Bull. Korean Math. Soc. **26**(1989), No. 2, pp. 151-158

EVALUATION OF SOME CONDITIONAL ABSTRACT WIENER INTEGRALS

DONG MYUNG CHUNG* AND SOON JA KANG**

1. Introduction and Preliminaries

Let (H, B, ν) be an abstract Wiener space where H is a separable Hilbert space with the inner product $\langle\cdot,\cdot\rangle$ and the norm $|\cdot|=\sqrt{\langle\cdot,\cdot\rangle}$, which is densely and continuously imbedded into a separable Banach space B with the norm $\|\cdot\|$, and ν is a probability measure on the Borel σ-algebra $\mathscr{B}(B)$ of B which satisfies

$$\int_B e^{i(y,x)}d\nu(x)=\exp\{-\frac{1}{2}|y|^2\} \text{ for every } y\in B^*,$$

where B^* is the topological dual of B and $(\cdot,\cdot)$ is the natural dual pairing between B and B^*. We will regard $B^*\subset H\subset B$ in the natural way. Thus we have $\langle y, x\rangle=(y,x)$ for all y in B^* and x in H. Let $\boldsymbol{R}^n$ and $\boldsymbol{C}$ denote the n-dimensional Euclidean space and the complex numbers respectively.

Let $(C[0,T], \mathscr{B}(C[0,T]), m_w)$ denote Wiener space, i.e. $C[0,T]$ denotes the Banach space $\{x(\cdot) : x \text{ is a real valued continuous function with } x(0)=0\}$ with the supremum norm and m_w denotes the Wiener measure on the Borel σ-algebra $\mathscr{B}(C[0,T])$ of $C[0,T]$ (see [10]). Let $C'[0,T]=\{x\in C[0,T] : x(s)=\int_0^s f(u)du,\ f\in L^2[0,T]\}$. Then it is a real separable infinite dimensional Hilbert space with inner product $\langle x_1, x_2\rangle=\int_0^T Dx_1(\tau)\cdot Dx_2(\tau)d\tau$ where $Dx=\frac{dx}{d\tau}$. As is known, $(C'[0,T], C[0,T], m_w)$ is one of the most important examples of abstract Wiener space [see [6]).

Let $\{e_j;\ j\geq 1\}$ be a complete orthonormal set in H such that e_j's are in B^*. For each $h\in H$ and $x\in B$, let

---

Received October 28, 1988.
* Research partially supported by KOSEF Grant.
** Research partially supported by the BSRIP, MOE, 1988.

$$(h, x)^{\sim}=\begin{cases}\lim_{n\to\infty}\sum_{j=1}^{n}\langle h, e_j\rangle\ (e_j, x), & \text{if the limit exists}\\ 0 & \text{, otherwise.}\end{cases}$$

Then for each $h(\neq 0)$ in H, $(h, \cdot)^{\sim}$ is a Gaussian random variable on B with mean zero, variance $|h|^2$ and $(h, x)^{\sim}$ is esentially independent of the choice of the complete orthonormal set used in its definition, and $(h, \lambda x)^{\sim}=\lambda(h, x)^{\sim}$ for all $\lambda>0$. It is known [2, 4, 9] that if $\{h_1, h_2, \dots, h_n\}$ is an orthogonal set in H, then the random variables $(h_i, x)^{\sim}$'s are independent, and that if $B=C[0, T]$, $H=C'[0, T]$, then

$$(h, x)^{\sim}=\int_0^T Dh(s)\ \tilde{d}x(s)$$

where $\int_0^T Dh(s)\ \tilde{d}x(s)$ is the Paley-Wiener-Zygmund integral of Dh.

Let A be a self-adjoint, trace class operator with eigenvalues $\{\alpha_k\}$ and corresponding eigenfunctions $\{e_k\}$. Let

$$(x, Ax)^{\sim}=\begin{cases}\lim_{n\to\infty}\sum_{j=1}^{n}\alpha_j[(e_j, x)^{\sim}]^2, & \text{if the limit exists}\\ 0 & \text{, otherwise.}\end{cases}$$

For more details, see [4, 5, 8].

Let X be a $\boldsymbol{R}^n$-valued measurable function and Y a $\boldsymbol{C}$-valued integrable function on $(B, \mathcal{B}(B), \nu)$. Let $\mathcal{F}(X)$ denote the σ-algebra generated by X. Then by the definition of conditional expectation, the conditional expectation of Y given $\mathcal{F}(X)$, written $E[Y|X]$, is any real valued $\mathcal{F}(X)$-measurable function on B such that

$$\int_E Y d\nu=\int_E E[Y|X]\ d\nu \text{ for } E\in\mathcal{F}(X).$$

It is well known that there exists a Borel measurable and P_X-integrable function ϕ on $(\boldsymbol{R}^n, \mathcal{B}(\boldsymbol{R}^n), P_X)$ such that $E[Y|X]=\phi\circ X$, where $\mathcal{B}(\boldsymbol{R}^n)$ denotes the Borel σ-algebra of $\boldsymbol{R}^n$ and P_X is the probability distribution of X defined by $P_X(A)=\nu(X^{-1}(A))$ for $A\in\mathcal{B}(\boldsymbol{R}^n)$. The function $\phi(\xi), \xi\in\boldsymbol{R}^n$ is unique up to Borel null sets in $\boldsymbol{R}$. Following Yeh [10] the function $\phi(\xi)$, written $E[Y|X=\xi]$, is called the *conditional abstract Wiener integral of* Y given X.

This paper continues the study of conditional abstract Wiener integrals previously given in [3]. Motivated by the work of Park and Skoug [7], we consider vector-valued conditioning functions and establish formulas for evaluating conditional abstract Wiener integrals for various functions on B. We then specialize our rsults to classical abstract Wieneer space $C[0, T]$ to evaluate various conditional Wiener integrals studied in [1, 7].

2. Evaluation of some conditional abstract Wiener integrals

In this section, we establish formulas for evaluating conditional abstract Wiener integral of various functions on B when the conditioning function is vector-valued.

THEOREM 2.1. *Let $\{g_1, g_2, \cdots, g_n\}$ be an orthonormal set in H. Let X and Z be measurable functions on $(B, \mathcal{B}(B))$ defined, respectively, by*

$$X(x)=((g_1, x)^{\sim}, (g_2, x)^{\sim}, \cdots, (g_n, x)^{\sim}) \tag{2.1}$$

and

$$Z(x)=(S^*h, x)^{\sim} \tag{2.2}$$

where $h\in H$ and S is a bounded linear operator on H. Then

$$E[Z|X=\xi]=\sum_{j=1}^{n} \langle h, Sg_j\rangle\xi_j, \quad \xi=(\xi_1, \xi_2, \cdots, \xi_n)\in \boldsymbol{R}^n. \tag{2.3}$$

Proof. We first note that $E[Z|X]$ exists since $E[|Z|]<\infty$. Let $S^*h=k$. Then k can be written as

$$k=\sum_{j=1}^{n} \langle k, g_j\rangle g_j+p, \quad p\in [g_1, g_2, \cdots, g_n]^{\perp}$$

where $[g_1, g_2, \cdots, g_n]^{\perp}$ stands for the orthogonal complement of the subspace of H spanned by $\{g_1, g_2, \cdots, g_n\}$.
Thus we have

$$E[Z|X]=\sum_{j=1}^{n} \langle k, g_j\rangle (g_j, x)^{\sim}+E[(p, x)^{\sim}|X].$$

Since $(p, x)^{\sim}$ and X are independent, $E[(p, x)^{\sim}|X]=E[(p, x)^{\sim}]=0$. Therefore we establish the equation (2.3) as desired.

THEOREM 2.2. *Let X be as in (2.1) and Z be a measurable function on $(B, \mathcal{B}(B))$ defined by*

$$Z(x)=(x, Ax)^{\sim} \tag{2.4}$$

where A is a self-adjoint, trace class operator on H. Then

$$E[Z|X=\xi]=TrA+<\sum_{j=1}^{n} \xi_j g_j, A(\sum_{j=1}^{n}\xi_j g_j)>-\sum_{j=1}^{n}\langle g_j, Ag_j\rangle \tag{2.5}$$

for $\xi=(\xi_1, \xi_2, \cdots, \xi_n)\in \boldsymbol{R}^n$, where TrA denotes the trace of A.

Proof. Let $\{e_m\}$ be the orthonormal eigenvectors and $\{\alpha_m\}$ be the corresponding eigenvalues of A. Let $\langle g_j, e_m\rangle =a_{mj}$. Since $(x, Ax)^{\sim}=\sum_{m=1}^{\infty} \alpha_m((e_m, x)^{\sim})^2$, a. e. $x\in B$, we have

$E[Z|X]=\sum_{m=1}^{\infty}\alpha_m E\left[((e_m, x)^{\sim})^2|X\right]$. To evaluate $E\left[((e_m, x)^{\sim})^2|X\right]$ we observe that each e_m can be written as

$$e_m=\sum_{j=1}^{n}\langle e_m, g_j\rangle g_j+p_m, \quad p_m\in[g_1, g_2, \cdots, g_n]^{\perp}.$$

Thus we have

$$((e_m, x)^{\sim})^2=(\sum_{j=1}^{n}a_{mj}(g_j, x)^{\sim})^2+2\sum_{j=1}^{n}a_{mj}(g_j, x)^{\sim}(p_m, x)^{\sim}+((p_m, x)^{\sim})^2.$$

Therefore, by noting that $E[(p_m, x)^{\sim}]=0$,

$$E[((e_m, x)^{\sim})^2|X]=(\sum_{j=1}^{n}a_{mj}(g_j, x)^{\sim})^2+E[((p_m, x)^{\sim})^2|X].$$

Since $(p_m, x)^{\sim}$ and $(g_j, x)^{\sim}$'s are independent, we have

$$E[((p_m, x)^{\sim})^2|X]=E[((p_m, x)^{\sim})^2].$$

But $E[((p_m, x)^{\sim})^2]=|p_m|^2=1-\sum_{j=1}^{n}a_{mj}^2$, so that

$$E[Z|X]=\sum_{j=1}^{n}\alpha_m\left[(\sum_{j=1}^{n}a_{mj}(g_j, x)^{\sim})^2+(1-\sum_{j=1}^{n}a_{mj}^2)\right]$$
$$=\mathrm{Tr}A+<\sum_{j=1}^{n}(g_j, x)^{\sim}\xi_j, A(\sum_{j=1}^{n}(g_j, x)^{\sim}\xi_j)>-\sum_{j=1}^{n}\langle g_j, Ag_j\rangle$$

Hence we establish the equation (2.5) as desired.

THEOREM 2.3. *Let X be as in (2.1) and Z be a measurable function on $(B, \mathcal{B}(B))$ defined by*

$$Z(x)=\exp\{\lambda(S^*h, x)^{\sim}\} \tag{2.6}$$

where $\lambda\in\boldsymbol{C}$, $h\in H$ and S is a bounded linear operator on H. Then

$$E[Z|X=\xi]=\exp\left\{\frac{\lambda^2}{2}\left(|S^*h|^2-\sum_{j=1}^{n}\langle h, Sg_j\rangle^2\right)+\lambda\sum_{j=1}^{n}\langle h, Sg_j\rangle\xi_j\right\} \tag{2.7}$$

for $\xi=(\xi_1, \xi_2, \ldots, \xi_n)\in\boldsymbol{R}^n$.

Proof. Since $E[|Z|]=\exp\left\{\frac{\lambda^2}{2}|S^*h|^2\right\}$ where $\lambda=\mathrm{Re}\,\lambda$, $E[Z|X]$ exists. Let $S^*h=k$. Then k can be written as

$$k=\sum_{j=1}^{n}\langle k, g_j\rangle g_j+p, \quad p\in[g_1, g_2, \cdots, g_n]^{\perp}.$$

Thus we have

$$\exp\{\lambda(k, x)^{\sim}\}=\exp\left\{\lambda\sum_{j=1}^{n}\langle k, g_j\rangle(g_j, x)^{\sim}\right\}\exp\{\lambda(p, x)^{\sim}\}.$$

Since $(p, x)^{\sim}$ and $(g_j, x)^{\sim}$'s are independent,

$$E[Z|X]=\exp\left\{\lambda\sum_{j=1}^{n}\langle k, g_j\rangle(g_j, x)^{\sim}\right\}E[\exp\{\lambda(p, x)^{\sim}\}].$$

By noting that $E[\exp\{\lambda(p, x)^{\sim}\}]=\exp\left\{\frac{\lambda^2}{2}|p|^2\right\}$, we have

$$E[Z\mid X]=\exp\left\{\lambda\sum_{j=1}^{n}\langle h, Sg_j\rangle (g_j, x)^{\sim}+\frac{\lambda^2}{2}\left(|S^*h|^2-\sum_{j=1}^{n}\langle h, Sg_j\rangle^2\right)\right\}.$$

Hence we establish the equation (2.7) as desired.

3. Examples

Let $S : C'[0, T]\to C'[0, T]$ be the linear operator defined by

(3.1) $$(Sg)(\tau)=\int_0^\tau g(u)\ du.$$

Then we see that the adjoint operator S^* of S is given by

(3.2) $$(S^*g)(\tau)=g(T)\tau-\int_0^\tau g(u)\,du$$

and that the operator $A=S^*S$ is given by

(3.3) $$(Ag)(\tau)=\int_0^T \min(\tau, s)\,g(s)\,ds.$$

Further, we see that A is a self-adjoint, compact operator on $C'[0, T]$ and that $\langle f, Ag\rangle=\langle Sf, Sg\rangle=\int_0^T f(s)g(s)\,ds$ for all $f, g\in C'[0, T]$ and so A is positive definite, i. e. $\langle f, Af\rangle\geq 0$, $f\in C'[0, T]$. It can be shown [6] that A is a trace class operator on $C'[0, T]$.

EXAMPLE 3.1. Let B be the classical Wiener space $C[0, T]$ with Wiener measure m_w and S be as in (3.1). Let us fix a partition $\{0=t_0<t_1<\cdots<t_n=T\}$ of $[0, T]$ and let $g_j\in C'[0, T]$ be defined by

$$g_j(\tau)=\frac{1}{\sqrt{t_j-t_{j-1}}}\int_0^\tau \underset{[t_{j-1},t_j]}{1}(u)\ du.$$

Then $\{g_1, g_2, \cdots, g_n\}$ is an orthonormal set in $C'[0, T]$. Let $h\in C'[0, T]$ be defined by $h(\tau)=\tau$. Then it is easily seen that

$$(S^*h, x)^{\sim}=\int_0^T x(s)\,ds.$$

By Theorem 2.1, we obtain

$$E\left[\int_0^T x(s)\,ds\,\Big|\,x(t_1)=\xi_1,\ \cdots,\ x(t_n)=\xi_n\right]$$

$$=E\left[(S^*h, x)^{\sim}\,\Big|\,(g_j, x)^{\sim}=\frac{1}{\sqrt{t_j-t_{j-1}}}(\xi_j-\xi_{j-1}),\ j=1, \cdots, n\right],\quad \xi_0=0$$

$$=\sum_{j=1}^{n}\langle h, Sg_j\rangle\,\frac{\xi_j-\xi_{j-1}}{\sqrt{t_j-t_{j-1}}}$$

$$=\int_0^T\int_0^s\sum_{j=1}^{n}\left(\underset{[t_{j-1},t_j]}{1}(u)\ \frac{\xi_j-\xi_{j-1}}{t_j-t_{j-1}}\right)\ du\ ds$$

$$=\sum_{j=1}^{n}\int_{t_{j-1}}^{t_j}\left(\xi_{j-1}+\frac{\xi_j-\xi_{j-1}}{t_j-t_{j-1}}(s-t_{j-1})\right)\,ds$$
$$=\frac{1}{2}\sum_{j=1}^{n}(\xi_j+\xi_{j-1})(t_j-t_{j-1})$$

which agrees with the results in [1, 7].

EXAMPLE 3.2. Let $B=C[0, T]$ and let $\{g_1, g_2, \cdots, g_n\}$ be as in Example 3.1. Let $S=I$ (identity operator) and let $h\in C'[0, T]$ be defined by $h(\tau)=\int_0^\tau 1_{[0,s]}(u)\,du$, where $t_{j-1}\leq s\leq t_j$. Then $(S^*h, x)^\sim=(h, x)^\sim=x(s)$. By Theorem 2.1, we obtain

$$\begin{aligned}&E[x(s)\,|\,x(t_1)=\xi_1, \cdots, x(t_n)=\xi_n]\\&=E[x(s)\,|\,x(t_{j-1})=\xi_{j-1}, x(t_j)=\xi_j]\\&=E\left[(h, x)^\sim\,\middle|\,(g_j, x)^\sim=\frac{\xi_j-\xi_{j-1}}{\sqrt{t_j-t_{j-1}}}\right]\\&=\langle h, g_j\rangle\frac{\xi_j-\xi_{j-1}}{\sqrt{t_j-t_{j-1}}}=\xi_{j-1}+\frac{\xi_j-\xi_{j-1}}{t_j-t_{j-1}}(s-t_{j-1}).\end{aligned}$$

EXAMPLE 3.3. Let $B=C[0, T]$ and let S and A be operators as in (3.2) and (3.3) respectively. Then we have $A=S^*S$ and $(x, Ax)^\sim=\int_0^T x^2(s)\,ds$ (see [3]). Let $\{g_1, g_2, \cdots, g_n\}$ be as in Example 3.1. Then by Theorem 2.2, we obtain

$$\begin{aligned}&E\left[\int_0^T x^2(s)\,ds\,\middle|\,x(t_1)=\xi_1, \cdots, x(t_n)=\xi_n\right]\\&=E\left[(x, Ax)^\sim\,\middle|\,(g_j, x)^\sim=\frac{1}{\sqrt{t_j-t_{j-1}}}(\xi_j-\xi_{j-1}), j=1, \cdots, n\right],\ \xi_0=0\\&=\mathrm{Tr}A+\left<\sum_{j=1}^{n}\frac{\xi_j-\xi_{j-1}}{\sqrt{t_j-t_{j-1}}}g_j, A\left(\sum_{j=1}^{r}\frac{\xi_j-\xi_{j-1}}{\sqrt{t_j-t_{j-1}}}g_j\right)\right>-\sum_{j=1}^{n}\langle g_j, Ag_j\rangle\\&=\frac{1}{2}T^2+\int_0^T\left(\int_0^s\sum_{j=1}^{n}\frac{\xi_j-\xi_{j-1}}{t_j-t_{j-1}}1_{[t_{j-1},t_j]}(u)\,du\right)^2ds\\&\qquad-\int_0^T\left(\int_0^s\sum_{j=1}^{n}\frac{1}{\sqrt{t_j-t_{j-1}}}1_{[t_{j-1},t_j]}(u)\,du\right)^2ds\\&=\frac{1}{2}T^2+\frac{1}{3}\sum_{j=1}^{n}(\xi_j^2+\xi_j\xi_{j-1}+\xi_{j-1}^2)(t_j-t_{j-1})\\&\qquad-\frac{1}{3}\sum_{j=1}^{n}(t_j+2t_{j-1})(t_j-t_{j-1})\end{aligned}$$

which agrees with the result in [7].

EXAMPLE 3.4. Let $B=C[0,T]$ and S be as in (3.2). Let $\{g_1, g_2, \cdots, g_n\}$ be as in Example 3.1. For $y\in L^2[0,T]$, let $h(\tau)=\int_0^\tau y(s)\,ds$, $\tau\in[0,T]$. Then $h\in C'[0,T]$ and $(S^*h, x)^\sim=\int_0^\tau y(s)x(s)\,ds$. Note that

$$|S^*h|^2=\int_0^T (h(T)-h(\tau))^2 d\tau=TY^2(T)-2Y(T)Z(T)+\|Y\|^2$$

and

$$\langle h, Sg\rangle=\int_0^T y(\tau)\tau\, d\tau=TY(T)-Z(T)$$

where $Y(\tau)=h(\tau)$, $Z(\tau)=\int_0^\tau Y(s)\,ds=Sh(\tau)$ and $\|Y\|^2=|Sh|^2$. Then by Theorem 2.3, we obtain

$$E\left[\exp\left\{\lambda\int_0^T y(s)x(s)\,ds\right\}\Big|\, x(t_1)=\xi_1, \cdots, x(t_n)=\xi_n\right],\quad \lambda\in\boldsymbol{C}$$

$$=E[\exp\{\lambda(S^*h, x)^\sim\}\mid (g_j, x)^\sim=\frac{1}{\sqrt{t_j-t_{j-1}}}(\xi_j-\xi_{j-1}),\ j=1,\cdots,n],\ \xi_0=0$$

$$=\exp\left\{\frac{\lambda^2}{2}\left(|S^*h|^2-\sum_{j=1}^n\langle h, Sg_j\rangle^2\right)+\sum_{j=1}^n\lambda\langle h, Sg_j\rangle\frac{\xi_j-\xi_{j-1}}{\sqrt{t_j-t_{j-1}}}\right\}$$

$$=\exp\left\{\frac{\lambda^2}{2}\left(|S^*h|^2-\sum_{j=1}^n\int_0^T y(s)\int_0^s\frac{1}{\sqrt{t_j-t_{j-1}}}\underset{[t_{j-1},t_j]}{1}(u)\,du\,ds\right)^2\right.$$

$$\left.+\lambda\int_0^T y(s)\int_0^s\sum_{j=1}^n\frac{\xi_j-\xi_{j-1}}{t_j-t_{j-1}}\underset{[t_{j-1},t_j]}{1}(u)\,du\,ds\right\}.$$

In particular when $\lambda=1$, $y(s)\equiv1$, we obtain

$$E\left[\exp\int_0^T x(s)\,ds\right\}\Big|\, x(t_1)=\xi_1,\cdots, x(t_n)=\xi_n\Big]$$

$$=\exp\left\{\frac{T^3}{6}\right\}-\frac{1}{8}\sum_{j=1}^n(t_j-t_{j-1})(2T-t_j-t_{j-1})^2$$

$$+\frac{1}{2}\sum_{j=1}^n(\xi_j+\xi_{j+1})(t_j-t_{j-1})$$

Further, if $n=1$, we obtain

$$E\left[\exp\left\{\int_0^T x(s)\,ds\right\}\Big|\, x(T)=\xi\right]=\exp\left\{\frac{T^3}{24}+\frac{1}{2}\xi T\right\}.$$

EXAMPLE 3.5. In Example 3.4, let $\lambda=1$, $S=I$ and $y\equiv1$. Let $h(\tau)=\int_0^\tau \underset{[0,s]}{1}(u)$ du where $t_{j-1}\le s\le t_j$. Then $(S^*h, x)^\sim=x(s)$. Hence we obtain

$$E[\exp\{x(s)\}\mid x(t_1)=\xi_1,\cdots,x(t_n)=\xi_n]$$

$$=\exp\left\{\frac{1}{2}|h|^2-\frac{1}{2}\sum_{j=1}^n\langle h, g_j\rangle^2+\sum_{j=1}^n\langle h, g_j\rangle\frac{\xi_j-\xi_{j-1}}{\sqrt{t_j-t_{j-1}}}\right\}$$

$$=\exp\left\{\frac{1}{2}s-\frac{1}{2}\sum_{j=1}^{n}\frac{1}{t_j-t_{j-1}}\left(\int_0^T \underset{[0,s]}{1}(u)\ \underset{[t_{j-1},t_j]}{1}(u)\ du\right)^2\right.$$
$$\left.+\int_0^T \underset{[0,s]}{1}(u)\sum_{j=1}^{n}\underset{[t_{j-1},t_j]}{1}(u)\frac{\xi_j-\xi_{j-1}}{t_j-t_{j-1}}\ du\right\}$$
$$=\exp\left\{\frac{1}{2}s-\frac{1}{2}\left[t_{j-1}+\frac{(s-t_{j-1})^2}{t_j-t_{j-1}}\right]+\xi_j+\frac{\xi_j-\xi_{j-1}}{t_j-t_{j-1}}(s-t_{j-1})\right\}$$

References

1. Kun S. Chang and Joo S. Chang, *Evaluation of Some Conditional Wiener Integrals*, Bull. Korean Math. Soc. **21**, 1984, No. 2, 99-106.
2. Dong M. Chung, *Scale-invariant measurability in abstract Wiener spaces*, Pacific J. Math., **130**, 1987, 27-40.
3. Dong M. Chung and Soon J. Kang, *Evaluation formulas for conditional abstract Wiener integrals*, Stochastic Analysis and Applications, **7**, 1989, 125-144.
4. L. Gross, *Abstract Wiener space*, Proc. 5th, Berkeley Sym. Math. Stat. Prob. **2**, 1965, 31-42.
5. G. Kallianpur, D. Kannan and R. L. Karnadikar, *Analytic and sequential Feynman integrals on abstract Wiener and Hilbert spaces and a Cameron-Martin formula*, Ann Institut H. Poincaré, **21**, 1985, 323-361.
6. H. H. Kuo, *Gaussian Measures in Banach Spaces*, Lecture Notes in Mathematics, No. 463, Springer, Berlin, 1975.
7. C. Park and D. Skoug, *A simple formula for conditional Wiener integrals with applications*, Pacific J. of Math., **135**, 1988, No. 2, 381-394.
8. M. A. Piech, *A fundamental solution of the parabolic equation on Hilbert space*, J. Func. Anal. **3**, 1969, 85-114.
9. I. E. Segal, *Tensor algebra over Hilbert spaces*, Trans. Amer. Math. Soc. **81**, 1956, 106-134.
10. J. Yeh, *Inversion of conditional Wiener integrals*, Pacific J. Math. **59**, 1975, 623-638.

Sogang University
Seoul 121-742, Korea
and
Cheonnam National University
Kwangju 500-757, Korea