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Simplified Collapse Analysis of Ship Transverse Structures

by

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선체 횡구조물의 단순화된 최종 강도 해석

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Abstract

In this paper, a theory for the static analysis of large plastic deformations of 3-dimensional frames, aiming at application to the collapse analysis of ship structures, is presented. In the frame analysis formulation, effects of shear deformations are included. A plastic hinge is inserted into the field of a beam end, and post-failure deformation of the plastic hinge is characterized by finite rotations and extensions. In order to model deep web frames of ship's structures into a framed structures, collapse of thin-walled plate girders is investigated. The proposed analysis method is applied to several ship structural models in the references.

요 약

본 논문에서는 선체구조의 최종강도 해석에 적용을 위하여 3차원 프레임의 대변형 탄소성 정적해석에 대한 것을 다루었다. 보의 해석에 관한 방법의 유도과정에 전단변형의 효과를 포함하고, 선체 횡구조의 강력늑골을 프레임 구조로 이상화 하기 위하여 박판거더를 최종강도 거동에 대한 연구를 수행하였다. 그리고 본 논문에서 제시된 해석방법을 참고 문헌에서 언급된 선체구조모델에 대해 적용하여 만족스러운 결과를 얻었다.

1. Introduction

The finite element method is probably one of the most useful general purpose analytical tools available to ship structural engineers. However, generally a ship structure is highly non-uniform, and requires a very complex finite element idealization if it is to be accurately modelled in its entirety. Such a procedure is an extremely large task from a practical

point of view, as it requires very large amounts of data preparation and takes much computing effort to get a solution. Thus, there is a need for more simplified procedures and a finite element method which is less sophisticated, but much cheaper and still accurate enough for practical purpose.

In this regard, linear frame analysis based on the finite element analysis has been widely used in structural problems as a simple and efficient tool since it was developed around 1960. It has been

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shown that the linear elastic analysis of ship structures has been successfully tackled using very rudimentary frame analysis models which are suitably tailored to the needs of the ship industry [1,2]. However, since frame analysis models normally represent only a limited part of the structure, the main problem in the analysis of frame models is to represent adequately influence of neighbouring structures, i.e. to determine boundary conditions of the structure. In some cases, in order to find actual boundary conditions for a frame analysis, a comprehensive finite element analysis including neighbouring structures has to be performed at first.

In the case of collapse analysis of structures, a nonlinear frame analysis with plastic hinge behaviour has received great attention. Since the principal load-carrying members have to collapse in order to form a mechanism, plastic hinges inserted into the collapsed parts of frame structure might offer significant reduction in computing efforts. There have been a number of theoretical and experimental studies on the nonlinear behaviour of frame structures. The elasto-plastic analysis of plane frames was considered by Jennings et al.[3] for the collapse analysis of multi-story buildings. In the reference [4,5] the elastic large deflection frame analysis was presented.

During the 1970s, interest in vehicle crashworthiness suggested a new area of application of the nonlinear frame analysis Miles[6] and McIvor et al.[7] developed frame analysis programs of large plastic deformations for a crashing analysis of vehicles. In contrast to most previous applications of the frame analysis, the determination of the collapse load itself is of little direct interest in this problem. It is the behaviour of structures during collapse, where much plastic energy is dissipated, that is the central issue.

In this paper, a structural theory is presented for the static analysis of large plastic deformations of 3-dimensional frames, aiming at application to the collapse analysis of ship structures. Including transverse shear deformation in beam analysis is essential if the beam element is used to model such

components of structure as short beams, web frames, or double bottoms in ship structures. Thus, in this frame analysis formulation, effects of shear deformations are included in the stiffness calculation.

In order to model deep web frames of ship's structures into a framed structure, collapse of thin-walled plate girders is investigated. Then, the proposed analysis program is applied to several structural models presented in the reference.

2. Theory of nonlinear frame analysis

2.1. Elastic stiffness matrix

In the development of a large displacement analysis for frame structures, it is necessary to evaluate deformations of the structures by an incremental solution procedure. If the loading path a structure is divided into a number of steps, the solution at each step is referred to the incremental solution.

A general 3-dimensional beam element is shown in Fig. 1.

The beam element is assumed to be straight and of constant cross-section having symmetry. It is assumed that plane sections of the beam element remain plane during deformation, but not necessarily perpendicular to the neutral axis of the beam, i.e. a constant shear is allowed. Here, increments of the translational displacements at a point in the centre line of the beam, parallel to x, y and z axes, are denoted by u_x, u_y , and u_z respectively. Rotations of

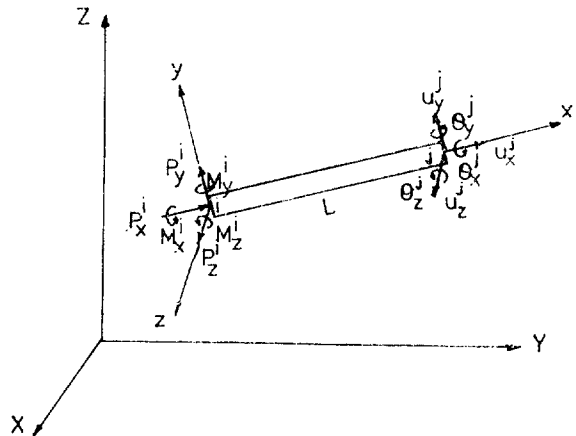


Fig. 1 Forces and displacement of beam element

the plane of the beam section with respect to x , y and z axes are denoted by θ_x , θ_y , and θ_z . Then, the above assumption for the deformation of plane sections leads to the following relation for the rotations of the beam section.

$$\beta_x = du_y/dx - \theta_z, \quad \beta_y = du_z/dx + \theta_x, \quad (1)$$

where β_x and β_y are the shear deformation angles with respect to z and y axes.

At each incremental step, the equilibrium relation for the beam element can be obtained by applying the principle of virtual displacement shown as the following equation.

$$\begin{aligned} & \int_0^L [p_x \delta(du_x/dx) + m_x \delta(d\theta_x/dx)] dx + \\ & \int_0^L [p_y \delta\beta_x + m_z \delta(d\theta_z/dx)] dx + \\ & \int_0^L [p_z \delta\beta_y + m_y \delta(d\theta_y/dx)] dx - \\ & \frac{1}{2} \int_0^L p_x \delta[(du_y/dx)^2 + (du_z/dx)^2] dx = \Delta R, \quad (2) \end{aligned}$$

where p_x , p_y , p_z , m_x , m_y , m_z are the incremental forces and moments, and

ΔR is the virtual work expression due to incremental external forces.

In the above equation the first integral term which is independent of the other remaining integral terms represents the incremental virtual work due to axial force and torsion of the beam. The second and the third integral terms represent the bending and shear deformation terms in the x - y and x - z planes respectively, and the last integral term represents the second order effects of bending due to axial forces.

The incremental nodal force vector and incremental nodal displacement vector are defined by the following equations using the forces and displacements shown in Fig. 1.

$$\begin{aligned} \{f\}^T &= (p_x^i p_x^j p_z^i m_x^i m_x^j p_z^j p_y^i m_x^i m_x^j m_z^i) \\ &= (\{f^i\}^T : \{f^j\}^T), \quad (3) \end{aligned}$$

and

$$\begin{aligned} \{u\}^T &= (u_x^i u_y^i u_z^i \theta_x^i \theta_y^i \theta_z^i u_x^j u_y^j u_z^j \theta_x^j \theta_y^j \theta_z^j) \\ &= (\{u^i\}^T : \{u^j\}^T) \quad (4) \end{aligned}$$

The incremental forces can be expressed by strains in the beam element as

$$\begin{aligned} p_x &= EA(du_x/dx), \quad p_y = \alpha_y GA\beta_x, \quad p_z = \alpha_z GA\beta_y \\ m_x &= GJ(d\theta_x/dx), \quad m_y = -EI_y(d\theta_y/dx), \\ m_z &= EI_z(d\theta_z/dx) \quad (5) \end{aligned}$$

where J : polar moment of inertia of the beam section

I_y, I_z : second moment of area of the beam section with respect to y and z axis respectively

α_y, α_z : the shear correction factor of the beam section for shear deformations parallel to y and z axis respectively.

Thus, the elastic stiffness matrix can be obtained by inserting eq. (5) into eq. (2) if suitable interpolation functions are available to relate the lateral displacement variables of the beam and displacements at the beam nodes. Since the transverse shear deformation is included in the formulation of the beam element, the usual Hermitian interpolation functions cannot be used in this study. Instead, the procedure of including transverse shear deformation into the displacement formulation using a cubic interpolation function, proposed by Severn[8], is adopted in this study.

The incremental stiffness equation of the virtual work equation (2) can be given by the following equation.

$$\{f\} = [K] \{u\} = ([K_b] + [K_G]) \{u\}, \quad (6)$$

where $[K_b]$ is the conventional stiffness matrix and $[K_G]$ is the geometrically nonlinear stiffness matrix.

2.2. Elasto-plastic behaviour of general frames.

When a frame is loaded beyond the elastic limit, the yielding occurs at out-fibres of the beam section, where bending moment exceeds the yield moment, and is spread into the beam section with increase of load. The behaviour of the yielded beam section is usually approximated by a plastic hinge. A collapse analysis of frame structures can be greatly simplified by considering the plastic hinges which may lead to a collapse mechanism.

In the application of frame analysis to collapse of ship structures, it should be noted that frame models of ship structure usually consist of thin-walled beams. Thus, collapse of the frame structure might be caused by local buckling in beam elements

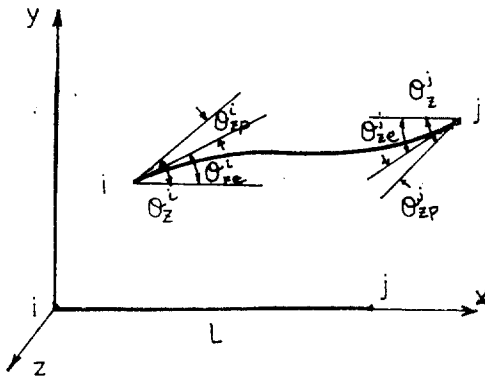


Fig. 2 Plastic deformation of beam element

as well as by forming plastic hinges due to yielding. However, the concept of the plastic hinge can be adopted as a useful idealization for failed parts of the frame models. When failure occurs at one or both ends of a beam element, a plastic hinge is inserted into the failed end, and post-failure deformation of the plastic hinge can be characterized by finite rotations and extensions of the beam. Practically, this is a very useful concept when a complete load-deformation relation is to be calculated such as in the problem of energy absorption of structures during a collision. Consider a beam ij , shown in Fig. 2. When both ends of the beam are yielded, discontinuities in slope exist at both ends of the beam. The relation between the rotation angles can be written as

$$\theta^i - \theta_z^i = \theta_p^i, \quad \theta^j - \theta_z^j = \theta_p^j \quad (7)$$

Where θ_z^i is the rotation angle of beam material at node i due to elastic rotation and rigid body rotation of the beam. The angle of discontinuity between θ_z^i and the rotation angle at beam node i is the plastic rotation angle θ_p^i . For the translational displacements, the relation between elastic and plastic displacements can be given by a similar equation as eq. (7). Thus, for the displacement vector at nodes i and j , the elastic and plastic displacements can be related by

$$\{u^i\} - \{u_e^i\} = \{u_p^i\}, \quad \{u^j\} - \{u_e^j\} = \{u_p^j\} \quad (8)$$

Naturally, the elastic displacements are related to

the incremental force by the following stiffness equation;

$$\{f\} = [K] \{u_e\}, \quad \text{or} \quad \begin{Bmatrix} f^i \\ f^j \end{Bmatrix} = [K] \begin{Bmatrix} u_e^i \\ u_e^j \end{Bmatrix} \quad (9)$$

In order to relate the incremental forces and total displacement increments, the plastic displacements in eq. (8) must be evaluated. The plastic displacement depends on behaviour of the plastic hinges, and the detailed behaviour can not, of course, be computed within the context of a structural theory. Therefore, the following approximation for the determination of plastic displacements is assumed. If yielding, or failure, at beam node i is initiated by the following equation;

$$\Psi^i \left(\frac{p_x^i}{p_x^c}, \frac{p_y^i}{p_y^c}, \frac{p_z^i}{p_z^c}, \frac{M_x^i}{M_x^c}, \frac{M_y^i}{M_y^c}, \frac{M_z^i}{M_z^c} \right) - 1 = 0 \quad (10)$$

where $p_x^c, \dots, M_x^c, \dots$ are the collapse forces and moments which can cause the yield or failure individually, and Ψ^i is a scalar function of yielding or failure criteria such as eq. (18),

the plastic deformations are assumed to be determined by the following equations.

$$\{u_p^i\} = \lambda_i \left\{ \frac{\partial \Psi^i}{\partial F^i} \right\}, \quad (11)$$

$$\text{Where } \{F^i\} = (p_x^i, p_y^i, p_z^i, M_x^i, M_y^i, M_z^i)$$

It should be noted that this procedure has analogy to the flow theory of plasticity through the Prandtl-Reuss equation. The λ_i in eq. (11) is a scalar which can be determined from the normality condition for the yield function;

$$d\Psi^i = 0 \quad (12)$$

For node j , the same equations can be written if subscript i is replaced by j in eqs. (11,12).

Since the nodes of the beam may not be failed simultaneously, different values of λ for the nodes of the beam have been used in the scaling of the plastic flow vector to determine the incremental plastic displacements. Using eqs. (10~12) gives the final form of the elasto-plastic stiffness matrix for a beam element after considerable algebraic manipulations.

2.3. Development of computer program for general frame analysis

In this study, a computer program for the general frame analysis of large plastic deformation has been developed based on the theory presented in the previous sections. Solving the governing nonlinear equations is central to nonlinear finite element analysis programs. From the review of previous works, it is observed that two classes of solution methods have been used for solution of the nonlinear finite element equation, i.e. incremental solution method and iterative solution method. However, in the frame analysis program with the plastic hinge behaviour, the iterative solution method is preferable to avoid large accumulation of error which may cause ill-conditioning of the solution procedure. Thus, the modified Newton-Raphson method is adopted in this analysis program.

In some cases, an analysis program might require capability of complete load-deflection history up to and/or beyond collapse of the structure. Thus, in view of the fact that load imposed on the structure can not be determined prior to an analysis, it is clear that a structural damage analysis program needs to adopt a displacement control method rather than a force control for an increment of solution step. The incremental displacement control method is adopted in this frame analysis program.

2.4. Numerical examples

In order to test the nonlinear analysis program developed in this study, two examples are presented. The first example is a square frame subjected to the two forces applied at the midpoints of a pair of opposite sides, Fig. 3 in order to test the elastic large deformation behaviour of the program. An analytical solution and experiment were provided by Kerr[9]. His results of the analysis and the experiment for the dimensionless displacements are shown in the figure. 4-beam elements are used to model a quadrant of the beam. 20-incremental steps are used in the analysis up to the displacement ratio $u/L=0.6$. The result of the present analysis is shown in Fig. 3. It can be seen that the present result is in good agreement with those obtained by the experiment and the exact solution.

A grid structure, shown in Fig. 4, under lateral load is considered in the second example. Experiments on the collapse of such structures were performed by Kim[10]. The grid is simply-supported at the boundaries and is free to pull in at all the supporting

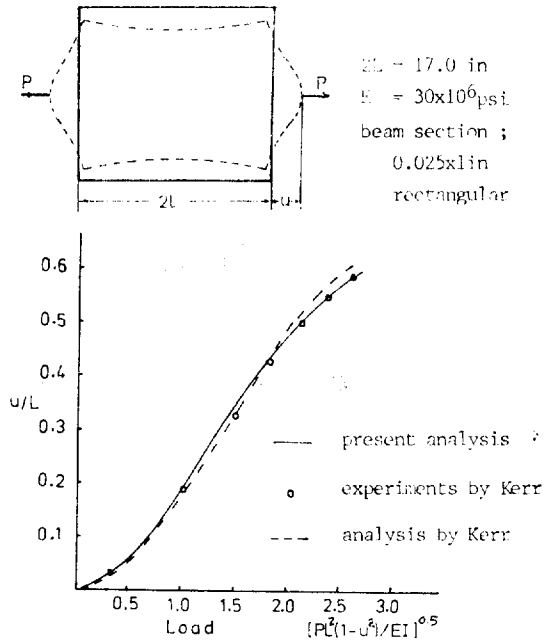


Fig. 3 Large deformation of square frame

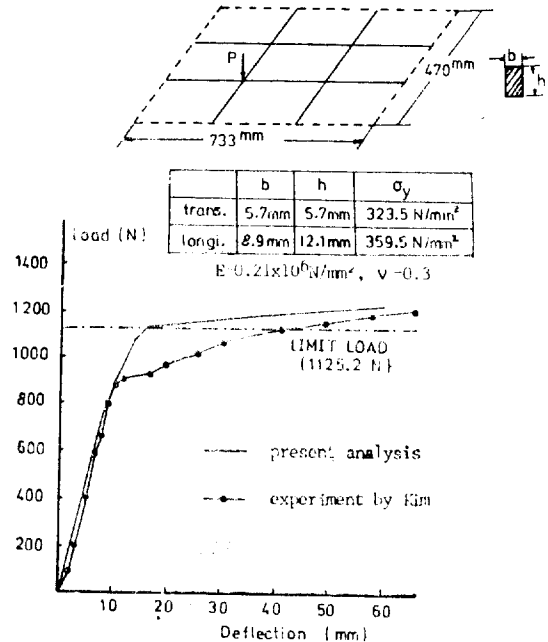


Fig. 4 Collapse of a grid structure

A grid structure, shown in Fig. 4, under lateral

points. 12-beam element are used and 30-incremental steps are adopted to analyze up to 60mm deflection at the loading point. The result of the analysis is shown in Fig. 4. In the load deflection relation of the present analysis, the grid structure shows a sudden collapse, while the experiment shows a gradual collapse, as might be expected. In this figure, the analytical collapse load by the limit analysis is also shown, and it can be seen that the present analysis predict a quite accurate limit load.

3. Collapse analysis of Ship Side Structures.

3.1. Collapse of web frames

Deep web beams are the structural component of transverse web frames, girders and stringers in ship structures. Analytical and experimental studies for the ultimate strength of transverse web frames were performed Ueda et al[11], and Suhara et al[12].

Meanwhile, during the last two decades, for the structural analysis of steel bridges the ultimate strength of welded plate girders has been the subject of numerous researches and experimental investigations[13]. As a result of their researches, the code of practice for the ultimate strength design of plate girders in steel bridges, BS5400 : part3, was introduced in the United Kingdom in 1982. It has been established that, when a thin walled plate girder is loaded by shear, failure will occur when the web plate yields under the joint action of the post-buckling membrane stress and the initial shear buckling stress of the web panel, Fig. 5. A simple formula to predict the ultimate shear load of a plate girder has been presented by Porter et al[13] considering

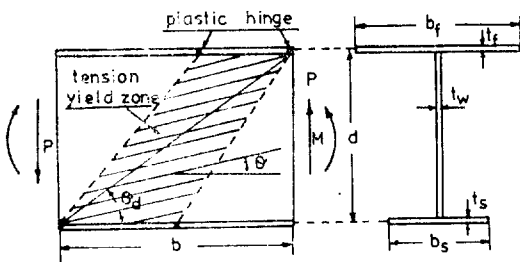


Fig. 5 Collapse mode of plate girder element

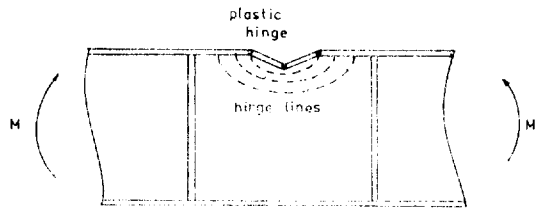


Fig. 6 Failure mechanism of plate girder by bending moment

a plastic mechanism method. In their theory, interaction between the web and the flanges of a plate girder was included. However, the plate girder is loaded primarily by bending, failure will occur by general yielding of the flanges or local buckling of the flanges, Fig. 6.

However, when the plate girder is subjected to shear and bending, it is necessary to allow for the influence of bending stresses on the buckling and post-buckling behaviour of the plate girder including in particular the influence of the bending stress on the stiffness of the flange members. In these instances, the accurate determination of the collapse strength involves a more complex solution than in the case of pure shear; nevertheless, as a result of an extensive parametric study presented by Evans et al[14], a simple method for predicting the failure load of plate girders under shear and bending has been presented. As a result of their studies, an interaction diagram for the ultimate strength of plate girders under shear and bending, shown in Fig. 7, was presented. In the figure, P_s represents the ultimate

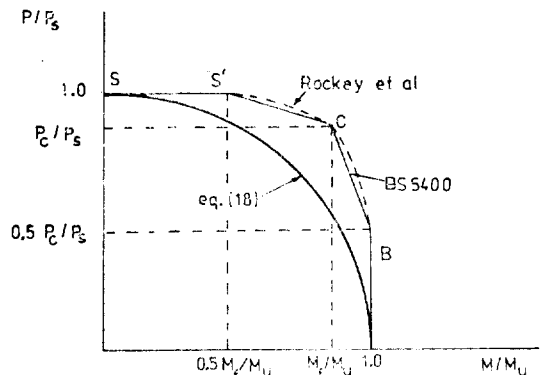


Fig. 7 Interaction diagram for collapse of plate girder

load in pure shear, M_u the ultimate bending moment and M_F the flange failure bending moment.

Since the construction of the complete interaction diagram is rather complicated, a simplified design procedure has been adopted in BS5400:part3 shown in Fig. 7. However, it may be difficult to adopt one of the interaction curves, as a collapse criterion of the frame analysis program, because many knuckle points are involved in the interaction curves. Thus, the following continuous function is adopted in the frame analysis program as a collapse criterion of plate girders.

$$\Psi = (P/P_s)^2 + (M/M_u)^2 = 1 \quad (18)$$

It is noted that the study performed by Sen[15] for assessment of the ultimate design rule of BS5400: part3 shows that 15 tests are inside the interaction curve of BS5400, among 113 collapse for plate girders reported in references, while it has been found that 6 tests are inside the proposed collapse curve of eq. (18) in this study.

The method for predicting the collapse of a thin-walled plate girder is tested for the following examples.

The first example of a plate girder is shown in Fig. 8, and the experiment for the collapse of the girder was performed by Ueda et al[11]. The girder is simply-supported at one end and highly restrained at the other end. A concentrated load is applied at the centre of the girder. The experimental results of the load-deflection relation are shown in Fig. 8. The frame analysis program developed in this study is used to analyze the collapse of the girder. It can be seen that the theoretical result correlates well with that of the experiment. In the analysis, the failure of the girder occurs by buckling of the two web plates at the centre and the web plate at the fixed end joint. This failure mechanism coincides with that of the experimental result.

The next example is a web frame model of a tanker, shown in Fig. 9. The experiments for the frame model are reported in the reference[12]. A concentrated load is applied at top of the centre strut. The failure mode of the web frame observed in the experiment is

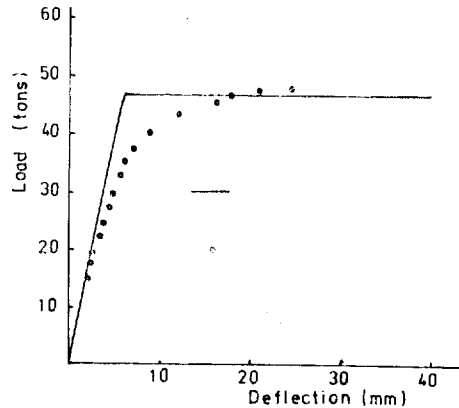
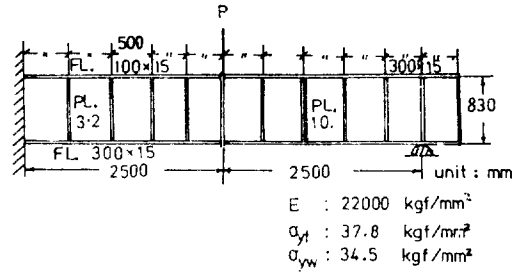


Fig. 8 Collapse analysis of a plate girder

- flange yielding at section 1, 2, 3, 4
- web buckling at upper corner section 3 and lower centre section 2.

The collapse load obtained in the experiment is 36.5 ton and the theoretical collapse load is 35.6 ton. The relation between load and vertical deflection at the loading point is shown in the figure together with the collapse sequence. It is noted that the analytical failure mode is yielding at the section 1, 2, 3, 4 by full plastic moments.

3.2. Damage analysis for model of a side structure

It is the purpose of this section to test the non-linear frame analysis program, developed in this study, for a side structure model of Hagiwara et al [16]. The structural model is shown in Fig. 10 and is an idealization of the side structure of a large tanker having a double hull construction, representing about one cargo hold of the tanker. In the structural model, the transverse frames were provi-

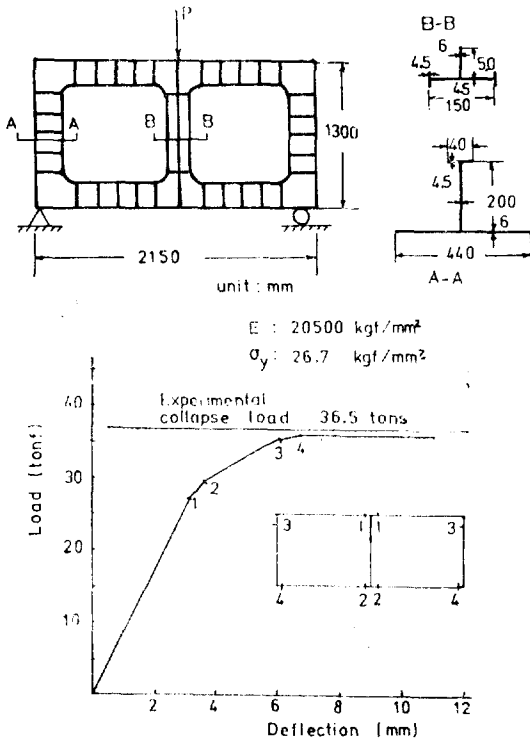


Fig. 9 Collapse analysis of a web frame

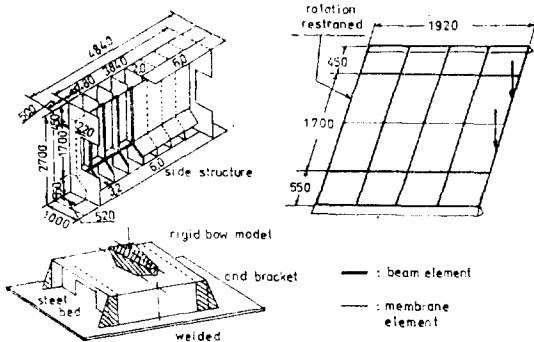


Fig. 10 Side structural model of Hagiwara et al[16]

ded in the manner similar to those of the actual ship, and the longitudinal stiffeners were treated as equivalent plate thickness under pure tension. The loading arrangement of the test is shown in Fig. 10, and it can be seen that a rigid-bow is used as a collision opponent.

A half of the structural model is idealized by a grillage model shown in Fig. 10. In the analysis,

it is assumed that the load is applied at the two points on the central transverse frame as shown in Fig. 10. Each transverse frame is modelled by beam elements. At the top and bottom of wing tanks there are longitudinal beam members, and deck and bottom plating are modelled by beam elements.

In the analysis of a cellular structure using a grillage technique, when the spacing of longitudinal webs is considerably greater than the of transverse webs, it becomes difficult to obtain an adequate representation of the longitudinal bending action by positioning grillage members at the webs only; such difficulties are due partly to errors introduced in the determination of the effective flange width[17]. However, in the large plastic deformation of the grillage, the longitudinal bending action may convert to membrane action. Thus, a membrane element is used instead of the fictitious bending element[17] in this study.

Hence, it is noted that width of the membrane elements are determined from the effective width concept presented in Interim Design Rules[18]. The resulting effective width ratio is approximately 0.25. Obviously, this procedure cannot rely on any theoretical basis, since the effective width concept is only valid for the elastic behaviour of a stiffened box girder. Therefore, the following two cases of width of the membrane elements are considered in this analysis, i.e. reduction to half of the effective width is assumed in case-1 study, while the full effective width is considered in case-2. So far, a very rough estimation has been introduced in the modelling of the membrane elements.

Therefore, these difficulties can be regarded as limitations and areas of further refinement of the frame analysis program.

The results of the analysis are shown in Fig. 11, and are compared with the experimental results. It can be seen in the comparison that there is considerable discrepancy between the analytical results and the experimental deformation of the outer hull plating in early stages of the load-deflection curves. This discrepancy may arise from a local denting of the web plate under the rigid bow which cannot be

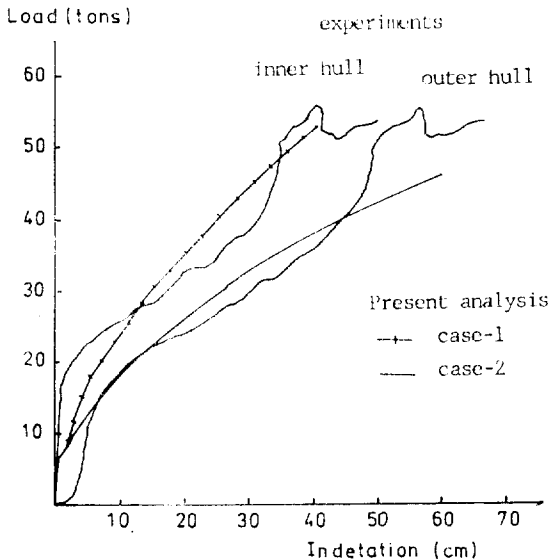


Fig. 11 Load-deformation relation of side structural model

obtained from the present frame analysis. However, as deformations of the structure increase, the results of the analysis for the case-1 show good correlation with the experimental result of the outer hull deformation, while the result of the case-2 correlates well with that of the inner hull deformation.

Since the width of the membrane elements in the case-2 analysis is twice as large as those of the case-1, the difference between the case-2 and case-1 of curves can be regarded as the influence of the longitudinal membrane action of the hull plating. It can be seen that the influence increases considerably as the deformations increase.

4. Conclusion

For the purpose of general collapse analysis of ship's side structures, a nonlinear 3-dimensional frame analysis program is developed in the study. Transverse shear deformations are included in the frame analysis formulation, since deep web beam elements are used in modelling such components of structure as web frames or double bottoms in ship structures. It is observed that the development of

the frame analysis program is easily accomplished, but a major difficulty lies in establishing a frame idealization of ship structures. Some modelling techniques of ship structures in frame analysis are summarized.

The frame analysis program is tested for collapse analysis of a grid structure, and it is concluded in this test that the present analysis program can predict efficiently the collapse load and collapse sequence of grid structures. For dealing with the collapse of thin-walled web frame, where effects of shear buckling strength of web plates are dominant, a simplified prediction method proposed by Rockey et al is adopted. In the comparison for the collapse strength of web frames, it is found that the results of the present frame analysis program correlate very well with the experimental results published in the references.

The present analysis program is applied to the collapse test of the side structural model of a tanker having a double hull. It is found that many difficulties and limitations are exercised in the modelling of double-hulled side structure into a framed structure, particularly when the longitudinal membrane action of hull plating between transverse web frames was modelled by membrane elements. Therefore, it is concluded that the present analysis program should be refined before being used in damage analysis of ship's side structures, although the resulting load-indentation relation of the present analysis correlates well with the experimental result in this case study.

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