〈研究論文〉

Journal of the Korean Institute of Industrial Engineers
Vol. 14, No. 1, June 1988

A Study on Class-based Turnover Assignment in Automated Storage/Retrieval Systems with Continuous Rectangular Rack Face in Time

Hark Hwang*
Sie Gheun Koh*
Gil Sang Jang**

Abstract

This paper studies class-based turnover assignment rule in terms of the expected travel time in automated storage/retrieval systems (AS/RS). With the rack face not necessarily in square in time, travel time models of two and three-class systems are developed, from which class partition values are determined for single command cycle. Also, the effects of the system parameters such as the rack shape factor and the skewness of the ABC curve are evaluated on the travel time through example problems.

1. Introduction

This paper addresses the problem of storage assignment for the scheduling of Automated Storage/Retrieval System (AS/RS) which use storage/retrieval (S/R) machine and palletized loads. Recently, many AS/RS related research papers have dealt with the storage assignment problem. Graves, Hausman and Schwarz (GHS) [3] considered three kinds of storage assignment policies (random, full turnover based and class based) in terms of the expected S/R machine travel time. Later, they extended the work to include interleaving, that is, the sequencing of storage and retrieve requests [2]. Bozer and White [1] studied the similar problem where the rack is rectangular in time and computed the expected travel times for random storage assignment under both single and dual command cycles. For multi-aisle S/R machine system (MASS) with random assignment rule, Hwand and Ko [4] determined the average travel time of S/R machine under both single and dual commands

* Department of Industrial Engineering, Korea Advanced Institute of Science & Technology
** Korea Institute for Defense Analyses

and proposed rack-class-based storage assignment rule.

This paper is an extension of the work by GHS [3] for class-based storage by relaxing the assumption that the rack is square in time. We derive the expected travel times of the S/R machine under two and three-class systems as function of the partitioning value R and then determine the optimal value R for various inventory distribution and rack shape factor b.

2. Development of the model

The following assumptions are made:

1) The rack is considered to be a continuous rectangular pick face where the input/output point (I/O point) is located at the lower left-hand corner.
2) The length and height of the rack as well as the S/R machine speed in the horizontal and vertical directions are known.
3) The S/R machine travels simultaneously in the horizontal and vertical directions. In calculating the travel time, constant velocities are used for horizontal and vertical travel.
4) Interleaving is ignored.
5) Pick-up and deposit (P/D) times associated with loading and unloading are ignored. The P/D time is generally independent of the rack shape and the travel velocity of the S/R machine.
6) Each pallet contains only one part number or item type, and all storage locations are the same size.
7) The turnover frequency of each item is known and constant through time.
8) Only the long-run average behavior of the system is considered.

2.1 Rack normalization

As stated earlier, the rack face is assumed to be a continuous rectangle in time with known dimensions. Following the work of Bozer and White, the rack face will be "normalized" by dividing the horizontal travel time t_h and the vertical travel time t_v by T, where

s_h=horizontal travel speed

s_v=vertical travel speed

L=rack length

H=rack height

$t_h = \frac{L}{s_h}$; time to reach the end of the rack

$t_v = \frac{H}{s_v}$; time to reach the top of the rack

$T = \text{Max}(t_h, t_v)$; denormalizing factor

$b = \text{Min}(t_h, t_v) / T, \; 0 \leq b \leq 1$

The factor, b, has been referred to as the "shape factor" for the rack. It will be assumed without loss of generality that $t_v \leq t_h$, and thus the rack has dimensions $1 \times b$ as shown in Figure 1. Note that if $t_h = t_v$, then $b=1$ and the rack is said to be square in time.

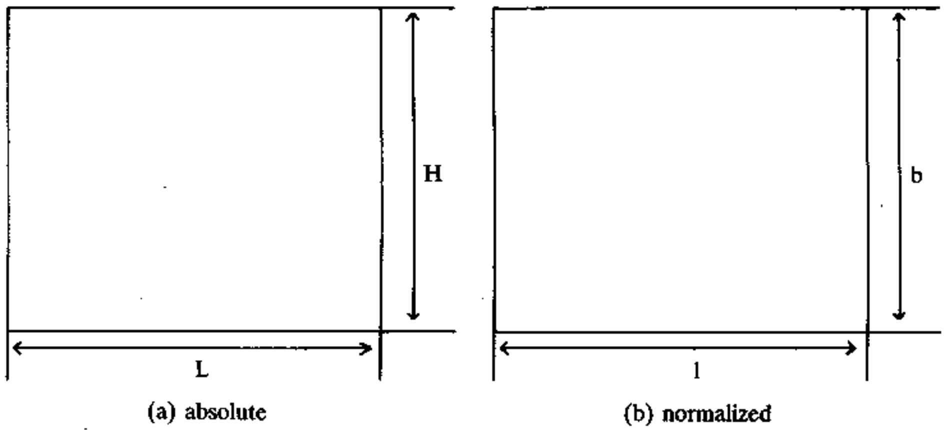


Figure 1. Continuous representation of storage rack

2.2 Turnover distribution

The turnover of a pallet is the number of times a given pallet requires strorage and retrieval during some time period. Assuming the basic EOQ model and the "ABC" phenomenon for inventories, GHS [3] derived the turnover of the j^{th} pallet, $\lambda(j)$, and

$$\lambda(j)=\left(\frac{2s}{K}\right)^{1/2}\left(j^{\frac{s-1}{s+1}}\right),\ 0<j\leq 1, \quad \cdots\cdots (1)$$

where s is the skewness parameter in the "ABC" curve and K is the ratio of ordering cost to holding cost which is assumed to be constant for all items. They represented the "ABC" curve by the function

$$G(i)=i^{s} \quad \text{for } 0<s\leq 1 \quad \cdots\cdots (2)$$

where demand is measured in pallet loads. Note that in (2) the total annual demand and the number of items are expressed in percentage. For more details, readers are advised to refer to [3].

2.3 Distance distribution

The continuous distance function $y(i)$ is defined as the time taken by the S/R machine to move from the I/O point to a location in the i^{th} fractile or percentile of distance distribution. Thus i^{th} percentile locations must be arranged in square for $0<i\leq b$ and in rectangular for $b<i\leq 1$ due to the assumption that the travel time to any point (x, y) is Max $[x, y]$. Therefore, the dimension of the square become $(bi)^{1/2}$ by $(bi)^{1/2}$ for $0<i\leq b$ and that of the rectangular, i by b for $b<i\leq 1$. From the above statements,

$$y(i)=\begin{cases}\sqrt{bi}, & 0<i\le b \\ i, & b<i\le 1\end{cases} \quad \cdots\cdots (3)$$

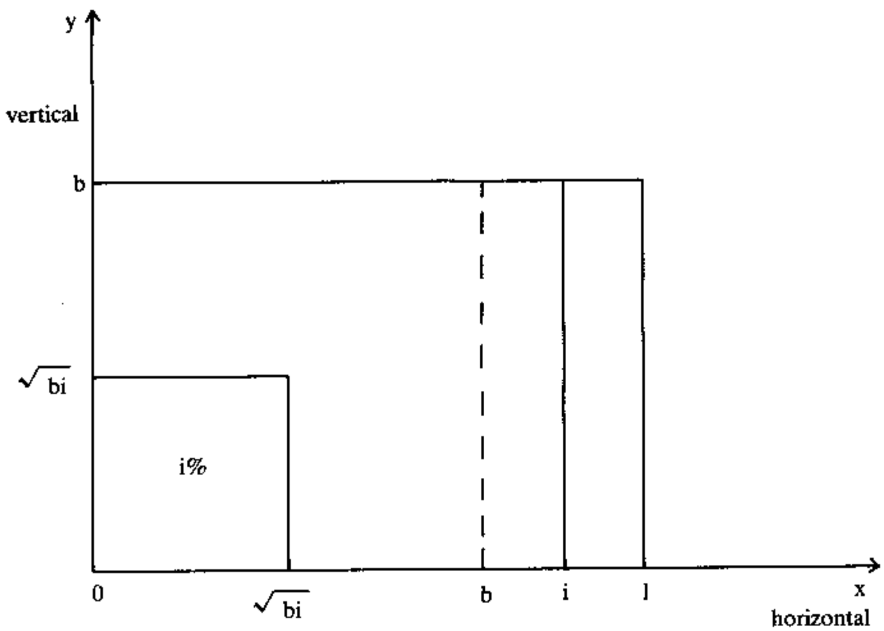


Figure 2. Continuous representation of storage rack

Now, three kinds of storage assignment rules, random, turnover-based, and class-based, are explained briefly as follows. In a random storage assignment rule, any pallet is equally likely to be stored in any of the rack locations. If the highest-turnover pallet is assigned to the closest location, we call the rule turnover-based assignment. The class-based turnover assignment means the racks and pallets are partitioned into K classes based on one way travel times and turnover rate, respectively. Pallets are then assigned to a class of storage according to their class of turnover such that closest-location class is assigned to highest-turnover class. Also, within any given class, a random storage assignment rule is adopted.

2.4 Results for random storage assignment

With (3), the expected one-way travel time, T_r, under random storge assignment is

$$T_r = E[y(i)] = \int_0^1 y(i)\,di \quad \cdots\cdots (4)$$

$$= \int_0^b \sqrt{bi}\,di + \int_b^1 i\,di \quad \cdots\cdots (5)$$

$$= \frac{1}{6}b^2 + \frac{1}{2} \quad \cdots\cdots (6)$$

2.5 Results of turnover-based assignment

For full turnover-based assignment, the expected one-way travel time, T_t, is

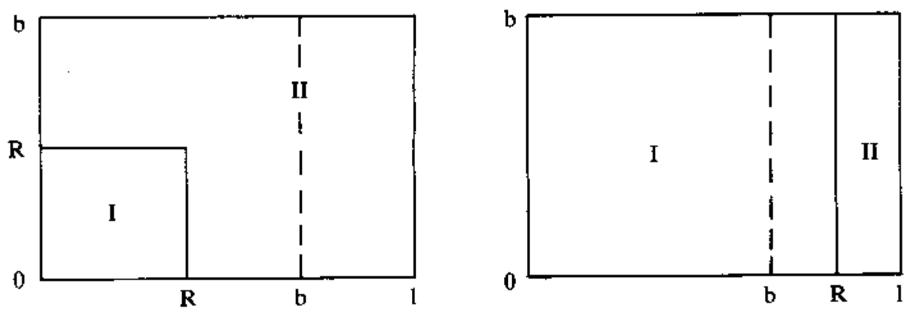
$$Tt = \frac{\int_{i=0}^{b} \lambda(i) y(i) di + \int_{i=b}^{1} \lambda(i) y(i) di}{\int_{i=0}^{1} \lambda(i) di} \quad \cdots\cdots (7)$$

Substituting (1) and (3) into (7) results

$$T_t = \frac{2s}{3s+1} \left(\frac{s+1}{5s+1} b^{3(s+1)/(s+1)} + 1 \right) \quad \cdots\cdots (8)$$

2.6 Class-based turnover assignment

(1) Two-class system



(a) case 1 (b) case 2

Figure 3. Areas for a two-class system

Let R be the location on the horizontal direction of storage rack, which divides the rack into two classes, that is, class I and II. The class I region is used for the higher turnover pallets, and the class II region for the lower turnover pallets (refer to Figure 3). The expected one-way travel time, T_2, under the two-class system depends on the partitioning value R and we want to find R which minimizes T_2. Representing T_2^1 as the expected travel time for $0 \leq R \leq b$, the following results are obtained.

case 1: $0 \leq R \leq b$

$$T_2^1 = \frac{\int_0^{R^2/b} \lambda(i)\, \overline{y}_I\, di + \int_{R^2/b}^{1} \lambda(i)\, \overline{y}_{II}\, di}{\int_{i=0}^{1} \lambda(i)\, di}, \quad \cdots\cdots (9)$$

where $\bar{y}_k$=average travel time to area K, K=I, II. It can be shown that

$$\bar{y}_I=\frac{1}{R^2/b}\int_{i=0}^{R^2/b}\sqrt{bi}\ di=\frac{2}{3}R, \quad (10)$$

$$\bar{y}_{II}=\frac{1}{1-R^2/b}\left[\int_{R^2/b}^{b}\sqrt{bi}\ di+\int_b^1 i\ di\right]$$

$$=\{b^3+3b-4R^3\}/\{6(b-R^2)\} \quad (11)$$

By substituting (1), (10) and (11) into (9), one can derive

$$T_2^1=\left[b^{(1-S)/(S+1)}R^{4S/(S+1)}(4R-b^2-3)+(b^3+3b-4R^3)\right]/\{6(b-R^2)\} \quad (12)$$

case 2: $b < R \leq 1$
Similarly,

$$T_2^2=\frac{\int_0^R \lambda(i)\bar{y}_I\,di+\int_R^1 \lambda(i)\bar{y}_{II}\,di}{\int_{i=0}^1 \lambda(i)\,di}, \quad (13)$$

where $\bar{y}_I=\frac{1}{R}\left[\int_0^b \sqrt{bi}\ di+\int_b^R i\ di\right]=R/2+b^2/(6R)$ (14)

and $\bar{y}_{II}=\frac{1}{1-R}\int_R^1 i\ di=(1+R)/2$ (15)

By substituting (1), (14) and (15) into (13),

$$T_2^2=R^{(S-1)/(S+1)}(b^2-3R)/6+(1+R)/2 \quad (16)$$

Due to the complexity of the functions, the optimal R^* values are found by numerical means for the s values and shape factors of interest. The results are given in Table 1.

In the Table, 20%/60% implies that 20% of the items in inventory represent 60% of the total demand, etc. The corresponding s value can be found from (2), i.e., $0.6=0.2^s$. It can be observed that for fixed s, as b is increasing, the value of R^* is also increasing. And for fixed b, with the decreasing value of s, R^* is also decreasing.

Table 1. Two class system

Shape Factors (b)	ABC curve			
	20%/60%	20%/70%	20%/80%	20%/90%
	R*	R*	R*	R*
0.10	0.244	0.208	0.172	0.100
0.20	0.280	0.200	0.198	0.128
0.30	0.300	0.258	0.210	0.132
0.40	0.312	0.272	0.216	0.136
0.50	0.320	0.280	0.230	0.150
0.60	0.336	0.300	0.240	0.156
0.70	0.350	0.308	0.252	0.154
0.80	0.368	0.320	0.256	0.160
0.90	0.396	0.342	0.270	0.180
1.00	0.400	0.360	0.280	0.180

Note: R*—optiml class partitioning value

(2) Three-class system

Similar approaches are applied to the three-class system. Let R_1 and R_2 be the locations on the horizontal direction of the rack face which partition the rack into three classes. The expected travel times, T_3^i, i=1,2,3, are computed, in which the superscript i represents the case defined in terms of the relative locations among R_1, R_2 and b.

case 1: $0 \leq R_1, R_2 \leq b$

$$T_3^1=\frac{1}{3}b^{-2s/(s+1)}\Big[2R_1^{(5s+1)/(s+1)}+2\,\frac{R_2^3-R_1^3}{R_2^2-R_1^2}\,(R_2^{4s/(s+1)}-R_1^{4s/(s+1)})$$
$$+\frac{b^3+3b-4R_2^3}{2\,(b-R_2^2)}\,(b^{2s/(s+1)}-R_2^{4s/(s-1)})\Big] \quad \cdots\cdots (17)$$

case 2: $0 \leq R_1 \leq b, b < R_2 \leq 1$

$$T_3^2=\frac{1}{6}b^{-2s/(s+1)}\Big[4R_1^{(5s+1)/(s+1)}+3(1+R_2)\{b^{2s/(s+1)}-(bR_2)^{2s/(s+1)}\}$$
$$+(b^3+3bR_2^2-4R_1^3)\{(bR_2)^{2s/(s+1)}-R_1^{4s/(s+1)}\}/(bR_2-R_1^2)\Big] \quad \cdots\cdots (18)$$

case 3: $b < R_1, R_2 \leq 1$

$$T_3^3=R_1^{2s/(s+1)}(b^2-3R_1R_2)/(6R_1)+R_2^{2s/(s+1)}(R_1-1)/2$$
$$+(1+R_2)/2 \quad \cdots\cdots (19)$$

It can be confirmed that the expected travel times of equations (6), (8), (12), (16), (17), (18) and (19) are consistent with those in [3] when b=1. Optimal R_1^* and R_2^* are found for various values of s and b and given in Table 2 from which we observe that with the fixed value of b, both R_1^* and R_2^* decrease gradually as s decreases.

Table 2. Three class system

Shape Factors	ABC curve							
	20%/60%		20%/70%		20%/80%		20%/90%	
(b)	R_1^*	R_2^*	R_1^*	R_2^*	R_1^*	R_2^*	R_1^*	R_2^*
0.10	0.126	0.458	0.106	0.428	0.072	0.348	0.034	0.248
0.20	0.138	0.432	0.108	0.380	0.078	0.326	0.042	0.256
0.30	0.150	0.426	0.122	0.386	0.092	0.344	0.052	0.284
0.40	0.172	0.446	0.144	0.414	0.104	0.366	0.054	0.294
0.50	0.192	0.470	0.152	0.430	0.108	0.378	0.054	0.302
0.60	0.198	0.484	0.158	0.444	0.112	0.394	0.058	0.318
0.70	0.204	0.502	0.164	0.462	0.116	0.408	0.060	0.330
0.80	0.214	0.524	0.170	0.482	0.122	0.428	0.062	0.344
0.90	0.222	0.546	0.178	0.504	0.126	0.446	0.066	0.362
1.00	0.234	0.574	0.188	0.530	0.134	0.470	0.070	0.382

Note: R_1^* and R_2^*—optimal class partitioning value ($R_1^* \leq R_2^*$)

3. Sensitivity analysis

The effects of b and s are investigated on the expected travel time through example problems. In this regard, 10 different configurations of the rack height and length are generated and listed in Table 3, each having the same rack area of 30624 ft^2 but different value of b. Also, four different values of s are chosen, i.e., 20%/60%, 20%/70%, 20%/80%, 20%/90%. Thus, we solve 40 problems with four storage assignment policies.

Table 3. Rack configuration

S. F (b)	Height	Length
0.10	29.33	1044.12
0.20	41.90	730.89
0.30	51.16	598.60
0.40	58.66	522.06
0.50	66.00	468.00
0.60	71.83	426.34
0.70	77.53	395.00
0.80	83.00	369.00
0.90	88.00	348.00
1.00	92.63	330.61

Suppose that the S/R machine is $s_h=356$ fpm and $s_v=100$ fpm. With H=88, L=348 and 20%/60%, we have s=0.317 from (2),

$t_h=L/s_h=348/356=0.9775$ min,
$t_v=H/s_v=88/100=0.8800$ min,
$T=\mathrm{Max}\{t_h, t_v\}==0.9775$ min,
and $b=\mathrm{Min}\{t_h, t_v\}/T==0.90$.

Utilizing T_2^i and T_3^i for the class-based storage assignment and equations (6) and (8), the expected round trip time under each assignment policy can be determined as shown in Table 4.

Table 4. The case with H=88, L=348 and 20%／60%

Assignment policy	Expected round-trip time
Random	1.241
Turnover	0.913
Two class	1.016
Three class	0.963

The above procedure is repeated for each problem and the results are listed in Table 5 for random assignment, in Table 6 for turnover-based assignment, in Table 7 for the two-class system and in Table 8 for the three-class system. To facilitate the understanding of the tables, the percentage improvement with a given value of s is determined for the case of square in time as follows:

Representing ERTT as the expected round-trip time,

$$\% = \frac{(\text{ERTT with the corresponding b value}) - (\text{ERTT with } b=1)}{\text{ERTT with } b=1}$$

The following observations can be made.

1) As expected, the expected travel time becomes minimum when the rack is square in time, i.e., b=1.
2) With any storage assignment policy with b given, it is apparent that ERTT is decreasing as s decreases, which is self evident in the light of the definition of ERTT.
3) Turnover-based assignment is known to give a minimum ERTT among all rules. With a given set of b and s values, ERTT becomes increasing under the following sequence of assignment rules, that is, turnover-based, three class, two class, and random. Considering that two class (three class) can be regarded as a special case of three class system (turnover-based), the observation above is consistent to our expectation.

Table 5. The expected round-trip time with random assignment

Shape Factor	Expected travel time
0.10	2.94268 (137.46)
0.20	2.08045 (67.89)
0.30	1.73189 (39.76)
0.40	1.54470 (24.65)
0.50	1.42419 (14.93)
0.60	1.34128 (8.24)
0.70	1.29073 (4.16)
0.80	1.25760 (1.48)
0.90	1.24146 (0.18)
1.00	1.23921 (0.00)

Table 6. The expected round-trip time with turnover-based assignment

Shape	ABC curve			
Factor	20%／60%	20%／70%	20%／80%	20%／90%
0.10	1.93940 (113.04)	1.60090 (105.23)	1.19285 (96.32)	0.68075 (86.02)
0.20	1.39970 (53.75)	1.16580 (49.45)	0.87905 (44.68)	0.50990 (39.34)
0.30	1.18900 (30.61)	0.99780 (27.91)	0.75945 (24.99)	0.44570 (21.79)
0.40	1.07860 (18.48)	0.91065 (16.74)	0.69800 (14.88)	0.41305 (12.87)
0.50	1.01175 (11.14)	0.85855 (10.06)	0.66190 (8.94)	0.39425 (7.73)
0.60	0.96480 (5.98)	0.82180 (5.35)	0.63610 (4.69)	0.38155 (4.26)
0.70	0.93760 (2.99)	0.80085 (2.67)	0.62170 (2.32)	0.37320 (1.98)
0.80	0.92105 (1.18)	0.78680 (0.87)	0.61305 (0.90)	0.36875 (0.77)
0.90	0.91275 (0.26)	0.78185 (0.23)	0.60880 (0.20)	0.36680 (0.18)
1.00	0.91035 (0.00)	0.78005 (0.00)	0.60760 (0.00)	0.36595 (0.00)

Table 7. The expected round-trip time with two class system

Shape Factor	ABC curve			
	20%/60%	20%/70%	20%/80%	20%/90%
0.10	2.183 (115.2)	1.910 (107.1)	1.564 (97.1)	1.087 (86.2)
0.20	1.569 (54.7)	1.390 (50.7)	1.169 (47.3)	0.849 (45.5)
0.30	1.339 (32.0)	1.202 (30.3)	1.022 (28.8)	0.746 (27.8)
0.40	1.220 (20.3)	1.101 (19.3)	0.940 (18.4)	0.688 (17.8)
0.50	1.141 (12.5)	1.033 (11.9)	0.884 (11.4)	0.648 (11.0)
0.60	1.085 (7.0)	0.984 (6.7)	0.844 (6.4)	0.620 (6.2)
0.70	1.050 (3.6)	0.954 (3.4)	0.820 (3.3)	0.602 (3.2)
0.80	1.027 (1.3)	0.934 (1.2)	0.803 (1.2)	0.590 (1.1)
0.90	1.016 (0.2)	0.924 (0.2)	0.795 (0.2)	0.585 (0.2)
1.00	1.014 (0.0)	0.922 (0.0)	0.794 (0.0)	0.584 (0.0)

Table 8. The expected round-trip time with three class system

Shape Factor	ABC curve			
	20%/60%	20%/70%	20%/80%	20%/90%
0.10	2.050 (113.3)	1.746 (105.5)	1.373 (97.1)	0.864 (86.1)
0.20	1.484 (54.3)	1.275 (50.1)	1.012 (45.2)	0.648 (39.6)
0.30	1.259 (30.9)	1.090 (28.3)	0.875 (25.6)	0.574 (23.5)
0.40	1.143 (18.9)	0.998 (17.5)	0.810 (16.3)	0.535 (15.3)
0.50	1.073 (11.6)	0.942 (10.9)	0.768 (10.2)	0.509 (9.6)
0.60	1.024 (6.5)	0.901 (6.1)	0.736 (5.7)	0.489 (5.3)
0.70	0.993 (3.3)	0.876 (3.1)	0.717 (2.9)	0.477 (2.8)
0.80	0.973 (1.2)	0.859 (1.1)	0.704 (1.0)	0.469 (1.0)
0.90	0.963 (0.2)	0.851 (0.2)	0.697 (0.1)	0.465 (0.1)
1.00	0.961 (0.0)	0.849 (0.0)	0.697 (0.0)	0.464 (0.0)

4. Concluding remarks

We showed that the storage assignment rules have substantial effects on the expected travel time of the S/R machine in an AS/RS. However, the current results must not be viewed as complete since actual AS/RS are operating with discrete pallets and discrete storage location rather than in continuous mode assumed in this paper. In addition to that, storage and retrieval interleaving (dual command) must be considered for class based rule which involves a lot of complicated computational efforts.

Acknowledgement

The authors wish to thank two anonymous referees for their valuable suggestions for improvement of the original version of this paper.

References

1) Bozer, Y.A. and White, J.A., "Travel-Time Models for Automated Storage/Retrieval Systems," *IIE Transactions* 16, 4, 329–338 (1984).
2) Graves, S.C., Hausman, W.H. and Schwarz, L.B., "Storage/Retrieval Interleaving in Automated Warehousing Systems," *Management Science* 23, 9, 935–945 (May 1977).
3) Hausman, W.H., Schwarz, L.B. and Graves, S.C., "Optimal Storage Assignment in Automated Warehousing Systems," *Management Science* 22, 6, 629–638 (Feburary 1976).
4) Hwang, H. and Ko, C.S., "A Study on Multi-aisle System Served by a Single Storage/ Retrieval Machine," *International Journal of Production Research* (To appear).