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A Graph-Theoretic Clustering Method Using Relaxation Labeling

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Abstract

A new graph-theoretic clustering method based on relaxation labeling is proposed. More accurate classification is achieved by using relaxation labeling which take advantage of contextual information. Moreover, threshold selection is not needed in this method.

1. Introduction

Clustering is a classification method which classifies pattern vectors in feature space without the aid of a training set of classified samples. Among the various kinds of clustering techniques the most widely used are the variants of k-means algorithm, such as ISODATA (Tou and Gonzalez, 1974). In conventional cases it is assumed that the grouping of pattern vectors from each class are globular or hyperellisoidal in shape. In those cases the algorithms are effective, but the results of the algorithm are unsatisfactory in other cases.

Graph-theoretic clustering techniques are developed to cope with the unconventional data structures(Zahn, 1971 ; Narendra, 1977). The basic idea is to generate a minimum spanning tree for the complete graph whose nodes are pattern vectors and whose weights are Euclidean distances in the feature spaces. By cutting all edges whose weights are greater than user specified threshold the tree is partitioned to subtrees, each of which represents a cluster. In some method the threshold is chosen as the sample mean of the

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edge weights plus a user specified number, γM, of sample standard deviation(Zahn, 1971). But it is difficult to choose a relevant threshold, and the performance is sensitive to the threshold.

In this paper a new graph-theoretic clustering method is proposed in which relaxation labeling is applied to the edges of the minimum spanning tree. This method requires no threshold selection, and the classification accuracy is improved by the relaxation labeling.

1.1 Relaxation labeling

The relaxation labeling enhances labeling accuracy by making use of the information embedded in context which is iterative and parallel. At each iteration certainty of a label at an object is increased or decreased according to the constraint relation. Let $A = \{a_1 \cdots a_n\}$ a set of objects to be labeled, and $\Lambda = \{\lambda_1 \cdots \lambda_m\}$ a possible labels. The constraint relation is embodied in $r_{ij}(\alpha, \beta)$ which indicates the compatibility of label α at a_i and label β at a_j.

Probabilistic labeling is an assignment of an n-dimensional vector P_i to each $a_i \epsilon A$, where $P_i(\lambda)$ is the estimated probability of label λ at a_i. Another probability vector associated with a_i is Q_i, degree of certainties of labels at a_i supported by its neighbors. It is approximated as follows.

$$Q_i^{(k)}(\alpha) = \sum_{j \epsilon N_i} c_{ij} \sum_{\beta \epsilon \Lambda} r_{ij}(\alpha, \beta) P_j^{(k)}(\beta)$$

Where c_{ij} is neighbor weight and N_i is a set of indexes for objects adjacent to a_i. The superscript (k) indicates an estimated value at k-th iteration.

The relaxation labeling may be expressed as follows.

```
repeat
  for each object a_i ε A
  begin
    calculate Q_i
    assign P_i to a_i, where
```

$$P_i(\alpha) = \frac{P_i(\alpha) * P_i(\alpha)}{\sum_{\lambda \epsilon \Lambda} P_i(\lambda) * P_i(\lambda)}$$

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  end
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until(total difference of labeling between iterations $<$ e)
where e is a prespecified limit value.

2. Clustering by relaxation of edge labels

The clustering procedure proposed in this paper is as follows :

1. Construct the minimum spanning tree.
2. Assign initial label to each edge.
3. Compute the compatibility.
4. Applying relaxation labeling.

A tree is a connected graph with no circuits, and a spanning tree of connected graph G is a tree in G which contains all nodes of G. Minimum spanning tree of graph G is a spanning tree whose weight is minimum among all spanning tree of G, and can be generated by Kruscal's algorithm(Horowitz, 1987). The basic idea is to include edges of G in ascending order of the weights if the edge under consideration do not form a circuit with those already chosen. This algorithm stops when (n−1) edges have been included where n is the number of nodes in G.

When the relaxation labeling is applied to partitioning the minimum spanning tree the objects to be labeled are the edges, and possible labels are STRONG(S) and WEAK(W). Groups of strong edges form clusters and weak edges form the boundaries between clusters.

Initial lavel is computed by

$$P_i^{(0)} = 0.5 + \frac{1}{8 * \sigma} (\mu - l_i)$$

Where l_i is length of i−th edge and μ and σ are mean and standard deviation of lengths of edges. Interval between mean value of 0.5 plus and minus $4 * \sigma$ is [0, 1]. If the computed value is less than 0 or greater than 1 they are truncated to 0 and 1 respectively.

$P(\alpha \mid \beta)$ is used as compatibility $r_{ij}(\alpha, \beta)$, and computed as

$$P(\alpha \mid \beta) = \frac{\sum_{i \epsilon E} \sum_{J \epsilon Ni} P_i^{(0)}(\alpha) * P_j^{(0)}(\beta)}{\sum_{i \epsilon E} \sum_{j \epsilon Ni} P_j^{(0)}(\beta)}$$

where E is the set of edges in the minimum spanning tree.

Degree of certainty of label at i-th edge supported by its neighboring edges is computed by

$$Q_i^{(k)}(S) = 1/n_i * \sum_{j \epsilon Ni}(P_j(S) * P(S \mid S) + P_j(W) * P(S \mid W))$$

$$= C1 + C2/n_i * \sum_{j \epsilon Nj} P_j(S)$$

Where $C1 = 1 - P(W \mid W)$ and $C2 = P(S \mid S) + P(W \mid W) - 1$ are constants, and n_i is the number of neighboring edges of i-th edge.

Finally the updating rule is computed as follows

$$P_i^{(k+1)}(S) = \frac{P_i^{(k)}(S) * Q_i^{(k)}(S)}{2 * P_i^{(k)}(S) * Q_i^{(k)}(S) + 1 - (P_i^{(k)}(S) + Q_i^{(k)}(S))}$$

3. An example

In order to demonstrate the performance of the proposed method an experiment is performed on simulated patterns. 100 patterns from three classes are generated according to the following rules(Fukunaga, 1972).

For class 1 and class 2 :

$$\mathbf{x1} = 20 * \cos \boldsymbol{\theta} + m_1 + \mathbf{n}_1$$

$$\mathbf{x2} = 20 * \sin \boldsymbol{\theta} + m_2 + \mathbf{n}_2$$

Where $\mathbf{n}_1$ and $\mathbf{n}_2$ are independent, identically distributed normal random variables with mean 0 and standard deviation 1.5. The term θ is normal with mean $\boldsymbol{\theta}_m$ and standard deviation of $\pi/4$.

$m_1 = 0$, $m_2 = 0$, and $\theta_m = \pi$ for class 1

$m_1 = 20$, $m_2 = 0$, and $\theta_m = 0$ for class 2

Class 3 is bivariate normal with mean vector M and covariance matrix Σ given by

$$M = [10.0 \quad 0.0] \quad \text{and} \quad \Sigma = \begin{bmatrix} 36.0 & 0.0 \\ 0.0 & 36.0 \end{bmatrix}$$

The progress and result are shown in figure 1 – figure 4.

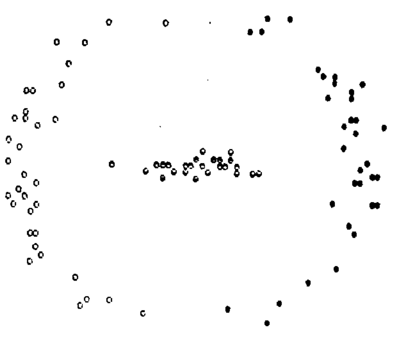

Fig. 1. Test Patterns.

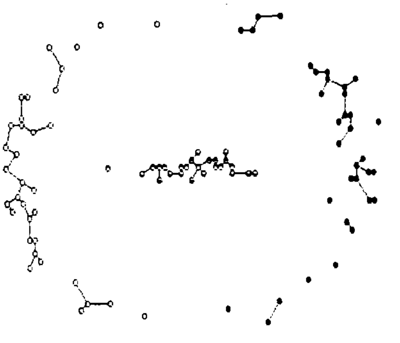

Fig. 2. Initial Labeling.

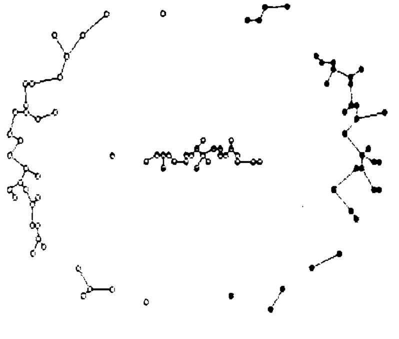

Fig. 3. Labeling at the 3rd Iteration.

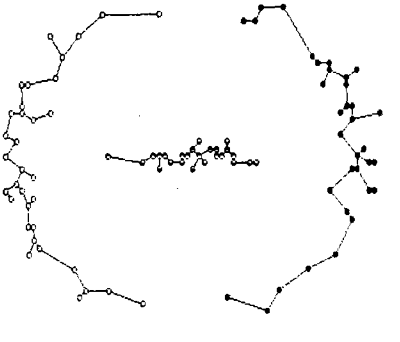

Fig. 4. Final Labeling at 15 th Iteration.

4. Conclusion

A new graph – theoretic clustering method based on relaxation labeling is proposed. More accurate classification is achieved by using relaxation labeling which take advantage of contextual information. Moreover, threshold selection is not needed in this method.

References

R. O. Duda and P. E. Hart, 1973, *Pattern classification and scene analysis,* New York : Wiely.

K. Fukunaga, 1972, *Introduction to statistical pattern recognition,* New York : Academic Press.

E. Horowitz and S. Sahni, 1987, *Fundamentals of data structures in PASCAL*, Computer Science Press.

P. M. Narendra and M. Goldberg, 1977, "A non parametric clustering scheme for Landsat," *Pattern Recognition*, Vol. 9, pp. 207 – 215.

A. Rosenfeld, R. Hummel, and Zucker, 1976, "Scene labeling by relaxation algorithms," *IEEE Tr. SMC,* Vol. 6, pp. 420 – 433.

J. T. Tou and R. C. Gonzales, 1974, *Pattern recognition principle.* Mass. : Addison – Wesley Publishing Company.

A. Touzani and J. G. Postaire, 1988, "Mode detection by relaxation," *IEEE Tr. PAMI,* Vol. 10, No. 6, pp. 970 – 978.

C. T. Zahn, 1971, "Graph theoretical methods for detecting and describing gestalt clusters," *IEEE Tr. Computer,* Vol. C – 20, No. 1.

S. Zucker and J. Mohamed, 1978 "Analysis of probabilistic relaxation labeling processes," in *Proc. IEEE Conf. PRIP, Chicago,* pp. 307 – 312.