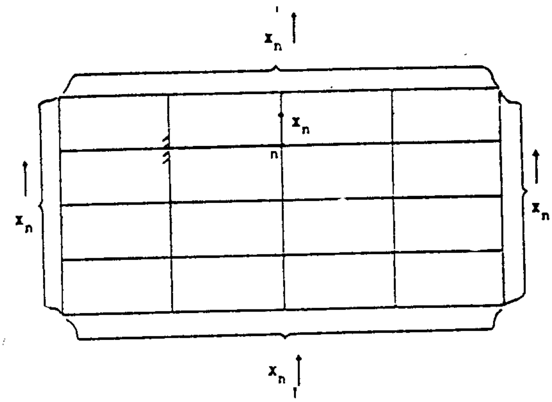

Figure 5 A Plate with Unit Displacements on Boundary for the Calculation of Bet

element of a general beam element stiffness matrix.

Once the effective plating breadth for vertical bending is obtained, determination of the corresponding vertical bending moment of inertia, I, of the equivalent beam is obvious. However, the equivalent torsional rigidity constant of the beam is not so obvious. The determination of the correct torsional rigidity constant of the equivalent beam, given the effective flange from 6), is described in the following.

The membrane analogy introduced by L. Prandtl, [11], establishes relations between the deflection surface of a uniformly loaded membrane and the distribution of stresses in a twisted bar. A simple solution of the twisting of a narrow rectangular cross section is obtained. [6, 12].

The torsional rigidity constant of the narrow rectangular cross section can be written as,

$$\sum_{i=1}^{n}\frac{1}{3}\ h\ t^{3}$$

Where h is the depth of the section and t is its thickness. This is the torsional rigidity constant of the section with zero warping restraint. However, the prevention of warping affects the angle of twist and the distribution of stresses significantly. For orthogonally intersecting plating stiffeners warping restraint exists since the plating cover must experience in-plane deformation at the stiffeners twist. In the case of I beams, channels and other thin-walled members of open section the prevetion of warping during twisting is accompanied by in-plane bending of the flange and may have considerable effect in reducing the angle of twist. The torsional center, or twist axis, about which the section rotates must, however, be known in order to evaluate the stiffening effect of the flange on twisting these cases.

The twist axis of the stiffener in this case is taken to be the straight line intersecting the neutral axes of the two bounding orthogonal equivalent beams. This is as depicted in Figure 6.

After the location of the torsion axis is found, the total torsional rigidity of the equivalent beam

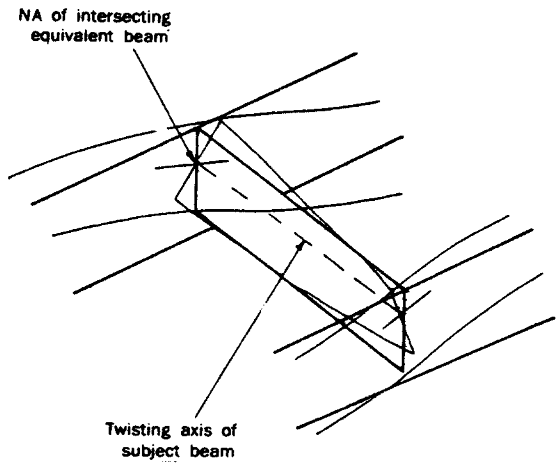

Figure 6 Twist axis

can be calculated by the membrane analogy and the warping restraint condition based on the calculated effective breadth for twisting, B_{et}

The torsional rigidity constant J of the equivalent beam is written,

$$J=\frac{1}{3}\ B_{et}\ t_{p}^{3}+\ \frac{1}{3}h\ t_{b}^{3}+\frac{12EI_{f}}{L^{2}G}\ \ d_{2}^{2} \qquad (9)$$

$$=\frac{1}{3}\ B_{et}\ t_{p}^{3}+\frac{1}{3}h\ t_{b}^{3}+\frac{Et_{p}B_{et}^{3}}{L^{2}G}\ \ d_{2}^{2}$$

The first two terms in (9) are for unrestrained warping from (8), the third term is the warping restraint contribution from the equivalent flange, [6]. Here, d_2 is the distance between the middle of the plate and the twist axis of the beam, and B_{et} is the effective breadth for twisting determined as described from the condensation procedure : (6). I_f denotes the inplane bending moment of inertia of the equivalent flange. Subscripts ***p*** and ***b*** denote plate and beam, respectively, and L is the length of the beam. Note that last term in (9) will be large compared with the other terms, implying the importance of the stiffness of the plating cover to the effective torsional rigidities of the equivalent stiffeners.

The torsional rigidities of the beams in the equivalent 2-D grillage model obviously affect the bending of the grillage, due to required equality of bending slope and twist angle at the joints of the orthogonal elements. There fore accurate determination of the torsional rigidity constants is in order in 2-D grillage modeling.

The 2-D grillage element section property reduction procedure described above is applied to the end nodal points of each equivalent grillage element, in turn. Section properties I, J of each grillage element are calculated. A complete 2-dimensional grillage which represents the stiffness of the original system is thus obtained.

The new equivalent stiffness method outlined is applied to a general orthogonally stiffened plated grillage structure in the following section for the purpose of demonstration.

Section 3

EVALUATION

As stated in the introduction, the objective of this paper is to develop a procedure for analyzing orthogonally stiffened plated grillage structures more precisely and efficiently. Demonstration of the success achieved in meeting this objective is conducted in terms of a model of a typical orthogonally stiffened plate.

2-D equivalent grillage models, with section properties based on several different procedures, including the new procedure developed here, are analyzed. Displacements are calculated for the grillage models and compared with the 3-D results.

Description of Numerical Model

Figure 7 shows a typical orthogonally stiffened plated grillage structure represented by beam and plate elements. The plate panels have been modeled with single 4-node plane stress plate elements. The stiffeners have been modeled with 2-noded three dimensional beam elements according to elementary beam theory, [13]. Section properties of the stiffener elements (independent of the plating) are tabulated in Table 1, according to the element numbers indicated on Figure 7. Note that the plating is taken as 0.25cm thick and that all plating panels are 40cm square. The dimensions of the stiffener cross section are 16x1cm, 12x1cm, respectively. The edges of the system are taken as clamped for the example calculation.

Numerical Results

Effective breadths of each equivalent beam for bending and twisting as caused by vertical loading of the structure are calculated by the procedure described in the foregoing.

The global stiffness matrix of the 3-dimensional model was assembled by the procedure described in Section 2. The corresponding equivalent condensed 2-D grillage is denoted as Grillage I. Figures 8 and 9 show the effective breadths for bending and twisting, respectively, approximately to scale. The effective breadth data is tabulated in Table 2. Section properties of the equivalent Grillage I are tabulated in Table 3. The torsional rigidity constant J of each equivalent beam is calculated by (9). The Grillage I section properties are compared on Table 4 with those for which the stiffener flange widths are set by the traditional 40t and 60t rules cited in the Section 1.

Vertical displacements were calculated by applying a concentrated load at the center of the models, as shown in Figure 10. The predictions of the midplate deflection by the various approximate methods are compared with the 3-D results. Figure 11 shows the deflection curves of the 3-D model and equivalent Grillage model I, and the 40t model, respectively. Figure 12 shows the same deflection curves, but with the 40t flange results replaced by the 60t results. The deflection data,

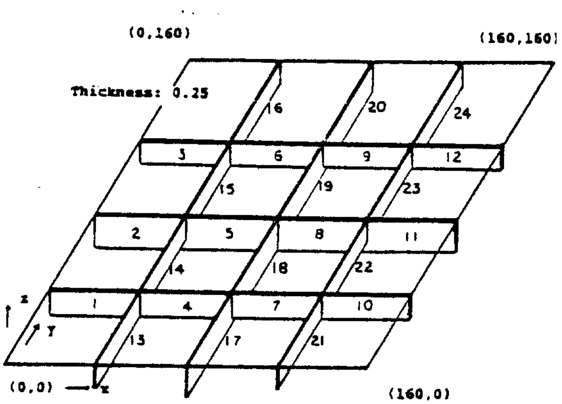

Figure 7 Typical Orthogonally Stiffened 3－D Plate Model

Table 1 Section Properties of the 3－D Model

a. Stiffener Elementss:					
Beam No.	Area	J	Iy	Iz	Length
1	1.20E01	4.00E00	1.44E02	1.00E00	4.00E01
2	1.60E01	5.30E00	3.41E02	1.33E00	4.00E01
3	1.20E01	4.00E00	1.44E02	1.00E00	4.00E01
4	1.20E01	4.00E00	1.44E02	1.00E00	4.00E01
5	1.60E01	5.30E00	3.41E02	1.33E00	4.00E01
6	1.20E01	4.00E00	1.44E02	1.00E00	4.00E01
7	1.20E01	4.00E00	1.44E02	1.00E00	4.00E01
8	1.60E01	5.30E00	3.41E02	1.33E00	4.00E01
9	1.20E01	4.00E00	1.44E02	1.00E00	4.00E01
10	1.20E01	4.00E00	1.44E02	1.00E00	4.00E01
11	1.60E01	5.30E00	3.41E02	1.33E00	4.00E01
12	1.20E01	4.00E00	1.44E02	1.00E00	4.00E01
13	1.20E01	4.00E00	1.44E02	1.00E00	4.00E01
14	1.20E01	4.00E00	1.44E02	1.00E00	4.00E01
15	1.20E01	4.00E00	1.44E02	1.00E00	4.00E01
16	1.20E01	4.00E00	1.44E02	1.00E00	4.00E01
17	1.60E01	5.30E00	3.41E02	1.33E00	4.00E01
18	1.60E01	5.30E00	3.41E02	1.33E00	4.00E01
19	1.60E01	5.30E00	3.41E02	1.33E00	4.00E01
20	1.60E01	5.30E00	3.41E02	1.33E00	4.00E01
21	1.20E01	4.00E00	1.44E02	1.00E00	4.00E01
22	1.20E01	4.00E00	1.44E02	1.00E00	4.00E01
23	1.20E01	4.00E00	1.44E02	1.00E00	4.00E01
24	1.20E01	4.00E00	1.44E02	1.00E00	4.00E01
b. Plate Elements:					
Thickness:0.25cm					
Plain stress 4－node elements 40×40cm					

according to node numer, is summarized on Table 5.

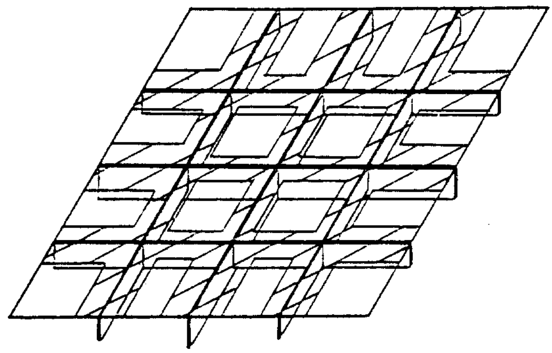

Figure 8 Effective Breadth for Bending

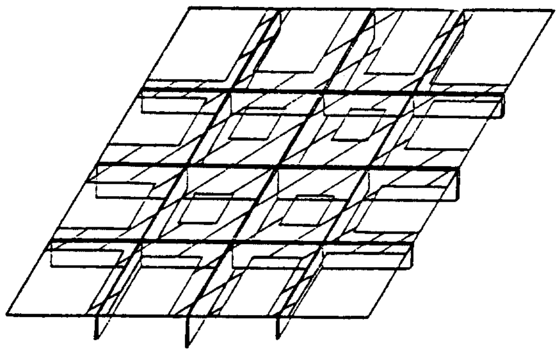

Figure 9 Effective Breadth for Twisting

The last figure, Figure 13, shows collected deflection curves. The 3－D predictions are included, along with the prediction from Grillage model I. Both the 40t and 60t model predictions are also included.

The remaining curve on Figure 13 is the center deflection from the 40t flange model, but with the more accurate torsional rigidity from the Grillage model I. This was included in order to allow an assessment of the treatment of beam element torsion.

Discussion of Numerical results

The new 2－dimensional grillage modeling method developed in this parer has heen indicated through example to offer improved accuracy in the analysis of orthogonally stiffened plate structures. Comparison of the deflections by the eq-

Table 2 Effective Breadth for Bending and Twisting Using the Golobal Stiffness Matrix

Beam Element Number	Effective Breadth For Bending	Effective Breadth For Twisting
1	24.277	8.458
2	24.533	17.759
3	24.277	8.458
4	24.079	21.115
5	24.003	24.922
6	24.079	21.115
7	24.079	21.115
8	24.003	24.922
9	24.079	21.115
10	24.277	8.458
11	24.533	17.759
12	24.277	8.458
13	24.277	8.458
14	24.079	21.115
15	24.079	21.115
16	24.277	8.458
17	24.533	17.759
18	24.003	24.922
19	24.003	24.922
20	24.533	17.759
21	24.277	8.458
22	24.079	21.115
23	24.079	21.115
24	24.277	8.458

Table 3 Section Properties of the Equivalent Grillage Based on Beam in Table 2

Beam Number	Equiv. Grillage Model		40t Model		60t model	
	Iy	Iz	Iy	Iz	Iy	Iz
1	295.2	299.1	221.6	21.8	251.2	71.3
2	634.1	308.9	484.1	26.2	541.9	71.6
3	295.2	299.1	221.6	21.8	251.2	71.4
4	295.4	290.7	221.6	21.8	251.2	71.4
5	629.5	289.4	484.1	26.2	541.9	71.6
6	294.4	290.7	221.6	21.8	251.2	71.3
7	294.4	290.7	221.6	21.8	251.2	71.3
8	629.5	289.4	484.1	26.2	541.9	71.6
9	294.4	290.7	221.6	21.8	251.2	71.3
10	295.2	299.1	221.6	21.8	251.2	71.3
11	634.1	308.9	484.1	26.2	541.9	71.6
12	295.2	299.1	221.6	21.8	251.2	71.3
13	295.2	299.1	221.6	21.8	251.2	71.3
14	294.4	290.7	221.6	21.8	251.2	71.3
15	294.4	290.7	221.6	21.8	251.2	71.3
16	295.2	299.1	221.6	21.8	251.2	71.3
17	634.1	308.9	484.1	26.2	541.8	71.6
18	629.5	289.4	484.1	26.2	541.9	71.6
19	629.5	289.4	484.1	26.2	541.9	71.6
20	634.1	308.9	484.1	26.2	541.9	71.6
21	295.2	299.1	221.6	21.8	251.2	71.3
22	294.4	290.7	221.6	21.8	251.2	71.3
23	295.4	290.7	221.6	21.8	251.2	71.3
24	295.2	299.1	221.6	21.8	251.2	71.3

uivalent 2-D grillage model developed here with those by conventional 2-dimensional grillage methods shows that the subject method provides better results. On the base of comparison against the fully 3-dimensional analysis, Figure 11, the 40t model gives highly overestimated deflections, 21.9% error. While the equivalent model gives significantly improved results, 7.7% error. The deflections by the 60t model are also highly over estimated values, 8.8% error, as shown in Fiqure 12.

The effective breadth for bending and that for twisting of the same equivalent beam in the equivalent grillage are calculated to be quite different, as shown in Table 2. Thus, the same value of the effective breadth should not be used in estimating bending and twisting rigidities. The values of effective breadth of the equivalent grillage elements for bending are very similar in magnitude, as shown in Table 2. This is not the case for twisting. The axial straining of the plating along the beam axes associated with vertical bending is more localized than the horizontal plate deformation associated with twisting. The effective plating flange for beam twisting should be expected to be more sensitive to the location of the stiffener relative to the model boundary, as Table 2 shows.

It is also found that the torsional rigidities of the beams in the equivalent grillage affects the solution for plating system lateral deflection. In Figure 13, deflections by the 40t model with the J according to the new grillage method are compared with the 40t flange used in calculating both I and J. Figure

Table 4 Comparison of section of Equivalent Grillage Ⅰ and 40t, 60t Model

Beam Number	Area	J	Iy	Iz	dl	dz
1	18.069	8.169	295.25	295.10	4.068	4.097
2	22.133	44.159	634.06	308.92	5.874	4.126
3	18.019	8.169	295.25	299.10	4.068	4.097
4	18.069	99.482	294.41	290.70	4.097	4.097
5	22.001	163.765	629.46	289.40	5.909	5.018
6	18.019	99.482	294.41	290.70	4.079	4.079
7	18.019	99.482	294.41	290.70	4.097	4.097
8	22.001	163.765	629.46	289.40	5.909	5.018
9	18.019	99.482	294.41	290.70	4.079	4.079
10	18.069	8.169	295.25	299.10	4.068	4.097
11	22.133	44.159	634.06	308.92	5.874	4.126
12	18.069	18.169	295.25	299.10	4.068	4.097
13	18.096	8.169	295.25	299.10	4.068	4.097
14	18.019	99.482	294.41	190.70	4.097	4.097
15	18.019	99.482	294.41	190.70	4.079	4.079
16	18.069	8.169	295.25	299.10	4.068	4.097
17	22.133	44.159	634.06	308.92	5.874	4.126
18	22.001	163.765	629.46	289.40	5.909	5.018
19	22.001	163.765	629.46	289.40	5.909	5.018
20	22.133	44.159	634.06	308.92	5.874	4.126
21	18.069	8.169	295.25	299.10	4.068	4.097
22	18.069	99.4821	294.41	190.70	4.079	4.097
23	18.069	99.482	294.41	190.70	4.079	4.097
24	18.069	8.169	295.25	299.10	4.068	4.097

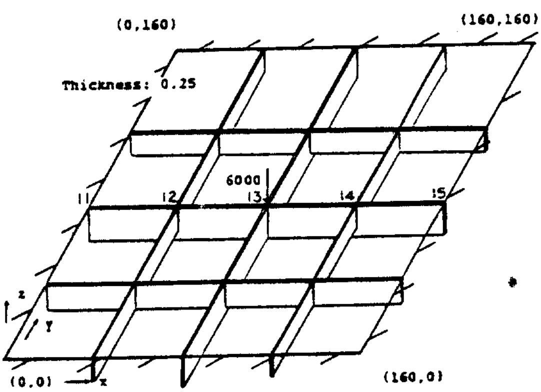

Figure 10 Orthogonally Stiffened Plate Model with Concentrated Load

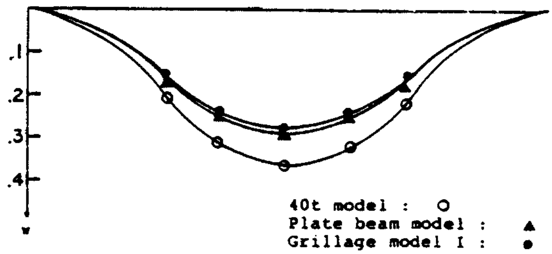

Figure 11 Deflection Curves of the Equivalent Grillage, Plate beam Model and Conventional 40t Model

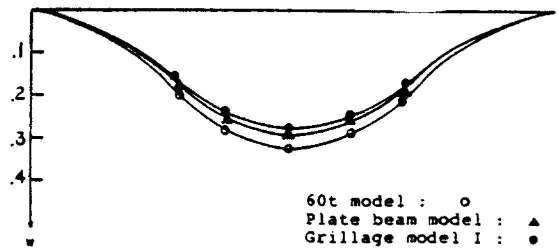

Figure 12 Deflection Curves of the Equivalent Grillage, Plate beam Model, and Conventional 60t Model

Table 5 Deflections by Equivalent Grillage Model, 40t, 60t Model

Modeling Procedure	Node Number				
	11	12	13	14	15
Plate beam Model	0.0	0.135E−1	0.297E−1	0.135E−1	0.0
Equivalent Grillage Ⅰ	0.0	0.130E−1	0.274E−1	0.130E−1	0.0
40t Model	0.0	0.171E−1	0.362E−1	0.171E−1	0.0
60t Model	0.0	0.152E−1	0.323E−1	0.152E−1	0.0
40t with Jeq Model	0.0	0.170E−1	0.357E−1	0.170E−1	0.0

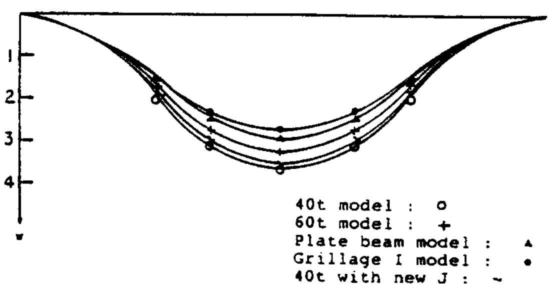

Figure 13 Deflection Curves of Various Models

13 implies that for accurate analysis of orthogonally stiffened plate structure accurate torsional rigidity constants of the grillage beam elements should be used.

Section 4

SUMMARY AND CONCLUSIONS

The main contribution of this paper is the development of a rational procedure for replacing the orthogonally stiffened plated grillage structure by an equivalent 2–dimensional grillage of 1–dimensional beam elements. The application of static condensation concepts for determining the effective breadths of plating on the basis of an equivalent stiffness concept represents a new approach to the problem. The approach determines more accurate bending rigidity constants for the equivalent grillage beam elements, as well as more accurate torsional rigidity constants.

Form the point of view of practical application, one possibility appears. Since the rational determination of the 2–D grillage beam section properties proposed requires the availability of the 3–D model stiffness matrix, essentially nothing is to be gained with the 2–D procedure when viewed simply as an analysis tool. However, significant value should be realizable from the method if applied in conjunction with the 3–D procedure to establish better element section properties of 2–D grillage models for use by structural designers.

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