論 文

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Nonlinear Perturbation Method for Dynamic Structural Redesign

by

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動的 構造 再設計를 위한 非線形 攝動法

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Abstract

Many mechanical systems including ships and/or offshore structures have poor dynamic response characteristics such as undesirable natural frequencies and undesirable mode shapes. It is mandatory to redesign the structure. In this paper a procedure for the dynamic redesign of an undamped structural system is presented. The method which uses a penalty function with a penalty term containing error in equilibrium for a given vibration mode may have a shortcoming. This method includes unconstrained eigenvector degrees of freedom as unknowns. In the work developed here, only constrained mode shape changes are used in the solution procedure, resulting in a reduction of the unnecessary calculations. Among the set of equations which characterizes the redesign of the structural systems, the under constrained problem is discussed here and formulated as an optimization problem, with an optimal criterion such as minimum change or minimum structural weight of the system. Four simple numerical applications illustrate the efficiency of the method. The method can be applied to the vibration problems of ships and/or offshore structures with an implementation of the commercial FE codes.

요 약

선체구조물이나 해양구조물의 동적응답중 원치 않는 고유진동수와 고유진동형태를 가지게 되는 경우가 있으며, 이러한 구조물은 동적 구조 재설계가 필수적이다. 본 소고에서는 비감쇄 구조물의 고유진동수와 진동형태를 기진력에 의한 특정한 진동수와 공진하지 않도록 또는 구조물의 중요한 부분이 특정 진동형태의 최대치에 오지않도록 구조물의 질량과 강성을 최적하게 변화시키는 방법에 대해 논의되고 있다. 이 방법은 기존의 방법에서 사용되는 모든 고유진동형태의 수식포함과 달리 구속된 고유진동형태만을 미지수로 수식중에 사용하여 불필요한 계산과정을 줄이고 있다. 동적 구조 재설계중 최적화문제에 중점을 두었으며 목적함수로는 구조물의 최소의 변화 또는 최소의 중량을 취하였고, 예제를 통하여 본 방법의 응용과 효율성을 입증하였다. 예제에서는 간단한 구조물을 다루었으나 본 방법은 상용 유한요소코드의 연계 이용으로 각종 선체구조물과 해양구조물의 진동문제해결에 응용될 수 있음은 자명한 일이다.

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1. Introduction

For the poor dynamic responsed structures, one can minimize the excitation forces or perform structural redesign. Since change or reduction of the exciting loads may be impossible in general, it is necessary to redesign the structure. Trial and error methods are often employed, however trial and error methods are expensive and sometimes inconclusive.

In the past the dynamic redesign problem has been studied and several typical procedures were proposed [1~4]. In more recent work, Sandstrom developed first order equations, that means the higher order terms were neglected in the solution procedures[1]. Kim and Anderson formulated the problem using the complete nonlinear dynamic equilibrium perturbation equation[2, 3]. A penalty function method, where the objective function was a minimum weight or minimum mass condition and the penalty term was a set of residual force errors, was employed. Their method is theoretically exact but may have a short coming. Hoff formulated the problem using an incremental formulation with a predictor-corrector solution[4]. This is a kind of iteration method. In the followings, fundamental theoretical background of redesign procedures are introduced and a new procedure for the problem is proposed.

2. The Perturbation Methods

The eigenvalue problem in the dynamic analysis of structures can be expressed as:

Baseline system $[k][\phi]=[m][\phi][\lambda]$ (1)

where $[k]$ and $[m]$ are the stiffness and mass matrices and $[\phi], [\lambda]$ are the eigenvectors and eigenvalues.

If the masses and stiffnesses are changed, the eigenvalues and eigenvectors also change. We call this system as objective system in comparison with the baseline system expressed in equation (1). The equilibrium equation for such a perturbed eigen system is:

Objective system $[k'][\phi']=[m'][\phi'][\lambda']$ (2)

The energy equations can be obtained by premultiplying equation (1) and (2) by $[\phi]^T, [\phi']^T$

$$[\phi]^T[k][\phi]=[\phi]^T[m][\phi][\lambda] \quad (3)$$

$$[\phi']^T[k'][\phi']=[\phi']^T[m'][\phi'][\lambda'] \quad (4)$$

Relationships between the baseline system and the objective system can be defined in terms of pertubation of the baseline system.

$$[k']=[k]+[\Delta k]$$
$$[m']=[m]+[\Delta m]$$
$$[\phi']=[\phi]+[\Delta\phi]$$
$$[\lambda']=[\lambda]+[\Delta\lambda] \quad (5)$$

Equations (2) can be expanded to show the nonlinearity in the perturbed terms.

$$([k]+[\Delta k])([\phi]+[\Delta\phi])$$
$$=([m]+[\Delta m])([\phi])([\lambda]+[\Delta\lambda]) \quad (6)$$

Terms up to the third order are shown in equation (6). In linear perturbation methods, terms involving Δ^2 and higher terms were neglected[1]. In subsequent work here, all terms in equation (6) are included.

Two approaches can be used. First, one can perturb the stiffness and mass of the system and solve for frequencies and mode shape changes which results.

This is called a forward perturbation. Second, one can specify desired changes in frequencies and mode shapes and determine the changes to the stiffness and mass required to cause the change. This is an inverse perturbation.

A practical interpretation can be given to the structural changes: $[\Delta k]$, $[\Delta m]$. By decomposing the system changes into L element changes, the structural changes are expressed as

$$[\Delta k]_{system}=\sum_{e=1}^{L}[\Delta k_e] \quad (7)$$

$$[\Delta m]_{system}=\sum_{e=1}^{L}[\Delta m_e] \quad (8)$$

Furthermore, each element change can be expressed as a fractional change from the baseline structural element.

$$[\Delta k_e]=[k_e]\alpha_e^k \quad (9)$$

$$[\Delta m_e]=[m_e]\alpha_e^m \quad (10)$$

where α_e^k and α_e^m represent the fractional change in the stiffness and the mass of the element e respectively. Thus we can get the expressions for the changes as

$$[\Delta k]_{\text{system}}=\sum_{e=1}^{L}[k_e]\alpha_e^k \tag{11}$$

$$[\Delta m]_{\text{system}}=\sum_{e=1}^{L}[m_e]\alpha_e^m \tag{12}$$

The perturbed eigenvalue is defined,

$$\lambda_i'=\lambda_i+\Delta\lambda_i=\omega_i^2+\Delta(\omega_i)^2 \tag{13}$$

The perturbed eigenvector in equation (5) can be expressed in terms of a single eigenvector change

$$\{\Delta\phi\}=\begin{Bmatrix}\Delta\phi^c\\ \Delta\phi^u\end{Bmatrix} \tag{14}$$

where $\{\Delta\phi^c\}$ and $\{\Delta\phi^u\}$ are the constrained (specified) and the unconstrained degrees of freedom respectively. Usually, the number of specified terms $\{\Delta\phi^c\}$ are fewer than $\{\Delta\phi^u\}$.

In some perturbation schemes, the perturbed mode can be represented as a linear combination of mode shapes obtained in the analysis of the baseline system [1]:

$$\{\Delta\phi\}=[\phi]\{C\} \tag{15}$$

This formal representation of the perturbed eigenvector, using a truncated set of eigenvectors as a basis, is purely a static relation. The perturbed eigenvectors may lack orthogonality.

The object of the redesign process is to get the solution of the equations (2) in terms of design variables α_e. To check whether these obtained design variables α can give the desired constrained modal characteristics of the system we may do reanalysis the system.

3. Nonlinear Perturbation with Penalty Function Formulation [2, 3]

This method has already shown good behavior for small systems of equations.

For the purpose of the comparison, this method is discussed here briefly.

The basic equations are the perturbed equation of motion (6) where all terms are included in the analysis. The unknowns are the mass and stiffness perturbations $[\Delta m]$, $[\Delta k]$ needed to create desired mode and frequency changes $[\Delta\phi]$, $[\Delta\lambda]$.

For the ith mode $\{\phi\}_i$, any approximate solution to the perturbed equilibrium equation has residual force error

$$\{R\}=[k']\{\phi'\}_i-[m']\{\phi'\}_i\lambda_i'\cong 0 \tag{16}$$

The penalty function is taken as

$$F(\{\alpha\},\ \{\Delta\phi^u\}_i,\ \Delta\lambda_i)=f(\{\alpha\})+\mu P(\{\alpha\},\{\Delta\phi^u\}_i,\Delta\lambda_i) \tag{17}$$

The penalty term $P(\{\alpha\},\ \{\Delta\phi^u\}_i,\ \Delta\lambda_i)$ has been found to be best chosen as a weighted norm of force unbalance at the nodal degrees of freedom:

$$P=\{R\}^T[\Gamma]_i\{R\} \tag{18}$$

where $\{R\}$ is the residual force error and the weighting matrix $[\Gamma]_i$ is diagonal and its jth component is:

$$(\Gamma_{jj})_i=\phi_{ji}^2 \tag{19}$$

that is j th component of the i th eigenvector.

The penalty function acts to minimize the error in energy in the particular mode of vibration. The method can be generalized to include more than one mode by including the corresponding error in the penalty term.

As can be seen in the equation (17), one short coming of the penalty function method is that this method need to include unconstrained eigenvector degrees of freedom as unknowns. As pointed out previously, the number of specified terms are extremely fewer than the number of unspecified terms. In the algorithm developed in this paper, the unspecified terms are excluded in the solution procedure, resulting in a reduction of considerable amount of associated calculations.

4. Solution Procedure for Redesign

Solution of equation (2) will provide the required structural changes to meet the modal objectives. The solution procedure to find the structural changes is based on splitting equation (2) for single mode into left and right side by dividing the new eigenvector into the baseline mode and mode shape change:

$$\{\phi'\}_i=\{\phi\}_i+\{\Delta\phi\}_i$$

In this work, we emphasize that in practice, only a few mode components are constrained while most of the components are unconstrained. For convenience, consider a single mode:

Let λ_i' be an eigenvalue of our objective system. Equation (2) for single mode is,

$$([k']-\lambda_i'[m'])\{\phi'\}_i=\{0\} \tag{20}$$

Here $\{\phi'\}_i$ can be decomposed as the sum of the baseline mode shape and the mode shape change relative to the baseline structure. We can normalize such that the maximum component of the objective mode is unity.

$$\{\phi'\}_i=\{\phi\}_i+\{\Delta\phi\}_i \tag{21}$$

Which can be expressed as,

$$\begin{Bmatrix} 1 \\ \phi_2' \\ \phi_3' \\ \vdots \\ \phi_n' \end{Bmatrix}=\begin{Bmatrix} 1 \\ \phi_2 \\ \phi_3 \\ \vdots \\ \phi_n \end{Bmatrix}+\begin{Bmatrix} 0 \\ \Delta\phi_2 \\ \Delta\phi_3 \\ \vdots \\ \Delta\phi_n \end{Bmatrix} \tag{22}$$

We can express equation (20) using equation (22)

$$([k']-\lambda_i'[m'])\{\Delta\phi\}_i=-([k']-\lambda_i'[m'])\{\phi\}_i \tag{23}$$

Let $([k']-\lambda_i[m'])=[A]$

As we can see, matrix $[A]$ is singular thus we can not calculate its inverse.

The rank of matrix $[A]$ is n-1 provided each eigenvalue is distinct.

Elimination of the first column of $[A]$ in the left hand side of eq. (23) which is multiplied by zero by no means changes the characteristics of the system. And since the rank of matrix $[A]$ is n-1 we only need n-1 equations for solving for $\{\Delta\phi\}_i$ in equation (23). Let's choose n-1 equations by discarding first equation. Then we get an $(n\text{-}1)\times(n\text{-}1)$ square matrix in the left hand side, which is not singular. Let this non-singular matrix be $[RA]$. Expressing the equation (23) with $[RA]$ after discarding the first column and row of matrix $[A]$:

$$\underset{(n-1)\times(n-1)}{[RA]}\ \underset{(n-1)\times 1}{\{\Delta\phi\}_i}=-[\underset{(n-1)\times 1}{RC}\ |\ \underset{(n-1)\times(n-1)}{RA}]\ \underset{n\times 1}{\{\phi\}_i} \tag{24}$$

Premultiplication of equation (24) by $[RA]^{-1}$ gives

$$\{\Delta\phi\}_i=-\{\phi\}_i-[RA]^{-1}\{RC\} \tag{25}$$

The mode shape change is composed of two parts:

$$\{\Delta\phi\}=\begin{Bmatrix} \Delta\phi^c \\ \Delta\phi^u \end{Bmatrix} \tag{26}$$

where $\{\Delta\phi^c\}$, $\{\Delta\phi^u\}$ are the constrained and unconstrained mode shape change, respectively.

Equations (25) and (26) give

$$\begin{Bmatrix} \Delta\phi^c \\ \Delta\phi^u \end{Bmatrix}=-\begin{Bmatrix} \phi^c \\ \phi^u \end{Bmatrix}-\begin{bmatrix} RA^c \\ RA^u \end{bmatrix}^{-1}\{RC\} \tag{27}$$

As we can see from equation (27), we only need RA^c part of $[RA]^{-1}$ corresponding to constrained mode shape change $\Delta\phi^c$. Noting that we do not need to calculate whole inverse of $[RA]$. We only need to calculate a small part of inverse of $[RA]$. This can be done using the definition of inverse of a matrix. That is.

$$[RA]^{-1}=\text{adj}[RA]/\det\,[RA] \tag{28}$$

Let $[[RA]]$ be the upper rows of $[RA]^{-1}$ corresponding to the constrained mode shape change $\Delta\phi^c$. Then $[[RA]]$ can be obtained by calculating only a few number of adj $[RA]$. Determinant calculation needs far less efforts than the inverse calculation needs.

Finally we get one equation which does not include unconstrained mode shape changes:

$$\{\Delta\phi^c\}=-\{\phi^c\}-[[RA]]\ \{RC\} \tag{29}$$

The system has a set of these equations according to how many mode shapes are constrained.

In the case of underconstrained problems, mathematical programming techniques can be used to achieve minimum weight or minimum change of the structure[5, 6]. Solving the equation (29) and the optimality criterion gives the desired design values α_e for the objective structural system.

The solution of these perturbation equations falls into three categories. They are overconstrained case, unique case, underconstrained case. The system is usually underconstrained (under-determined), i.e., there is more than one physical redesign, one requires either minimum weight or the least structural change from the original design. For minimum weight, the objective function is

$$f(\alpha_i)=\sum_{i=1}^{L}\alpha_i \tag{30}$$

The α_i can be α_e^k or α_e^m. For some cases α_e^k coincides with α_e^m. Rather than using this criterion, one can use minimum structural change from the baseline design. The objective function is then taken as,

$$f(\alpha_i)=\sum_{i=1}^{L}\alpha_i^2 \tag{31}$$

For the practical implementation of the redesign technique, various commercial FE code should be incorporated with the algorithm. However in this paper the author would like to show the procedure and four simple examples are illustrated using the perturbation method described in this paper.

5. Numerical Examples of Redesign

Example 1

For the simplicity, the mass-spring system with three degrees of freedom is used to illustrate the method. In this example, both frequency and mode shape are constrained for the first mode. The goal of redesign is to change mode 1 such that axial displacement at node 1 is 0.372 instead of the baseline value of 0.445. The first frequency squared will be 0.260 instead of the baseline value of 0.198. The design variables are the changes of stiffness and mass at the same rate, that is $\alpha_e^k=\alpha_e^m$. In the example problems, the Generalized Reduced Gradient Methods is used for the constrained minimization[7]. The problem is formulated as an optimization problem with an optimal criterion of a minimum weight. And the baseline structure ($\alpha_i=0$, $i=1$ to L) is used for the initial starting point.

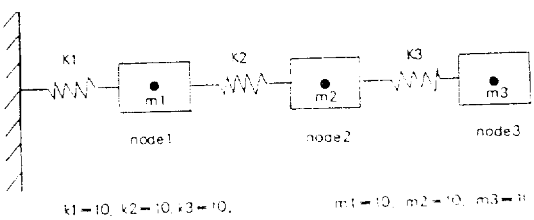

Fig. 1 Example problem for redesign

The obtained stiffness and mass with design variables of $\alpha_1=1.0$, $\alpha_2=0.5$, $\alpha_3=0.0$, gives exact values of desired frequency and mode shape.

It should be noted that this example is contrived, in the sense that the goals were set to coincide with

Table 1 First frequency and mode shape, case 1

	Baseline	Desired	Predicted
Eigenvalue	0.198	0.260	0.260
Eignervector	0.445	0.372	0.372
	0.802	0.739	0.739
	1.000	1.000	1.000

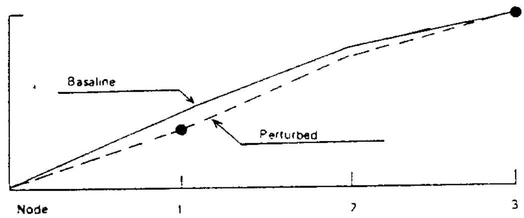

Fig. 2 First mode of perturbed mass-spring system, case 1

integer constants.

Example 2

In this example the mode shape change is tripled from that of first example. The goal of redesign is to change mode 1 such that axial displacement at node 1 is to be 0.227 and first frequency squared is to be 0.284. The percentage change for the mode shape and the frequency of first mode will be −48.9 %, 19.8%m, respectively. The obtained stiffness and mass with design variables of $\alpha_1=2.0$, $\alpha_2=\alpha_3=0$, gives exact values of desired shown in Table 2.

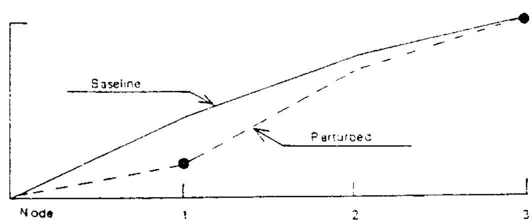

Fig. 3 First mode of perturbed mass-spring system, case 2

Table 2 First frequency and mode shape, case 2

	Baseline	Desired	Predicted
Eigenvalue	0.198	0.284	0.284
Eignervector	0.445	0.227	0.227
	0.802	0.715	0.715
	1.000	1.000	1.000

Example 3

The cantilever beam model shown in Fig. 4 can be used to demonstrate the potential of the method developed here. The stiffness matrix and consistent mass matrix for a beam element are:

$$[k]=EI/L^3\begin{bmatrix}12 & 6L & -12 & 6L\\ 6L & 4L^2 & -6L & 2L^2\\ -12 & -6L & 12 & -6L\\ 6L & 2L^2 & -6L & 4L^2\end{bmatrix}$$

$$[m]=\rho AL/420\begin{bmatrix}156 & 22L & 54 & -13L\\ 22L & 4L^2 & 13L & -3L^2\\ 54 & 13L & 156 & -22L\\ -13L & -3L^2 & -22L & 4L^2\end{bmatrix}$$

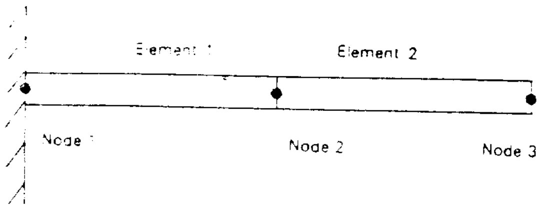

Fig. 4 Cantilever beam model

Data assumed for the beseline model are given in Table 3. The baseline vibration characteristics are given in Table 4.

Suppose that the goal of redesign is to change mode 1 such that the translation of node 2 of mode 1 is 0.349 instead of the baseline value of 0.339. The problem is formulated as an optimization problem with an optimal criterion of a minimum change from the baseline system. That is:

$$\min \sum_{e=1}^{L}\alpha_e^2$$

For a starting point the baseline system is used. Here α_e refers α_e^k and masses are assumed not to change.

The important characteristics of the solution procedure are:

1. Both element will be allowed to change
2. Changes in the bending stiffness of each beam (I_1, I_2) will be used to accomplish this change
3. Frequencies are not constrained for this example

Table 3 Baseline beam element properties

	Element 1	Element 2
I —Moment of Inertia	1.0	1.0
A —Area	1.0	1.0
L —Length	0.5	0.5
E —Young's Modulus	1.0	1.0
ρ —Density	1.0	1.0

Table 4 Baseline beam modal characteristics (mode 1)

Frequency (rad/sec)	2.87
Node 1 Translation	0.0
Node 1 Rotation	0.0
Node 2 Translation	0.339
Node 2 Rotation	1.163
Node 3 Translation	1.000
Node 3 Rotation	1.376

Result by this new method gives the following baseline system modifications.

I_1—decreased by 1.80%

I_2—increased by 53.87%

The modified system yields the node 2 translation of the first mode of 0.349.

The value represents the exact node 2 translation desired.

Table 5 Desired and predicted modal characteristics, case 3

	Baseline	Desired (goal)	Predicted
Translation of node 2 of mode 1	0.339	0.349	0.349

Example 4

In this example, the mode shape change is doubled from that of previous example.

The goal of redesign is to change mode 1 such that the translation of node 2 of mode 1 is 0.359 instead of the baseline value of 0.339. The problem is also formulated as an optimization problem with an optimal criterion of a minimum change from the baseline sytem.

Result using this method gives following baseline system modifications.

I_1—decreased by 3.49%

I_2—increased by 228.64%

Table 6 Desired and predicted modal characteristics, case 4

	Baseline	Desired (goal)	Predicted
Translation of node 2 of mode 1	0.339	0.359	0.359

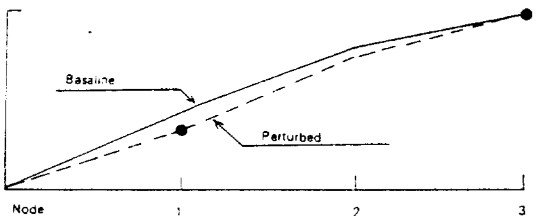

Fig. 5 First mode of cantilever beam model 2

The modified system yields the node 2 translation of the first mode of 0.359.

This represents the exact node 2 translation desired.

These final results indicate the method has worked very well for these cases.

6. Conclusions

The main contribution of this paper is the development of an efficient procedure for dynamic redesign of the structural system. The elimination of the unspecified modal degrees of freedom in the solution procedure in the present method can be proved to be effective by comparison with the competing method introduced in the contents. Eventhough numerical examples are very simple, the method is proved to work quite well.

For the redesign problems of more complicated larger degrees of freedom system, various commercial FE programs is to be incorporated and this works are recommended for future research work. Evidently the method can be applied to the vibration problems of ships and/or offshore structures with an implementation of the commercial FE codes. Work on the inverse perturbation method is continuing and we feel that the method is of significant value to engineers.

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