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Evaluation by Fuzzy Checklist

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Abstract

Checklist method is rapid and comprehensive to evaluate in practice. Check items are commonly rated by subjective utility function; i.e., evaluator's significant judgment.

Since human judgment includes fuzziness (vagueness) inherently in spite of its significance, fuzzy set theory is useful in this case. The paper illustrates a evaluation method using fuzzy checklist where check items are rated as fuzzy numbers. Pairwise comparison data is used to determine the weights of check items, since it has comparative advantage for human's fuzzy judgment. Sample of BASIC program is provided for microcomputer. When uncertainty is due to subjectivity or imprecision of data, this method can be applied to practical problems widely.

1. EVALUATION BY CHECKLIST

The checklist usually contains a list of significant items to be considered and these items are checked or evaluated one by one. Checklist method is simple but it serves as the useful guideline for the following purposes; (1) evaluating or analyzing the subjects, (2) checking the action guides not to do erroneous actions, and (3) gathering data. Checklists are used in action guide in hazardous operations, chair evaluation, proposal evaluation, plant layout evaluation, personnel evaluation, commodity evaluation, organization evaluation, safety analysis, public opinion polling, market research, beauty contest and so on.

Though there may be many variations and purposes in checklist method, we deal with evaluation problem. Checklist method is rapid and useful to evaluate the effectiveness of evaluation subject (sometimes called entity) which is the thing, or system to be evaluated.

Measuring effectiveness which affects the utility for the evaluator is the basis for evaluation. It can be considered in positive way or negative way. Positive way means desirabil-

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ity, attractiveness, worth, and positive valuation; the larger evaluation value the greater preference. Negative way means undesirability, unattractiveness, lack of merit, and negative valuation, for example, risk level. Effectiveness is broken down into many check items (evaluation criteria).

However, we deal with evaluating by human judgement rather than by complicated and analytic model. In this case, check items are commonly rated by subjectivity or evaluator's significant judgement. The basic philosophy of this method is on human's ability to summarize the information, hence to judge significantly.

When each check item has relative importance differently, weight is given. We call it the weighted checklist, which will be discussed in this paper. Check items are evaluated one by one, hence rating scores are obtained. And then final score, that represents whole effectiveness or whole utility of evaluation subject, is obtained by weighted average or weighted summation of rating scores. The equation for final score is, assuming weighted average,

$$u = \sum_i w_i \cdot u_i, \qquad \Sigma\, w_i = 1 \tag{1}$$

where u_i and w_i are rating score and weight, respectively, for i-th check item.

Checklist evaluation method is, actually, a simple tool for multiple attribute decision making (MADM) or multiple attribute utility (MAU) in decision analysis. There is not enough space to discuss the philosophical background of MADM and MAU in detail, and they are theoretically well explained in [8] [10] [11].

Check items and their weights may be commonly prepared for specific areas in advance (ready-to-use) by experts related to that area, and this checklist is used conveniently as needed.

Checklist method is highly flexible, yet it is reasonable, even though its accuracy is based on a series of judgement and estimations. If the normalized weight of check items and their ratings are known exactly, evaluation is performed easily. However, there are some difficulties.

First, assigning weights is important. It is difficult to assign by absolute judgement, but there may exist fuzzy nature. Human ability to compare two things at a time on a relative basis far exeeds the absolute judgement ability. We will discuss later about determining weights from pairwise comparisons.

Second, there may exist fuzzy nature in rating step because of evaluator's subjective judgements. In most of the real situations, there are many qualitative items in checklist, and evaluator is forced to take judgement on the basis of ill-defined variable or imprecision of data. Evaluator's judgement and rating score cannot be avoided including vagueness (fuzziness) in both case of (1) rating a qualitative item (beauty, comfort, etc.) and (2) rating a quantitative item (mileage, price, etc.), when data is imprecise and ill-defined and/or utility function itself is fuzzy (though for exact data). Therefore the ratings can be expressed as numbers with extended range or as linguistical expressions like high, medium, low, etc.

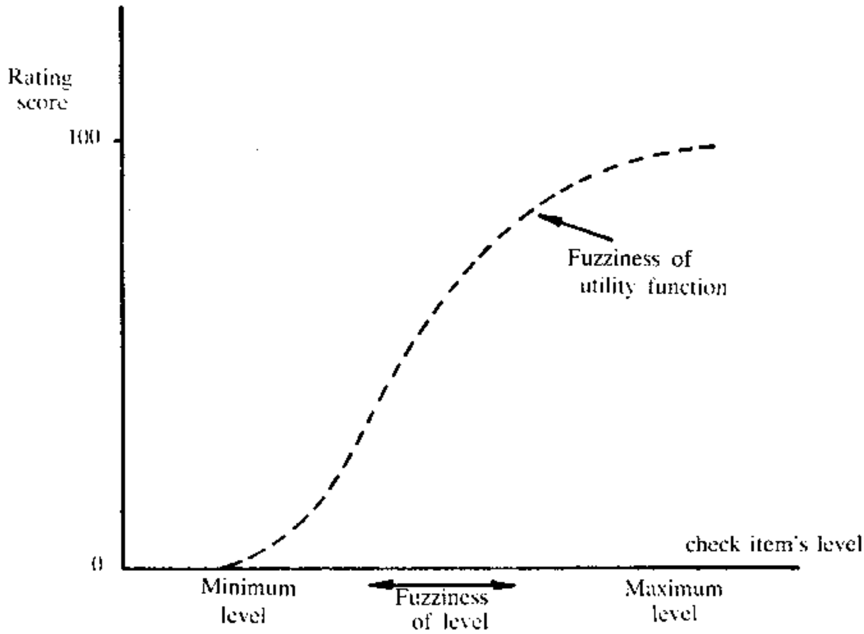

Fig. 1 Fuzzey nature in rating a quantitative item

Further, fuzziness depends on evaluator's ability and quality of data. Even exellent expert's judgement also has fuzziness though it is smaller than inexperienced person's.

Conventional checklist requires the deterministic rating score, but it is not well suited for dealing with fuzziness. For example, when some items were very imprecise as 'unknown,' evaluator assigned the middle score (50 in 100 point scale) or she ignored and excluded such items.

It is reasonable that ratings includes fuzziness, therefore the problem is how to handle this fuzziness. This fuzziness is quite different from probabilistic randomness. If the fuzziness is due to the subjectivity or imprecision of data, the tools of statistics are not appropriate. Fuzzy set theory makes it possible to consider this fuzziness as a whole. The strength of fuzzy sets theory lies in the ability to quantify and manipulate qualitative statements, vagueness, lack of data, or subjectivity of opinion.

This paper first reviews brief preliminaries of fuzzy sets. Then a simple techniques of obtaining the weights from pairwise comparison will be represented. Next we represent the method of converting the fuzzy ratings to final fuzzy score, and simple example will be shown.

2. PRELIMINARIES OF FUZZY SETS

Notation

X, Y, Z, ... Crisp support sets (of universe of discourse),

$\tilde{A}$, $\tilde{B}$, $\tilde{C}$, ... fuzzy sets (may be fuzzy numbers),
x, y, z, ... element of a set,
$\mu_A(.)$ membership function of $\tilde{A}$,
$\mu_{A,L}(.)$, $\mu_{A,R}(.)$ left (increasing) part and right (decreasing) part of membership function of $\tilde{A}$,
$f^{-1}(.)$ inverse function if f is a function,
$x/\mu_A(x)$ ordered pair; element of fuzzy set $\tilde{A}$ in X.

Fuzzy set theory as the foundation for a formalized logic that reflects the vagueness inherent in human reasoning was introduced originally by Zadeh[15]. Since Zadeh formulated the initial theory of fuzzy sets, there has been a flood of propositions to adopt this abstract mathematical concept to a variety of fields. Recent advances in the theory of fuzzy sets make it possible to study the complex system and ill-defined systems (and concepts) where uncertainty is due to fuzziness or degree of vagueness. Fuzzy sets theory is explained in much of literature. Much of the theoretical literature have been developed in depth[6].

Fuzzy set theory is based on a recognition that certain sets have imprecise boundaries. The imprecision of fuzzy sets deals with the shades of membership of an object in a set with imprecise boundaries, whereas the randomness of probability theory deals with the uncertainty regarding the occurrence or nonoccurrence of some event.

Zadeh[15] defines a fuzzy set as "a class of objects with a continuum of grades of membership." The transition from membership to nonmembership in a subset of a reference set is gradual rather than abrupt. Thus it is permissible for elements to be only partly elements of a set.

Let $X=\{x\}$ denote a space of points (objects), with a generic element of X denote by x. The X, denumerable or not, is a nonfuzzy support set of a universe of discourse. X may be a real n-dimensional Euclidean space R^n. A fuzzy set $\tilde{A}$ in X is characterized by a membership function $\mu_A(x)$ which associates with each point in X. The central concept of fuzzy sets theory is the membership function whose value is a real number in the interval [0,1], representing the grade of membership of x in $\tilde{A}$. And $\mu_A(x)$ may be interpreted as the degree of possibility of x given concept $\tilde{A}$.

Then, formally, a fuzzy set $\tilde{A}$ in X is a set of ordered pairs, x and $\mu_A(x)$, we denote $\tilde{A}=\{x/\mu_A(x)\}$. In many applications, the degree of membership $\mu_A(x)$ may be interpreted as the degree of compatibility of x with the concept represented by $\tilde{A}$.

Example: Let the universe of discourse X be the set of all girls in a particular college. Let $\tilde{A}$ be a fuzzy set of 'beauty'. Then, $\tilde{A}$='beauty'= {..., Anne/0.8, Barbara/0.7, ..., Cathy/0.9, ...}.

Example: Let Y be the interval (0,300) representing the weight of people in kilograms. Let $\tilde{B}$ be equated to linguistic value 'heavy'. Then, $\tilde{B}$ = 'heavy'= $\{y/\mu_B(y)\}$, where

$$\mu_B(y)=\begin{cases}(1/30)\,y-2\,, & y\in[60,90]\\ 0, & y\in(0,60)\cup(90,300).\end{cases}$$

A fuzzy singleton is a fuzzy set which has only one supporting point x: $\tilde{A}=\{x/\mu\}$. The α-level set of a fuzzy set $\tilde{A}$ in X is defined a the crisp set A_α for which the degree of membership exceeds the level α, $0\le\alpha\le 1$:

$$A_\alpha = \{x : \mu_A(x) \geq \alpha\}. \tag{2}$$

Two fuzzy sets $\tilde{A}$ and $\tilde{B}$ are equal ($\tilde{A}=\tilde{B}$) if and only if $\mu_A(x)=\mu_B(x)$ for all x in X. Fuzzy set $\tilde{A}$ is a subset of $\tilde{B}$ or contained in $\tilde{B}$ if and only if $\mu_A(x) \leq \mu_B(x)$ for all x in X.

The calculus of fuzzy sets is based on three reasonable proposition:

$$\text{Intersection, } \tilde{A} \quad \tilde{B};\ \mu_{A \cap B}(x)=\min\{\mu_A(x),\ \mu_B(x)\} \tag{3}$$

$$\text{Union, } \tilde{A} \quad \tilde{B};\ \mu_{A \cup B}(x)=\max\{\mu_A(x),\ \mu_B(x)\} \tag{4}$$

$$\text{Complement, } \tilde{A}';\ \mu_{A'}(x)=1-\mu_A(x) \tag{5}$$

A fuzzy set is convex if and only if its α-level sets are convex for all $0 \leq \alpha \leq 1$. An equivalent definition is: for all $x_1, x_2 \in X$ and $\lambda \in [0,1]$,

$$\mu_A(\lambda x_1 + (1-\lambda) x_2 \geq \min(\mu_A(x_1),\ \mu_A(x_2)). \tag{6}$$

If $\tilde{A}$ and $\tilde{B}$ are convex, so is their intersection[15].

Fuzzy sets induced by mappings is obtained by extension principle. This is originally suggested by Zadeh[16]. The basic concept is that the joint occurrence is possible to the extent to which the least possible individual occurrence is possible.

Extension principle (Zadeh's)

Let $\tilde{A}_1, \ldots \tilde{A}_m$ be m fuzzy sets in $X_1, \ldots, X_m$, respectively. Let $y=f(x_1, \ldots, x_m)$ be a mapping from $X_1 \times \quad \times X_m$ to a universe Y. The fuzzy set $\tilde{B}$ induced from m fuzzy sets $\tilde{A}_i$ through f is on Y such that

$$\mu_B(y) = \max_{y=f(x_1, \ldots, x_m)} \min\{\mu_{A1}(x_1), \ldots, \mu_{Am}(x_m)\}. \tag{7}$$

$\tilde{B}$ with Eq. (7) is represented simply and conveniently as

$$\widetilde{B}=f(\widetilde{A}_1, \ldots, \widetilde{A}_m). \tag{8}$$

Specially if f is one to one, $y=f(x_1)$, Eq. (7) becomes

$$\mu_B(y) = \mu_{A_1}(f^{-1}(y)). \tag{9}$$

Dubois & Prade[5] and Mizumoto & Tanaka[13] presented various definitions of fuzzy number and theorems for basic operations of fuzzy numbers based on extension principle. Fuzzy number, or more generally fuzzy sets of the real line, are convenient concept for the representation and arithmetic manipulation of ill-known numerical quantities.

Fuzzy number

A real fuzzy number $\tilde{N}$ is more precisely described as any fuzzy set of the real line (i.e., universe set is R), whose membership function μ, is

$$\begin{aligned} &(1)\ 1 \quad \text{on } [b, c] \\ &(2)\ \text{increasing, let } \mu_L, \text{ on } [a, b] \\ &(3)\ \text{decreasing, let } \mu_R, \text{ on } [c, d] \\ &(4)\ 0, \quad \text{otherwise, i.e., on } (-\infty, a] \cup [b, +\infty). \end{aligned} \tag{10}$$

This definition based on [5] includes convexity (Eq. (6)). When we say about fuzzy number, unless otherwise specified it is commonly convex. A fuzzy set with discrete supports cannot be convex fuzzy number. If $a=-\infty$ or $d=+\infty$, this fuzzy number is 'unbounded.' A fuzzy number is said to be positive if $0 < a$.

Let the fuzzy number denote (a, b, c, d) $_{\tilde{N}}$, or (a, b, c, d) unless confusion.

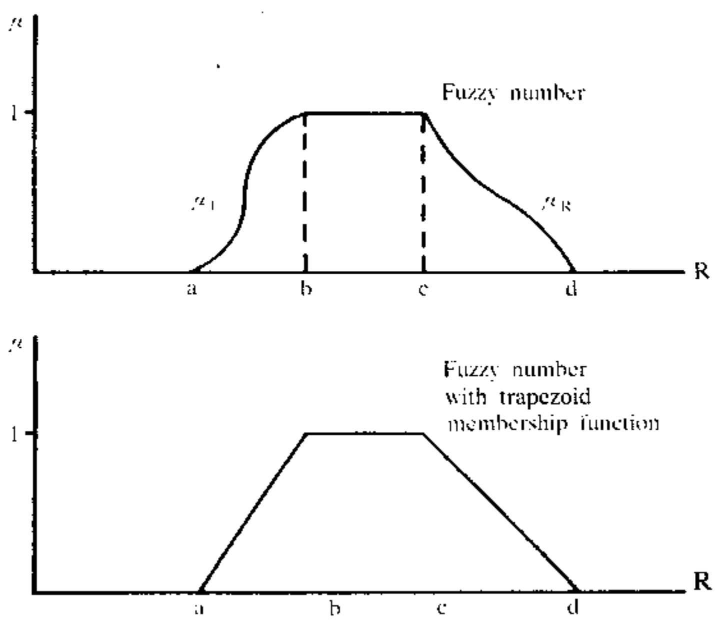

Fig. 2 Fuzzy number

The sophistication of the shape of the membership function of fuzzy number is useless because of the imprecision. The linear membership function (i.e., trapezoid) is used widely to the fuzzy number and to the their operations instead of complicated shape. Because it manage the vagueness of human judgement and ill-known numeric quantities anyhow, there is no severe error[5].

3. ASSESSING THE WEIGHTS FROM PAIRWISE COMPARISON

Each check item requires information about the relative importance. It is given by a set of weights. A set of all weights may be assigned directly at a time by human's judgement and estimation, but it is not easy. However, the pairwise comparison data may be easier to estimate their relative important ratio, w_i / w_j.

Assuming pairwise comparison data is available, eigenvector method and weighted least-square method can be used [10]. These methods have comparative advantage to derive the weights from human's fuzzy judgements as compared with traditional techniques. Particularly, weighted least-square method has two advantages; (1) it is conceptually easier than eigenvector method, and (2) it involves the solution of a set of simultaneous linear algebraic equations [4].

Assume evaluator judges the relative importance of two items. The number of judgement is ${}_nC_2 = n(n-1)/2$, where n is number of check items. Since the precise values of w_i/w_j may not be known in general, let the estimate of it be a_{ij}, i.e.,

$$a_{ij} \cong w_i/w_j \tag{11}$$

As the results, pairwise comparison matrix, A, is determined;

$$A = [a_{ij}]_{n \times n} \tag{12}$$

It is assumed matrix A has reciprocal property,

$$a_{ij} = 1/a_{ji} \tag{13}$$

If $a_{ij} = w_i/w_j$ exactly, then A has the consistency property as well as reciprocal property,

$$a_{ij} = a_{ik}/a_{jk} \tag{14}$$

From $a_{ij} \cong w_i/w_j$,

$$a_{ij} \cdot w_j \cong w_i \tag{15}$$

Basic concept of obtaining the weights is to minimize the error sum of square of Eq. (15), $a_{ij} \cdot w_j - w_i$. The set of weights is obtained by solving the constrained optimization problem.

$$\begin{aligned} &\min z = \sum_{i=1}^{n} \sum_{j=1}^{n} (a_{ij} \cdot w_j - w_j)^2 \\ &\text{sub. to } \Sigma\ w_i = 1. \end{aligned} \tag{16}$$

Reciprocal property, here, may not be essential requirement. Then the number of pairwise judgements will be $n(n-1)$. Eq. (16) is solved by classical optimization method, Lagrangian multiplier method. Eq. (16) induces following simultaneous linear equations from differentiating the Lagrangian function with respect to w_k,

$$\sum_{i=1}^{n} (a_{ik} w_k - w_i)\, a_{ik} - \sum_{j=1}^{n} (a_{kj} w_j - w_k) + \lambda = 0,\ k = 1, 2, \ldots, n, \tag{17}$$

where λ : Langrangian multiplier. Or

$$\mathbf{B} \cdot \mathbf{w} = \mathbf{m}, \tag{18}$$

where $\mathbf{w} = (w_1, \ldots, w_n, \lambda)^T$,

$\mathbf{m} = (0, \ldots, 0, 1)^T$, and

$\mathbf{B} = [b_{ij}]$; $(n+1) \times (n+1)$ matrix, where

$b_{ii} = (n-1) + \sum_{j \neq i} a_{ji}^2$, $1 = 1, \ldots, n$

$b_{ij} = -(a_{ij} + a_{ji})$, $i, j = 1, \ldots, n$

$b_{i,n+1} = b_{n+1,i} = 1$, $i = 1, \ldots, n$

$b_{n+1,n+1} = 0$.

Since the solution $\mathbf{w} = \mathbf{B}^{-1} \cdot \mathbf{m}$ and m is unit vector, the weights are obtained simply,

$$\mathbf{w} = (\beta_{1,n+1}, \ldots, \beta_{n,n+1}, \beta_{n+1,n+1}) \tag{19}$$

where β_{ij} is (i, j) element of $\mathbf{B}^{-1}$

This property makes the computer program more simple using a standard subroutine for inverse matrix.

4. WEIGHTED FUZZY CHECKLIST

Assume there are n check items and their weights are obtained from the method in section 3 or given in advance.

When the rating scores including fuzziness are represented as fuzzy sets, specially fuzzy number, let the rating scores be denoted as $\tilde{R}_i$. And let interval U_i be universe set for i-th item, this means $\tilde{R}_i$ has supports, u_i, in U_i. It is assumed that $\tilde{U}_i = [0,100]$ for all. We can write

$$\tilde{R}_i = (a_{Ri}, b_{Ri}, c_{Ri}, d_{Ri}) \text{ in } U_i = [0,100], \ 1 = 1, \ldots, n. \tag{20}$$

More simply, the triangle form can be used and it may be more practical. Triangle membership function is special form of trapezoid membership function, i.e. $b_{Ri} = c_{Ri}$ in Eq. (20).

Fuzzy rating scores can be expressed linguistically. Linguistic expressions are applied similarly; the corresponding membership functions must be determined. Although humans are quite unsuccessful in quantitative predictions, they may be comparatively efficient in qualitative forecastings. In fact, the knowledge of experts usually consists of qualitative variables stated verbally, as evidenced by recent developments in knowledge-based expert systems[12]. For example, we suggest five templates (basic qualitative evaluation) as in Fig. 3. Notice the membership functions are triangular.

Very high = (75, 100, 100, 100)
High = (50, 75, 75, 100)
Medium = (25, 50, 50, 75)

Low $=(5,\ 25,\ 25,\ 50)$

Very low $=(0,\ 0,\ 0,\ 25)$

'Unknown' (or 'not clear') is available;

Unknown$=(0,\ 0,\ 100,\ 100)$.

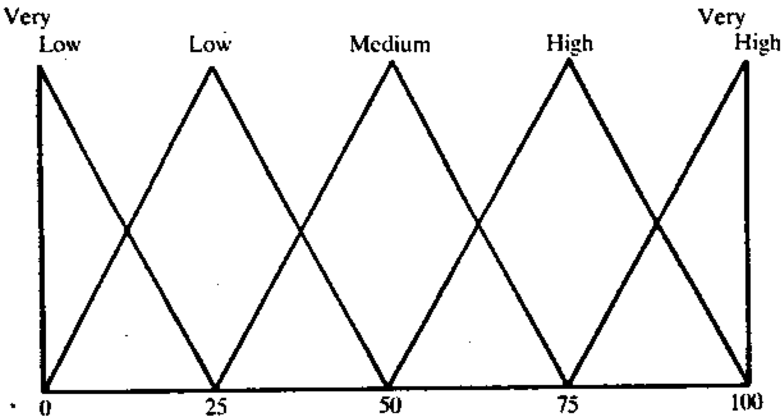

Fig. 3 Example of fuzzy sets for linguistic expressions

Intermediate fuzzy expressions are available such as 'between high and medium'=(37.5, 62.5, 62.5, 87.5) when it is difficult to say either one or the other,

If the fuzziness is larger, then more than one templates may be included. For example, 'medium to high'=(25, 50, 75, 100). Notice the membership function is trapezoid here. The templates are suggestive; they can be changed for practical problems.

Here we will use trapezoid fuzzy number ratings rather than linguistic expressions for convenient explanations.

We must find the final score based on Eq. (1), linear aggregation or weighted average. However, the ratings are not deterministic crisp numbers but fuzzy numbers, $\tilde{R}_i$. Then final score is another fuzzy set, let $\tilde{T}$, by extension principle in fuzzy set theory as stated in section 2. $\tilde{T}$ is in $U=[0,\ 100]$, and its membership function is;

$$\mu_T(u)=\max_{u=\Sigma_i^n w_i \cdot u_i} \min \{\mu_{R2}(u_2),\ \ldots,\ \mu_{R2}(u_2),\ \ldots,\ \mu_{Rn}(u_n)\}. \tag{21}$$

$\tilde{T}$ may be denoted simply,

$$\tilde{T}=w_1\tilde{R}_i+\ \ldots\ +w_n\tilde{R}_n,\ \text{or}$$

$$=\sum_{i=1}^{n} w_i\tilde{R}_i. \tag{22}$$

We can find $\tilde{T}$ and μ_T by theorem of fuzzy number addition. Dubois & Prade[5] and Mizumoto & Tanaka[13] presented fuzzy number's arithmetic operations based on Zadeh's extension principle. They are different concept from fuzzy set operation as union, intersection, and complement. They means obtaining fuzzy set induced by arithmetic operations of supports which are real numbers and come from different universe sets each other. The theorems in [5] [13] describe the shape of the resulting membership function using inverse function.

In some basic theorems, we only need one for addition.

[Theorem] Let $\tilde{M}$, $\tilde{N}$ be two fuzzy numbers, (a_1, b_1, c_1, d_1) in real line **R** and (a_2, b_2, c_2, d_2) in R respectively. And let μ_1 and μ_2 be their membership functions which need not to be trapezoid. Then, $\tilde{A}=\tilde{M}+\tilde{N}$ is fuzzy number in **R**, and whose membership function is

$$\mu(x)=\begin{cases}\mu_L(x), & x \in [a_1+a_2,\ b_1+b_2]\\ \mu_R(x), & x \in [c_1+c_2,\ d_1+d_2]\\ 1 & , x \in [b_1+b_2,\ c_1+c_2]\\ 0 & , x \in (-\infty,\ a_1+a_2] \quad [d_1+d_2,\ +\infty)\end{cases} \tag{23}$$

where $\mu_L^{-1}(u)=\mu_{1,L}^{-1}(u)+\mu_{2,L}^{-1}(u)$, and $\mu_R^{-1}(u)=\mu_{1,R}^{-1}(u)+\mu_{2,R}^{-1}(u)$, $0< u\ 1$.
Proof: See [5].

For the trapezoid fuzzy number in particular, the addition of two fuzzy numbers becomes another trapezoid fuzzy number.

Since weighted sum is extended form of simple addition, $\tilde{T}$ is easily obtained by sequential applying the above theorem.

$$\tilde{T}=(a_T,\ b_T,\ c_T,\ d_T) \text{ in } U=[0,\ 100], \tag{24}$$

where $a_T=\sum_{i=1}^{n} w_i \cdot a_{Ri}$, ..., $d_T=\sum_{i=1}^{n} w_i \cdot d_{Ri}$.

And its membership function or Eq. (21) is also trapezoid.

Expressing final score as fuzzy number, $\tilde{T}$, is more reasonable and comfortable than as punctual crisp number. Whole utility is estimated linguistically according where this final score is on the universe set [0, 100]. Sometimes the expression as "high" is more comfortable than punctual crisp number to manager. We suggest the method what basic template in Fig. 3. is fitted to this final score. We use a measure of unconformity between two fuzzy sets using the integrals of inverse functions of two left sides and two right sides of membership functions.

$$I\ (\tilde{A},\tilde{B})=\left|\ \int_0^1[\mu_{AL}^{-1}(u)-\mu_{BL}^{-1}(u)]du+\int_0^1[\mu_{AR}^{-1}(u)-\mu_{BR}^{-1}(u)]du\ \right| \tag{25}$$

This measure is simpler and more discriminative than Yager's Hamming distance[14]. For two trapezoid fuzzy numbers in particular,

$$I\ (\tilde{A},\tilde{B})=\left|\ (a_1-a_2)+(b_1-b_2)+(c_1-c_2)+(d_1-d_2)\ \right| /2. \tag{26}$$

When the fuzzy final scores are obtained for many alternatives, ranking problem may be arisen. The comparing or ranking fuzzy sets seems to be simple conceptually, but it has a delicate feature. Apart from combining a fuzzy sets, another crucial issue is to compare them, namely which one is greater than another[7]. Many rules have been suggested, but they are controversial as reviewed in [3]. Refer the recent papers for this prolem, Dubois & Prade [7] and Bortolan & Degani [3]. We suggest a new simple rule that uses a measure of comparison which is similar to measure of unconformity, namely, the value without absolute symbol.

$$V(\tilde{A}, \tilde{B}) = \int_0^1 [\mu_{AL}^{-1}(u) - \mu_{BL}^{-1}(u)]du + \int_0^1 [\mu_{AR}^{-1}(u) - \mu_{BR}^{-1}(u)]du \qquad (27)$$

If $V(\tilde{A}, \tilde{B}) > 0$, we decide $\tilde{A}$ is greater than $\tilde{B}$.

On the other hand, the area surrounded with membership function is a measure of fuzziness based on evaluator's skill and experience. The larger area means the poor judgement.

$$F = \int_a^d \mu(x)dx. \qquad (28)$$

For trapezoid fuzzy number,

$$F = [(d-a)+(c-b)]/2. \qquad (29)$$

A suggestive expressions about fuzziness is as follows;

0– 10%	fairly exact,
10– 25%	a little fuzzy,
25– 50%	fuzzy,
50– 75%	more fuzzy,
75–100%	very fuzzy.

Evaluator may be team that consist of more than one (group evaluation or multiple evaluator). In this case, the fuzzy rating score is simply summation of each person's fuzzy ratings. If each person's weight exists (according to experience and ability, for example, by super evaluator), weighted sum is applied similar manner.

If checklist has hierarchical structure—a check item may have sub-items, similar procedure can be applied sequentially.

Weights may be assigned as fuzzy sets (specially positive fuzzy numbers). In this case direct normalized weights are impossible from fuzzy numbers operations (addition, multiplication, etc.). In comparing and ranking problem, fuzzy weights are available without normalization. However, weighted average has more complex feature than weighted sum. See [1] for the theoretical implications of both fuzzy weights and fuzzy ratings.

5. AN EXAMPLE FOR SAFETY ASSESSEMENT

We will show the numerical example for safety assessment. Safety assessment is important field of safety engineering. There are several approaches to the assessment of safety [2] [9], some of which are quantification ones with respect to probabilistic method as fault tree analysis.

However, probabilistic quantification is difficult. For example, fault tree analysis is time consuming to establish the structure, and its computation time is extensive. The large efforts are used for theoretical verification of safety level instead of eliminating, lessening or controlling the practical existing hazard. It is difficult to initiate the model, there are lack of data, and many assumptions are not guaranteed. Since the risk is a concept that is not absolutely objective in nature, but rather relative and subjective, the notion of risk must be looked upon in the interaction between the environment and the human [12].

Quick methods are provided by rating the factors (check items) considered to affect the safety as illumination, ventilation, operating method, maintenance, guarding, supervision, and the attitude of the top executive [2]. Since rating has vagueness, fuzzy checklist method is useful.

Assume safety engineer considers the three significant factors, ventilation, illumination, and noise. Their relative importance data, i.e., pairwise comparison data is;

ventil : illumi $= w_1/w_2 = 2$

ventil : noise $= w_1/w_3 = 1.5$

illumi : noise $= w_2/w_3 = 1$

The results (normalized weights) from above date are .4639898, .2486608, and .2873495.

Their fuzzy ratings are shown in Table 1. Notice the factors are rated as negative way; the more value the more risky. Each membership functions are trapezoid and universe sets are [0, 100] (in other word, % dissatisfaction is used).

Table 1. Risk factor's estimates

unit : %

Factors	Rating
Ventilation	$\tilde{R}_1 = (55, 60, 65, 70)$
Illumination	$\tilde{R}_2 = (45, 50, 60, 65)$
Noisy	$\tilde{R}_3 = (60, 65, 70, 80)$

Let final score induced from Table 1 be $\tilde{T}$ in [0, 100].

Then, by section 4.,

$a_T = .4639898\ (50) + .2486608\ (45) + .2873495\ (60)$
$= 53.95.$

Similarly, $b_T = 58.95$, $c_T = 65.19$, and $d_T = 71.63$. or

$$T = \sum_{i=1}^{n} w_i \cdot A_i.$$
$$= (53.95,\ 58.95,\ 65.19,\ 71.63)$$

For the linguistic expressions, the basic templates are converted as very safe, safe, medium, risky, and very risky. We compute the measure of unconformity, then I ($\tilde{T}$, Medium)= 49.72, I ($\tilde{T}$, Risky)=50.28, I ($\tilde{T}$, 'Between Medium and Risky')=0.28, and I ($\tilde{T}$, 'Medium to Risky')=0.28. On the other hand, the fuzziness of $\tilde{T}$ is 11.96, that of 'Between Medium and Risky' is 25, and that of 'Medium of Risky' is 50. Therefore, we can say "this system's safety is 'Between Medium to Risky' approximately, and this assessement is 'a little fuzzy'."

We provide a simple BASIC program for fuzzy checklist evaluation. The program list is shown in appendix which includes the subroutine of solving simultaneous linear equations to obtain weights; we modified standard subroutine for inverse matrix. This program can be applied to expert system with appropriate developments.

6. CONCLUSION

Checklist method is rapid and useful to evaluate the effectiveness of evaluation subject. The basic philosophy is on human's ability to summarize the information and to judge significantly. However, human judgment includes vagueness or fuzziness inherently in spite of its significance. Ratings of the check items may become imprecise and somewhat vague. Fuzzy set theory makes it possible to consider the fuzzy evaluation. In this paper, we show evaluation methos using weighted fuzzy checklist where check items are rated as fuzzy numbers.

To obtain the weights, we use pairwise comparison data, since this method has comparative advantages for human's fuzzy judgment, and solution procedure.

Final score is also fuzzy number. It is more reasonable than punctual crisp number. From this final score, evaluator can judge the evaluation subject approximately. Membership function of final score includes the fuzziness of evaluator's judgment. We suggest some measures; a measure of unconformity to find appropriate linguistic expression, a measure of comparison to find greater fuzzy number, and a measure of fuzziness.

Fuzzy checklist method provides useful tool to decision making, and it can be applied to practical problems widely, for example, handling uncertainty in expert system. Sample program is provided to apply to microcomputer.

Appendix : Example of BASIC program

```
10  '*** fuzzy checklisting (FUZZYCHE. BAS)
20  INPUT "Number of Check items=", N: N1=N+1
30  DIM ITM$ (N), A (N, N), W (N), B (N1, 2*N1)
40  PRINT "Description of the check items..."
50  FOR I=1 TO N: PRINT "ITM$ ("; I;")=";: INPUT ITM$ (I): NEXT
60  PRINT "Relative importance of check items by pairwise comparison."
70  FOR I=1 TO N: FOR J=I TO N: IF I=J THEN A (I, J)=1: GOTO 90
80  PRINT ITM$ (I); " : "; ITM$ (J) ;"=w ("; I;") / w ("; J;")=";: INPUT A (I, J)
90  A (J, I)=1/A (I, J) : NEXT J, I
100 FOR I=1 TO N: SS=0 '*** Simult Eq Coeff
110 FOR J=1 TO N: B (I, J)=-A (I, J)-A (J, I) : SS=SS+A (J, I)^2: NEXT
120 B (I, I)=N-1+SS-1: B (I, N1)=1: B (N1, I)=1
130 NEXT I: B (N1, N1)=0: GOSUB 360
140 PRINT: PRINT "Normalized weights of the check items to be aapplied"
150 FOR I=1 TO N: PRINT ITM$ (I) ;" : w ("; I;")="; W (I): NEXT
160 '
170 '*************** fuzzy fuzzy
180 DIM R (N, 4), T (4)
190 WINDOW (-100, -1) - (200, 2)
200 INPUT "Number of evaluation subjects to be evaluated", M
210 FOR J=1 TO M : CLS : PRINT "evaluation subject no.", J
220 FOR I=1 TO N
```

```
230 PRINT "Fuzzy rating of"; ITM$ (I);" (% on [0, 100], 4 points) : ";
240 INPUT R (I, 1), R (I, 2), R (I, 3), R (I, 4) : PRINT
250 NEXT I
260 'Final score & drawing it
270 FOR P=1 TO 4: T (P)=0: FOR I=1 TO N: T (P)=T (P)+W (I)*R (I, P) : NEXT
    I, P
280 CLS:PRINT "Fuzzy final score=("; T (1) ; ", "; T (2) ; ", "; T (3) ; ", "; T (4) ; ")"
290 LINE (0, 0)-(100, 0) : LINE (0, 0)-(0, 1)
300 LINE (T (1), 0)-(T (2), 1) : LINE (T (2), 1)-(T (3), 1) : LINE (T (3), 1)-(T (4),
    0)
310 PRINT "Press any key to continue"
320 K$=INKEY$ : IF K$="" THEN 320
330 NEXT J
340 END
350 '
360 FOR I=1 TO N1 : FOR J=1 TO N1 : B (I, J+N1)=0: NEXT: B (I, I+N1)=1:
    NEXT
370 FOR K=1 TO N1 : IF K=N1 THEN 410 ELSE L=K
380 FOR I=K+1 TO N1 : IF ABS (B (I, K)) > ABS (B (L, K)) THEN L=I
390 NEXT I : IF L=K THEN 410
400 FOR J=K TO 2*N1 : SWAP B(K, J), B (L, J) : NEXT
410 FOR J=K+1 TO 2*N1 : B (K, J)=B (K, J) / B (K, K) : NEXT : IF K=1 THEN 440
420 FOR I=1 TO K-1 : FOR J=K+1 TO 2*N1 : B (I, J)=B (I, J)-B (I, K)*B (K, J)
430 NEXT J, I : IF K=N1 THEN 470
440 FOR I=K+1 TO N1 : FOR J=K+1 TO 2*N1 : B (I, J)=B (I, J)-B (I, K)*B (K,
    J)
450 NEXT J, I
460 NEXT K
470 FOR I=1 TO N : W (I)=B (I, 2* N1) : NEXT : RETURN
```

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