Technical Paper

大 韓 造 船 學 會 誌
第26卷 第4號 1989年 12月
Journal of the Society of
Naval Architects of Korea
Vol. 26, No. 4, December 1989

Design Equation for Predicting the Residual Strength of Damaged Tubulars Under Combined Axial Compression and Hydrostatic Pressure

by

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축 압축력과 수압하의 손상된 원통의 잔류 강도 추정을 위한 설계식

조 상 래*

Abstract

Recently the residual strength of damaged tubulars under axial compression has extensively been investigated. However, in spite of the possibility of damage onto underwater members of offshore structures as results of collisions, dropped objects and other accidental impacts occurring in service or during fabrication or installation, no research works on the structural behaviour of damaged tubulars under combined loadings including hydrostatic pressure have been reported in the literature.

In this paper, a numerical method has been proposed to estimate the residual strength of damaged tubulars under combined loadings, and then the proposed method has been substantiated with corresponding test data. A simple design equation has been derived based upon the results of the parametric study using the proposed method. The accuracy of the predictions using the derived equation is found to be a 10.1% COV(Coefficient of Variation) together with an 1.037 mean comparing with the test data.

요 약

최근 배와의 충돌이나 중량물의 낙하 또는 건조, 설치 및 운영 과정 중 예기치 않은 충격 하중으로 인한 해양 구조물의 손상 과정이나, 손상 후의 잔류 강도에 관한 연구가 많은 관심을 끌고 있다. 잔류 강도의 경우, 축 압축력 하의 손상된 원통의 거동에 관한 연구는 많이 이루어진 편이나 수면하 부재가 손상을 입는 경우, 즉 수압을 포함한 복합하중 하의 손상된 원통의 거동에 관한 연구 결과는 거의 발표되지 않은 상태이다.

본 논문에서는 수압과 축 압축력에 대한 손상 원통의 잔류 강도를 추정할 수 있는 수치 해석 방법을 제안하고, 관련된 실험 결과들과 비교하여 검증하였다. 또한 이 방법을 사용한 수치 해석 결과를 바탕으로 간편한 설계식을 유도하였다. 이 설계식에 의한 추정 값과 실험 결과를 비교한 바, 그 비의 평균이 1.037이고 COV(coefficient of variation)는 10.1%가 되었다.

본 논문은 1989년도 대한조선학회 춘계연구발표회에서 발표된 논문임
Manuscript received: May 1, 1989. Revised Manuscript received: July 19, 1989

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Notation

B length of the flattened part of a damaged tubular

C_c correction factor for shell effects defined as eqn (18)

D diameter to mid-thickness of a tube

E Young's modulus

E_{eff} 'effective' modulus defined as eqn(12)

L length of a tube

M bending moment

M_H bending moment per unit length due to hydrostatic pressure

M_p plastic bending moment capacity of an intact cross-section of a tube

M_{pd} plastic bending moment capacity of a dented cross-section of a tube

M_z bending moment about z-axis, increment dM_z

M_1 linear limit bending moment

P_{ext} externally applied axial force

P_H axial force due to hydrostatic pressure, $\pi/4\ Q_H (D+t)^2$

P_t total applied axial force, $P_{ext}+P_H$, increment dP_t

P_Y axial force at fully yield condition of a tube's cross-section, $\pi\ \sigma_Y\ D\ t$

Q_H applied hydrostatic pressure

Q_{Hcr} elastic buckling pressure of a 'long' tube under hydrostatic pressure, $2E/(1-v^2)/(D/t)^3$

Q_{ij} element of tangent stiffness matrix $[Q]$ defined as eqn(15)

R mean radius of a tube or radius of curvature of a finite shell element before denting deformation

R' radius of curvature of a finite shell element after denting deformation

d_d depth of dent at the point of impact

d_0 out-of-straightness of a damaged tube

m non-dimensionalised value for $M_z, M_z/M_p$

m_{pc} non-dimensionalised value for M_{pc}, M_{pc}/M_p

m_1 non-dimensionalised value for M_1, M_1/M_p

p non-dimensionalised value for P_{ext}, P_{ext}/P_Y

q non-dimensionalised value for Q_H, Q_H/Q_{Hcr}

t thickness of a tube

x co-ordinate axis along the tube, see Fig. 3

y co-ordinate axis normal to the tube, see Fig. 3

y' distance from the middle surface of a tube: (+); outwards, (−); inwards

z co-ordinate axis normal to the tube, see Fig. 3

Φ curvature of a cross-section

Φ_Y curvature at initial yield state of an intact cross-section, $2\sigma_Y/E/D$

Φ_z curvature with respect to z-axis, increment $d\Phi_z$

δ_d non-dimensionalised depth of dent at the point of impact of a tube, d_d/D

δ_0 non-dimensionalised out-of-straightness of a damaged tube, d_0/L

ε_x axial strain, increment $d\varepsilon_x$

ε_{x0} axial strain on z-axis, increment $d\varepsilon_{x0}$

$\varepsilon_{\theta r}$ circumferential residual strain due to denting damage

ϕ non-dimensionalised curvature with respect to z-axis, Φ_z/Φ_Y

ϕ_0 non-dimensionalised curvature with respect to z-axis due to external axial force and/or hydrostatic pressure

ϕ_1 non-dimensionalised curvature with respect to z-axis corresponding to m_1

λ reduced slenderness ratio of a column, $\sqrt{\sigma_Y/\sigma_{cr}}$

λ_{PR} Perry-Robertson 'imperfection' parameter

ν Poisson's ratio

σ_{cr} Euler column buckling strength

σ_e von Mises equivalent stress, $\sqrt{\sigma_x^2+\sigma_\theta^2-\sigma_x\sigma_\theta}$

σ_u ultimate strength of a column under combined axial compression and hydrostatic pressure

σ_x axial stress, increment $d\sigma_x$

σ_Y static yield stress

$\sigma_{\theta H}$ circunferential stress due to hydrostatic pressure

$\sigma_{\theta r}$ circunferential residual stress due to denting damage

1. Introduction

There are basically two models suggested for the evaluation of the ultimate strength and postultimate strength behaviour of damaged tubular members subjected to axial compression. One is proposed by

Taby, Moan and Rashed[1], and the other is by Smith, Somerville and Swan[2]. However, it seems difficult to adopt any of these two models for combined axial compression and hydrostatic loading not only because both of these models involve the empirical factors based upon test results of axially compressed damaged tubulars but because hypothetical stresses were used in the analyses rather than real occurring ones. In order to incoporate hydrostatic pressure in the analysis it seems necessary to develop a method by which real occurring stresses can be determined. In this study, therefore, the geometric configuration of dented portion is realistically simulated in the analysis by using the equations obtained based on the lateral impact test results[3] and and the circumferential residual stresses due to denting deformation are considered. In other words the damaged tubular is treated as a beam-column having varing cross-section and residual stresses.

For a long or intermediate length beam-column having initial crookedness the effect of lateral deflection which magnifies the primary moments by the axial load cannot be ignored in the analysis. Therefore the ultimate strength of the beam-column should generally be determined from the stand point of load-deflection analysis. On top of that if the column fails beyond the elastic limit of the material the problem becomes more complicated and, thus, recourse must be made to numerical methods to obtain solutions.

The numerical method proposed in this paper involves two separate phases of calculations:

o The moment-external axial compression-hydrostatic pressure-curvature ($M-P_{ext}-Q_H-\Phi$) relationships for damaged cross section are derved;

o then, using the relationship the residual strength of the damaged tubular is determined.

The $M-P_{ext}-Q_H-\Phi$ relationships are computed using the tangent stiffness formulation[4] and the approximate equation for the relationships are obtained by fitting the computed data to non-linear multiple regression models. The ultimate strength is computed by using the Newmark's integration method[5].

The predictions using the proposed method are compared with available experimental results to demonstrate their validity and accuracy. Using the developed method a rigorous parametric study is performed to calculate the residual strengths of damaged tubulars under combined axial compression and hydrostatic pressure. And then a design equation is derived based on the parametric study results where the Perry formula[6] is adopted as a basis of the formulation.

2. $M-P_{ext}-Q_H-\Phi$ Relationships for Dented Tubular Sections

The $M-P_{ext}-Q_H-\Phi$ or generalised stress-strain relationships may be computed every time in need in the ultimate strength solution scheme. However, by using close-form approximate expressions for the relationships instead of computing the relationship in the solution scheme the computing time can considerably be reduced. As a starting point of the ultimate strength analysis, therefore, approximate equations are derived for a dented tubular cross-section subjected to a given value of external axial force and hydrostatic pressure.

Using the test results obtained from the sections deformed due to lateral impact the geometric configuration of a dented tubular cross-section having a given non-dimensionalised depth of dent is mathematically described straightforwardly in ref. 3. In the description a dented section is assumed to consist of one flattened segment, two segments of reduced radius and one segment of enlarged radius.

2.1. Residual Stresses

It seems not easy to accurately express the longitudinal and circumferential residual stresses in a tubular caused by the local denting and overall bending deformation due to lateral impact. In the present analysis, however, the residual stresses in circumferential direction only are simply approximated. By assuming that the denting of the cross-section is the results of irreversible and inextentional circumferential bending deformations, the circumfe-

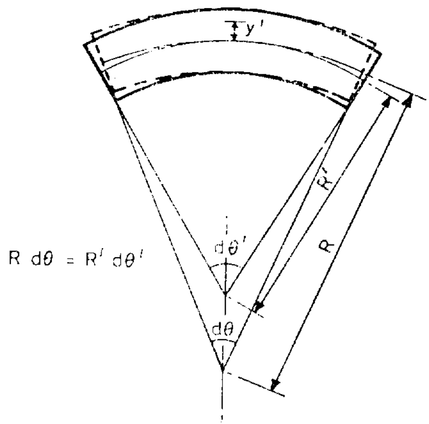

Fig. 1 Inextentional circumferential bending deformation of a tube segment

rential strain can be obtained by the following equation (see Fig. 1).

$$\varepsilon_{\theta r}=\frac{(R'+y')\frac{R}{R'}d\theta'}{(R+y')d\theta}-1=\frac{y'\left(\frac{R}{R'}-1\right)}{R+y'} \qquad (1)$$

where $\varepsilon_{\theta r}$: circumferential residual strain due to denting damage

R : radius of curvature of the finite shell element before denting

R' : radius of curvature of the finite shell element after denting

y' : distance from the middle surface of the tube (+) ; outwards, (−) ; inwards

$d\theta$: central angle of the finite element before denting

$d\theta'$: central angle of the finite element after denting

Consequently, the circumferential residual stress ($\sigma_{\theta r}$) due to denting damage can be obtained from eqn (2).

$$\sigma_{\theta r}=\begin{cases}\sigma_Y & ;\ \sigma_{\theta r}\geq\sigma_Y\\ E\dfrac{y'}{R+y'}\left(\dfrac{R}{R'}-1\right) & ;\ |\sigma_{\theta r}|<\sigma_Y\\ -\sigma_Y & ;\ \sigma_{\theta r}\leq-\sigma_Y\end{cases} \qquad (2)$$

2.2. Effect of Hydrostatic Pressure

Since hydrostatic pressure does not introduce any bending moment along the length, it does not contribute to the deflection and consequently not influence the theoretical elastic buckling strength of the column It is accordingly necessary to distinguish the external axial compression from the end force due to hydrostatic pressure. However, the axial and hoop stresses due to hydrostatic pressure may indirectly influence the failure load of a column in inelastic range.

Because of the lack of symmetry in the cross-section of a dented tubular the resultant hoop stress produced by hydrostatic pressure applies eccentrically causing an additional moment with respect to the middle surface of the wall. Furthermore, the eccentrically applied hoop stress can magnify the cross-sectional geometric imperfection, which in turn increase the bending stress in the circumference. In order to consider the magnification effect of hydrostatic pressure in the analysis it is assumed that the circumferential deformation are inextensional and the internal circumferential forces in the dented tubular of unit length renduce to a constant circumferential force and a bending moment. The constant circumferential force and the bending moment can be obtained from eqns (3) and (4) respectively.

$$S=Q_H\frac{D}{2} \qquad (3)$$

$$M_H=Sw_0\frac{1}{1-Q_H/Q_{Hcr}} \qquad (4)$$

where S : circumferential force per unit length due to hydrostatic pressure

M_H : bending moment per unit length due to hydrostatic pressure

w_0 : radial deviation of the dented cross-section from the perfect circle, $D/2-\sqrt{y^2+z^2}$ (see Fig. 2)

In the equations the out-of-roundness is defined as the radial deviation of the dented section from a

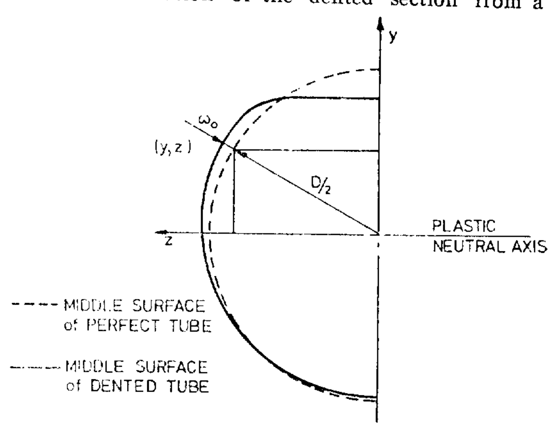

Fig. 2 Radial deviation of dented cross-section from a perfect circle

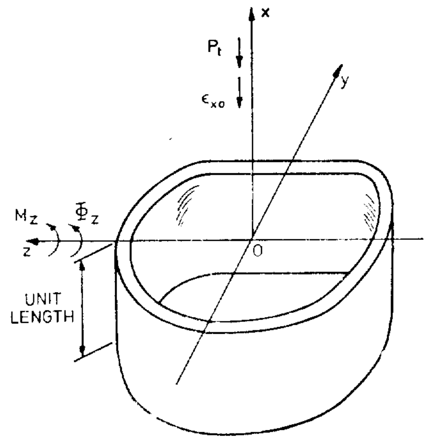

Fig. 3 Positive vectors of generalised stresses and generalised strains

perfect circular form, and the magnification of the geometric imperfection due to the hydrostatic pressure is considered by multiplying the well-known amplification factor, $1/(1-Q_H/Q_{Hcr})$. Finally, the circumferential stress due to hydrostatic pressure can be calculated from eqn(5).

$$\sigma_{\theta H}=\frac{Q_H D}{2t}+w_0\frac{1}{1-Q_H/Q_{Hcr}}\frac{6y'Q_H D}{t^3} \tag{5}$$

2.3. Tangent Stiffness Formulation

In the tangent stiffness method the cross-section is divided into many small elements and the total axial force(P_{ext}) and bending moment(M_z) can be obtained by summing up the effects of axial stresses

$$P_t=\int_A \sigma_x dA \tag{6}$$

$$M_z=\int_A \sigma_x y dA \tag{7}$$

The generalised stresses and strains are shown in Fig. 3 in positive direction, where z-axis coincides with the plastic neutral axis of the cross-section. By assuming that plane remains plane after deformation the axial strain at a point in the cross-section can be expressed in a linear form as

$$\varepsilon_x=\varepsilon_{x0}+y\Phi_z \tag{8}$$

where ε_{x0} : axial strain on z-axis
Φ_z : curvature with respect to z-axis

Because of the non-linear character of the material property (the material is assumed to be elastic-perfectly plastic) it is necessary to establish incremental generalised stress equations. Changing eqns(6) and (7) into incremental form eqns(9) and (10) can be obtained.

$$dP_t=\int_A d\sigma_x \, dA \tag{9}$$

$$dM_z=\int_A d\sigma_x y \, dA \tag{10}$$

The rate of change of axial stress is then given as eqn(11) by introducing the von Mises criteria.

$$d\sigma_x=E_{eff} \, d\varepsilon_x \tag{11}$$

$$E_{eff}=\begin{cases} E \, ; \, |\sigma_e|<\sigma_Y \\ 0 \, ; \, |\sigma_e|\geq\sigma_Y \end{cases} \tag{12}$$

where σ_e : von Mises equivalent stress, $\sqrt{\sigma_x^2+(\sigma_{\theta r}+\sigma_{\theta H})^2-\sigma_x(\sigma_{\theta r}+\sigma_{\theta H})}$

The equation for axial strain change rate is

$$d\varepsilon_x=d\varepsilon_{x0}+y \, d\Phi_z \tag{13}$$

By carrying out substitution eqns(9) and (10) yield the following incremental relationship in matrix form:

$$d\begin{Bmatrix} M_z \\ P_t \end{Bmatrix}=\begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} d\begin{Bmatrix} \Phi_z \\ \varepsilon_{x0} \end{Bmatrix} \tag{14}$$

where $[Q]$ is called the tangent stiffness matrix whose elements Q_{ij} are defined as

$$\begin{aligned} Q_{11}&=\int_A E_{eff} y^2 dA \\ Q_{12}=Q_{21}&=\int_A E_{eff} y dA \\ Q_{22}&=\int_A E_{eff} dA \end{aligned} \tag{15}$$

Once the tangent stiffness matrix $[Q]$ corresponding to a given state of stress can be evaluated, the path of generalised strains for a given path of generalised stresses can be determined through a step-by-step incremental calculation and an iteration procedure. For a given state of increments of external forces the corresponding increments of deformations may be approximately obtained from eqn(14) when all the information of stress and strain and the tangent stiffness matrix of the current state are known. However, the solution for a partly yielded section may deviate considerably from the exact value because the tangent stiffness matrix is that before the increments occur. Therefore an iteration procedure must be employed for inelastic problems. In this study the step-by-step iterative technique is adopted proposed in ref. 7.

2.4. $M-P_{ext}-Q_H-\Phi$ Data Generation

Based on the equations formulated a computer program was developed to provide numerical results from which approximate equations can be derived for damaged tubulars under hydrostatic pressure. Using the computer program computations have been conducted for the following values of parameters:

$D/t=20.0,\ 40.0,\ 60.0$

$\delta_d=0.00,\ 0.01,\ 0.02,\ 0.05,\ 0.10,\ 0.15,\ 0.20$

$Q_H/Q_{Hcr}=0.0,\ 0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5,\ 0.6$

$P_{ext}/P_Y=0.0,\ 0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5,\ 0.6,\ 0.7,\ 0.8,\ 0.9$

where P_Y : axial load at fully yielded condition of a section, $\pi\ \sigma_Y\ D_t$

In the computation a half of the damaged tubular cross-section was divided into fibres as shown in Fig. 4 and diameter, Young's modulus and yield stress were assumed to be $50MM$, $210,000N/MM^2$ and $350N/MM^2$ respectively. Bending moment M_z was increased by 1% of the fully plastic moment M_Y when external axial force P_{ext} was less than 0.8 P_Y and the increment was reduced to 0.5% M_p when P_{ext} was greater than or equal to 0.8 P_Y.

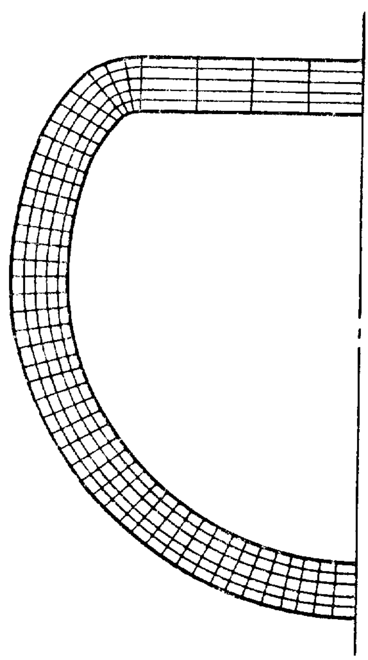

Fig. 4 Division of damaged cross-section into fibres

Iteration was continued until both the unbalaced values for $P_t(=P_{ext}+P_H)$ and M_z were less than 0.01% of P_Y and M_{pd} respectively. M_{pd}, fully plastic bending moment of the dented section, was calculated numerically in the program. Fully plastic state of the section was defined when the determinant of the tangent stiffness matrix $[Q]$ was not positive or when the curvature Φ_Z was greater than fifty times of Φ_Y.

2.5. Derivation of Approximate Equations

The analysis of damaged tubular beam-column problem may considerably be simplified if an analytical expression can be found to reasonably approximate the numerically computed $M-P_{ext}-Q_H-\Phi$ relationships. Using non-dimensionalised quantities,

$$q=Q_H/Q_{Hcr},\quad p=P_{ext}/P_Y,\quad m=M_Z/M_p,\quad \phi=\Phi_z/\Phi_Y \tag{16}$$

were M_p : plastic bending moment capacity of an intact tube, $\sigma_Y D^2 t$

Φ_Y : curvature at initial yielding, $2\sigma_Y/E/D$

the non-linear moment-curvature relationships may be approximately represented by:

$$m=\begin{cases}0 & (\phi\le\phi_0)\\ a(\phi-\phi_0) & (\phi_0\le\phi\le\phi_1)\\ m_{pc}-(m_{pc}-m_1)\exp[f(\phi)] & (\phi_1<\phi)\end{cases} \tag{17}$$

where a : slope of the linear part

m_1 : non-dimensionalised linear limit bending moment, M_1/M_p

m_{pc} : non-dimensionalised fully plastic bending moment, reduced for the presence of axial load, M_{pc}/M_p

$$f(\phi)=-c_1(\phi-\phi_1)\cdot c_2$$

$$\phi_1=\frac{m_1}{a}+\phi_0$$

The parameters a, ϕ_0, c_1, c_2, m_1, and m_{pc}, which are functions of $D/t, \delta_d, q$ and p, were determined using the computed results of the moment-curvature relationships. In the derivation the values of a, ϕ_0, m_1 and m_{pc} for each moment-curvature curve were first determined from the computed data and then a regression analysis was carried out for each of them. The fitted approximate equations for those values are given in ref. 8 and the equations together with some of the computed retults are illustrated in Fig. 5.

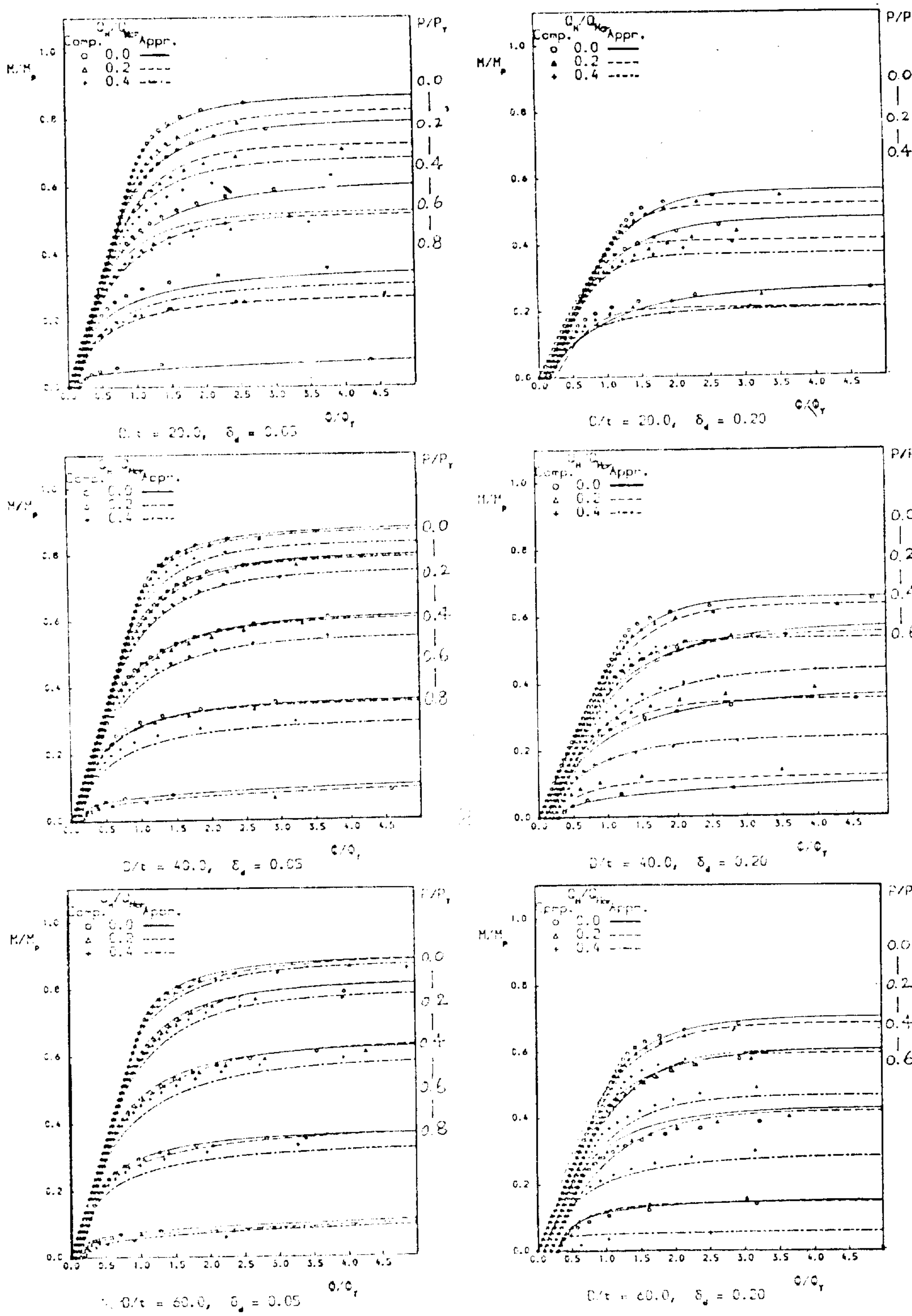

Fig. 5 Approximate equations for $M-P_{ext}-Q_H-\Phi$ relationships of damaged tubulars as derived from computed data

3. Residual Strength

3.1. Effect of Local Shell Deformation

In the derivation of the moment-curvature relationships for dented tubular sections the dented cross-section was assumed not to change, i.e. no further local deformation was considered. For deeply dented thin tubes a notable local shell deformation at damaged part, probably in the form of growth of dent depth, may occur before ultimate state and consequently the ultimate strength can be reduced.

The results are illustated in Fig. 6 of pure bending tests on damaged tubulars (compression in dent) given in ref. 9 where M_u is the experimental ultimate bending moment and M_{pd} is the fully plastic bending moment of the dented section. As clearly be seen in the figure the fully plastic capacity of damaged tubulars under pure bending can be reduced further for the thinner and more deeply dented ones. This is probably due to the local shell deformation at damaged part, which can be exhibited through the growth of dent depth. Therefore, a modification must be made of the moment-curvature relationships which were derived neglecting the change of the cross- section in order to account for such a deteriorating effect.

For that purpose the moment-curvatiure relation-

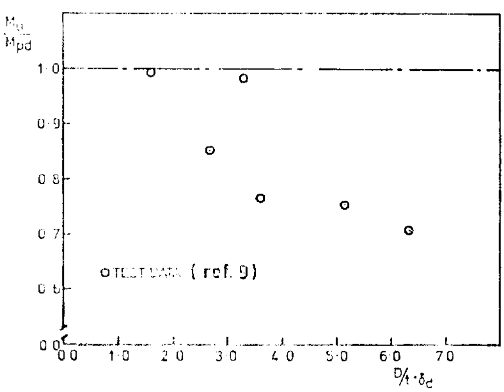

Fig. 6 Dependence of ultimate strength of damaged tubulars under bending moment (compression in dent) on diameter to thickness ratio (D/T) and depth of dent (δ_d)

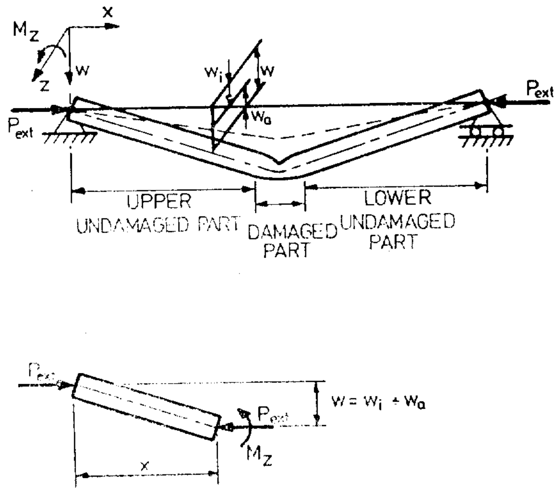

Fig. 7 Simply supported damaged tubular

ships, which were derived using the tangent stiffness method and then approximated by regression, have been modified. By multiplying the correction factor C_s by m_1 and m_{pc}, i.e. reducing both the linear limit moment and the non-linear part by C_s, the modified moment-curvature relationships have been obtained.

$$C_s = \begin{cases} \exp(0.44-0.011D/t-1.6\delta_d) & C_s<1 \\ 1 & C_s\geq 1 \end{cases} \tag{18}$$

3.2. Newmark's Integration Method

Having obtained the modified $M-P_{ext}-Q_H-\Phi$ relationships for dented tubular sections, the residual strength of damaged tubulars are determined by using the Newmark's integration method, which was initially proposed particularly for the determination of buckling loads of bars of variable cross-section and which has recently been employed successfully for the ultimate strength analysis of fabricated tubular columns[10].

The calculation steps of the Newark's numerical procedure described in the following to determine the residual strength of a damaged tubular having simply supported boundries.

- Procedure of Calculation:

1) Divide the upper undamaged part, damaged part and lower undamaged part. The nodal points are called stations. Describe the initial out-of-straightness at all stations.

2) Assume an additional deflection at every station,

3) Compute bending moment M_z (see Fig. 7) at all stations due to the given axial load P_{ext}.

4) Compute curvature at all stations from the $M-P_{ext}-\Phi$ relationships of the section.

5) Determine the deflection at all stations using the Newmark's integration method.

6) Compare the new deflections with the assumed additional deflections (check convergence). If they show an acceptable agreement the assumed additional deflections are the correct additional deflection of the member for the given load. If not, repeat steps 2-5 until the deflected shape converges into a prescribed error bound. For that case the new deflections can be new assumed additional deflections.

7) Increase the axial load and repeat steps 2-6 until the resultant deflections diverge, at which the axial load exceeds the ultimate strength of the member.

4. Correlation Study

Based on the analysis procedure decsribed above a computer program was written for determining the residual strength of a damaged tubulars subjected to combined axial compression and hydrostatic pressure. Using the program a correlation study has been performed with available test data in order to validate the proposed method. A summary of the correlation study results is given in Table 1 and a plot of the actual to predicted strength ratios against

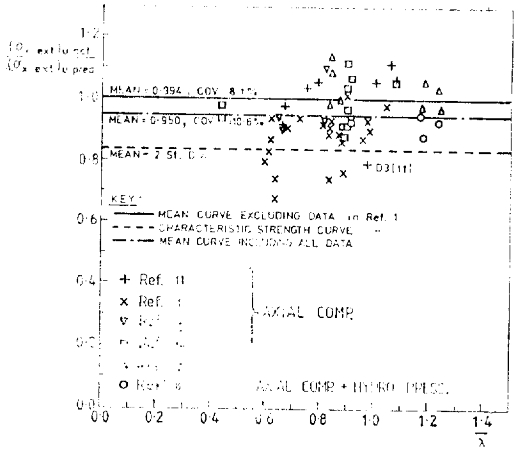

Fig. 8 Comparison of actual to predicted strength using proposed theoretical methiod for damaged tubulars under combined axal compression and hydrostatic pressure

the reduced column slenderness ratio is provided in Fig. 8. The actual to predicted ratio for a total of fifty seven test data give a 10.6% COV together with a 0.950 mean. However, twenty one Trondheim test data[1] give a much smaller mean than those of the other sources, which is probably because the dent depth was measured relative to the upper generatrices of the undamaged part of the tube wall in which the measured value can easily be too small [13]. When excluding these data the COV and mean are improved to 8.1% and 0.994 respectively.

Comparing these values with an 8.2% COV and 0.992 mean obtained by analytical predictions[13]

Table 1 Summary of correlation study

Ref. No.	Loading Type	Number of Tests	Actual to Predicted Strength Ratio Mean	COV
[11]	Axial Comp.	8	1.005	10.6%
[1]	ditto	21	0.872	9.8%
[2]	ditto	4	0.968	9.3%
[12]	ditto	12	0.984	7.4%
[3]	ditto	8	1.034	5.8%
	sub-total (Axial Comp.)	53	0.949	10.9%
[8]	Axial Comp.+Hydro. Press.	4	0.953	7.5%
	total (for all data)	57	0.950	10.6%
	(excluding the data in ref. 1)	36	0.994	8.1%

of forty four Trondheim test data and an 11% COV and 1.01 mean obtained using a non-linear finite beam-column element computer program[12] for fifty seven test data in refs. 1, 2, 11 and 12 it seems that the proposed theoretical method provides reasonably reliable and at the same time accurate estimates of residual strength for damaged tubulars. According to the COV and mean excluding the Trondheim test data, only one data, model *D*3 in ref. 11, is on the unsafe side of the characteristic strength defined as mean minus 2 standard deviation.

5. Derivation of Design Equation

Using the developed method described above a rigorous parametric study has been performed to calculate the residual strengths of the damaged tubulars under pure axial compression and under combined axial compression and hydrostatic pressure for the following values of parameters.

D/t $=20.0,\ 40.0,\ 60.0$

λ $=0.25, 0.50,\ 0.75,\ 1.00,\ 1.25$

δ_d $=0.00,\ 0.01,\ 0.05,\ 0.10,\ 0.15$

δ_0 $=0.0005,\ 0.001,\ 0.005,\ 0.01,\ 0.02$

$Q_H/Q_{Hcr}=0.0\ 0.1\ 0.2\ 0.3$

The Perry formula, eqn[19], is adopted as the basis of the proposed design equation to predict the residual strength of damaged tubulars under combined axial compression and hydrostatic pressure.

$$(\sigma_Y-\sigma_u)(\sigma_{cr}-\sigma_u)=\lambda_{PR}\sigma_{cr}\sigma_u \tag{19}$$

where σ_u : failure stress

σ_{cr} : Euler column buckling strength

λ_{PR} : Perry-Robertson 'imperfection' parameter

Using the parametric study results the Perry-Robertson 'imperfection' parameter was evaluated. Before deriving an expression for λ_{PR}, it was assumed that λ_{PR} consists of three parts namely

$$\lambda_{PR}=\lambda_{PRO}\ \lambda_{PRL}\ \lambda_{PRH} \tag{20}$$

where λ_{PRO} : overall straightness imperfection parameter

λ_{PRL} : equivalent imperfection parameter for local denting

λ_{PRH} : equivalent imperfection parameter for hydrostatic pressure

The equation finally derived are as follows

$$\begin{aligned}
\lambda_{PRO}&=22.2(\delta_o\lambda)^{0.7}\\
\lambda_{PRL}&=1.0+1.26\delta_d^{1.3}(D/\mathrm{t})^{0.6}\\
\lambda_{PRH}&=\exp[0.025(Q_H/Q_{Hcr})^2\lambda^{0.5}(D/t)^{-0.5}\delta_o^{-1}]
\end{aligned} \tag{21}$$

Having derived the expression for λ_{PR}, eqn[20] together with eqn[21], the residual strength of damaged tubulars under combined axial compression and hydrostatic pressure can be estimated using eqn [22] which is the lower root of eqn[19].

$$\sigma_u=\frac{\sigma_Y+(1+\lambda_{PR})\sigma_{cr}}{2}-\sqrt{\left\{\frac{\sigma_Y+(1+_{PR})\sigma_{cr}}{2}\right\}^2-\sigma_{cr}\sigma_Y} \tag{22}$$

6. Discussion

Adopting the Perry formula as the basis and then deriving the Perry-Robertson design formula, eqn [22] together with eqns [20] and [21], has been obtained to predict the residual strength of simply supported damaged tubulars having a 'sharp' dent at mid-length under combined axial compression and hydrostatic pressure. In comparison with the prediction accuracy of the theory, i.e. 10.6% COV for all of the available test results and 8.1% excluding the results given in ref. 1, the accuracy of the prediction using the proposed formula is found to be a little bit worse. Despite the fact that the location of damage and shape of dent were not considered in the calculation, the accuracy of the predictions, however, is in the range accepted as a well formulated one, say less than 13%.

The effects of the damage location and dent shape on the residual strength were investigated using the proposed methods, and the results are illustrated in Figs. 9(a) and 9(b) respectively. As can be seen in Fig. 9(a), the residual strength can be increased by some 8% and 16% when the damage location changes from $x_d/L=0.5$, midspan, to $x_d/L=0.2$ and 0.1 respectively with negligible differences depending on hydrostatic pressure. These results are similar to those of the experimental findings in ref. 12. The figure also shows that the difference in the residual strength is insignificant if the centre of

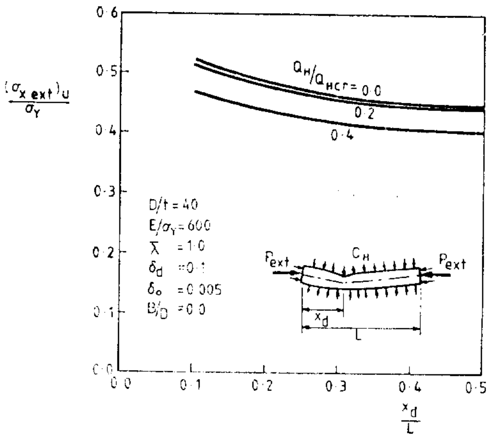

Fig. 9 (a) Effect of damage location on residual strength of damaged tubulars under combined axial compression and hydrostatic pressure

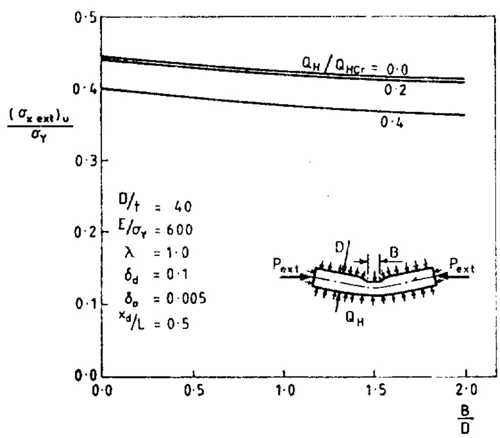

Fig. 9 (b) Effect of dent shape on residual strength of damaged tubulars under combined axial compression and hydrostatic pressure

damage is in the middle half of the tube. According to Fig. 9(b) when the length of flattened part (B) is twice of the diameter the reduction of strength upon that of sharp dent can be about 8% for Q_H/Q_{Hcr}=0.0 and about 11% for Q_H/Q_{Hcx}=0.4. Unlike the case for damage location a little bit further reduction of residual strengh can be expect for higher hydrostatic pressure.

The influences of extent of damage, depth of dent and out-of-straightness, diameter to thickness ratio and hydrostatic pressure on the residual strength of simply supported damaged tubulars having a 'sharp' dent at mid-length under combined axial compression and hydrostatic pressure are demonstrated in Figs. 10(a)-(d). The influence of extent of damage on the residual strength is most significant, while that of hydrostatic pressure is negligible when Q_H/Q_{Hcr} =0.2 (which is corresponding to approximately 150M water depth when D/t=40) and when Q_H/Q_{Hcr}=0.4 (which is corresponding to approximately 300M water depth when D/t=40) the loss of strength due to hydrostatic pressure is at most about 7% for a damaged tube of δ_d=0.1, δ_o=0.005 and D/t=40. However, the strength reduction due to the presence of hydrostatic pressure can be increased to some 13% when δ_d=0.15 and δ_o=0.005.

The proposed formula is based on the results of the parametric study of damaged tubulars having simply simply supported boundaries. Obviously, the end restraints of offshore unstiffened tubulars are different from that of simply supported. However, for undamaged tubular column the effect of end conditions is normally accosunted for by means of the effective length concept. But direct application of the effective length approach for undamaged tubulars to damaged ones can give conservative results especially for severely damaged cases [14, 15]. On top of that in the case of bracing members supported by chords the end restraint may be influenced not only by the flexural rigidities of chord members but also by local flexibility of chord walls. Therefore in order to improve the prediction accuracy it seems necessary to modify the effective length calculated for the corresponding undamaged tubulars in which the local flexibility of chord walls is also considered.

7. Conclusions

An analytical method has beed developed to estimate the residual strength of damaged tubular members under combined axial compression and hydrostatic pressure. Using the developed method a

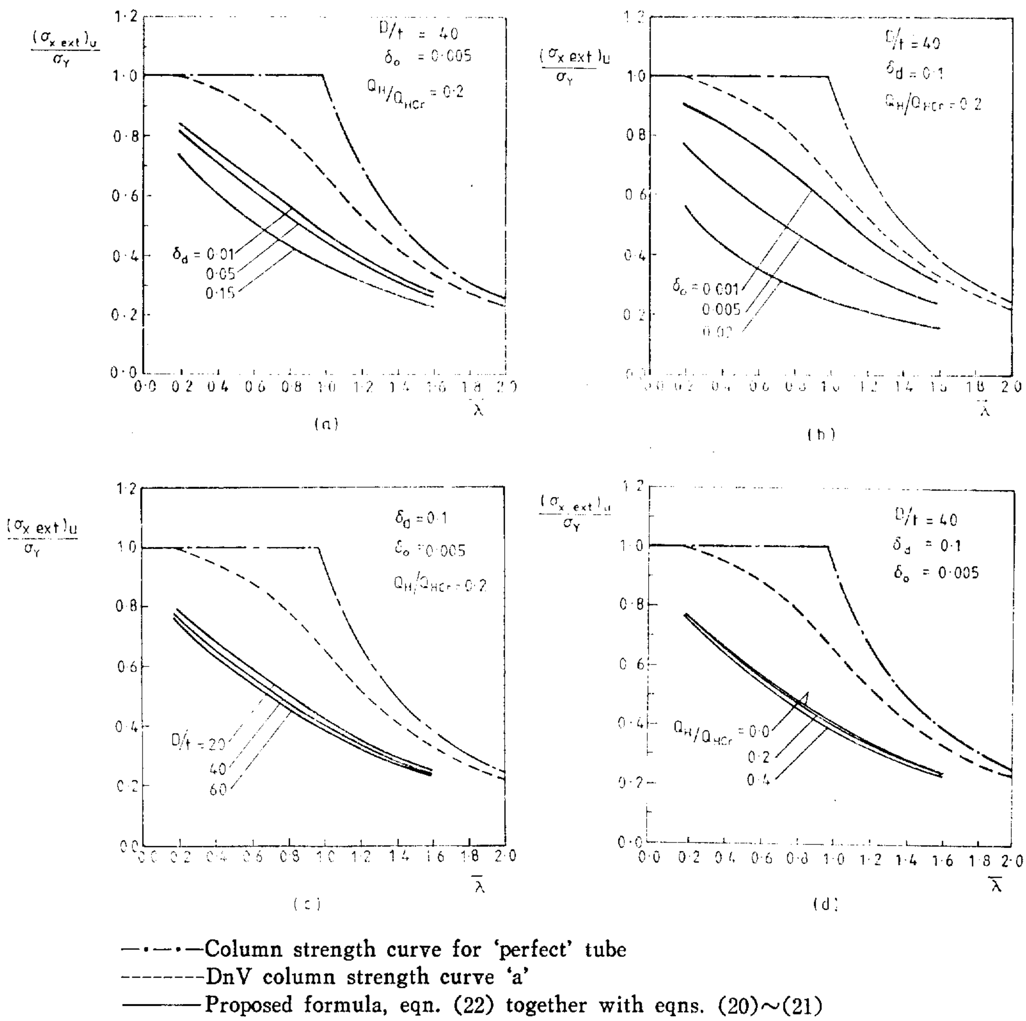

Fig. 10 Influences of parameters on residual strength of damaged tubulars under combined axial compression and hydrostatic pressure: (a) depth-of-dent (δ_d), (b) out-of-straightness(δ_0), (c) diameter to thickness ratio (D/t), (d) hydrostatic pressure (Q_H/Q_{Hcr})

correlation study with avaliable test data and parametric studies have been performed. With reference to the results obtained from these studies the following conclusions can be made:

a) The actual to predicted residual strength ratios for a total of fifty seven test data available give a 10.6% COV together with 0.950 mean. When excluding twenty one Trondheim test data the COV and mean are improved to 8.1% and 0.994 respectively. It seems that the proposed theoretical method provides reasonably reliable and at the same time accurate estimates of residual strength for damaged tubulars;

b) for thinner and deeply dented tubulars it is necessary to consider the local shell deformation in the analysis in order to safely estimate the residual strength of damaged tubulars;

c) the effects of damage location and dent shape on the residual strength is not significant, therefore from practical design veiw point these factors can be ignored.

d) the influence of the extent of damage on the residual strength is most significant, while that of hydrostatic pressure is less significant.

The Perry formula was adopted as the basis of the design equation derived in this study and then an expression for the Perry-Robertson 'imperfection' parameter was obtained based on the parametric study results. The following are the findings:

a) Eqn [22] together with eqns[20] and [21] can be used to predict the residual strength of simply supported damaged tubulars under combined axial compression and hydrostatic pressure;

b) the correlation of all the available test results with predictions using the proposed design equation gives a 12.8% COV together with a mean of 0.983.

In order to improve the prediction accuracy of the derived design equation it seems necessary to modify the effective length calculated for the corresponding undamaged tubulars in which the local flexibility of chord walls is also considered.

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