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Periodic Review Inventory Model for Deteriorating Items with Partial Returns and Additional Orders

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Abstract

A periodic review inventory model for deteriorating items in which time is treated as a discrete variable is developed. The model is developed under deterministic but time dependent demands and instantaneous delivery. Deterioration is assumed to be a constant fraction of the on hand inventory and partial returns are allowed for the deteriorated items. The solution procedures for obtainting the optimal order quantities which maximize the total profit in the scheduling period are presented for the cases of back orders and lost sales.

Finally, when the additioal orders are allowed, an efficient solution algorithm determining the initial and additional order quantities and additional ordering time is developed. Some numerical examples are also presented to illustrate the results.

1. Introduction

Efforts in analysing mathematical models of inventory in which a constant or variable proportion of the on hand inventory deteriorates per time unit have been made. Ghare and Schreder [1] have developed an EOQ model for exponentially decaying inventory. Covert and Philip [2] and Philip[3] have devloped EOQ model for items with variable rate of deterioration by assuming Weibull density function for the time of deteriortion of an item. This work has been generalized by Shah [4] by allowing shortages and considering general deterioration function. Misra [5] has developed a deterministic model with a finite production rate which has been generalized by Shah and Jaiswal [6] to allow shortages. Some probabilistic models have been developed by Shah and Jaiswal [7].

In all the above models, time is treated as a continuous variable which is not exactly the case in practice. In real life problems time is often treated as a discrete variable, i.e. in

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terms of complete units of days, weeks, months, etc. Dave [8] has developed a discrete-in-time order-level inventory model for deteriorationg items and Rengarajan and Vartak [9] have generalized Dave's model by considering a variable known demand.

In all the previous models, the remaining value for deteriorated items is not considered. Sorai, Arizono and Ohta [10] have generalized the single period inventory model known as the newsboy probem. Even though it was not deteriorating inventory model, they have considered the partial returns and additional orders.

In this paper a periodic review inventory model for deteriorating items in which time is treated as a discrete variable is devloped. The model is devloped under deterministic but time dependent demands and instantaneous delivery. Deterioration is assumed to be a constant fraction of the on hand inventory and the partial returns are allowed for the deteriorated items. The solution procedure for the obtaining the optimal order quantities which maximize the average total profit per unit time are presented for the cases of back orders and lost sales.

Finally, when the additinal orders are allowed, the efficient solution algorithm determining the initial and additional order quantities and additional ordering time is derived.

2. Mathematical Model

The following assumputions are made to develop the model :

1. A finite scheduling period is devided into some sub-intervals of unit duration each.
2. The demand rates in each sub-intervals is known.
3. There is no replenishment lead time.
4. A constant fraction of on hand inventory at the begining of each time units deteriorates per unit time.
5. The deteriorated items are returned or removed at the end of each sub-intervals.
6. The limit of partial returns are proportional to the order quantitits.

The following notations are used throughout this paper :

T : Scheduling period (T, a positive integer)
θ : deteriorating rate ($0 \leqq \theta \leqq 1$)
D_j : demand in j^{th} sub-interval ($j = 1, 2, \cdots, T$)
C : cost per unit item
C_v : selling price per cuit ($C_v > C$)
C_h : inventory holding cost per unit per unit time
C_s : shortage cost per unit per unit time
C_r : returning value per unit ($C_r < C$)
a : limit of returning proportion ($0 \leqq a \leqq 1$)
S_i : inventory level at time point i ($i = 1, 2, \cdots, T$)

2.1 Back orders case

As shown in Fig. 1, the system starts with the inventory level of So and this amount is reduced by the demand and deterioration. The inventory level comes to zero at time $t = t_1$ and the demands occuring after the time t_1 are backlogged and are fulfilled by a new procurement. Since the order quantity $Q(t_1)$ should raise the initial inventory level to So,

$$Q(t_1) = S_o + \sum_{j=t_1+1}^{T} D_j \tag{1}$$

At time t_1, the inventory level is zero,
i.e.

$$S_{t_1} = 0.$$

But

$$S_{t_1} = (1-\theta) S_{t_1-1} - D_{t_1}$$

i.e.

$$S_{t_1-1} = D_{t_1}(1-\theta)^{-1}.$$

Similarly we can write

$$S_{t_1-2} = D_{t_1}(1-\theta)^{-2} + D_{t_1-1}(1-\theta)^{-1}$$

$$S_{t_1-3} = D_{t_1}(1-\theta)^{-3} + D_{t_1-1}(1-\theta)^{-2} + D_{t_1-2}(1-\theta)^{-1}$$

$$\vdots$$

$$S_0 = D_{t_1}(1-\theta)^{-t_1} + D_{t_1-1}(1-\theta)^{-(t_1-1)} + \cdots + D_1(1-\theta)^{-1}$$

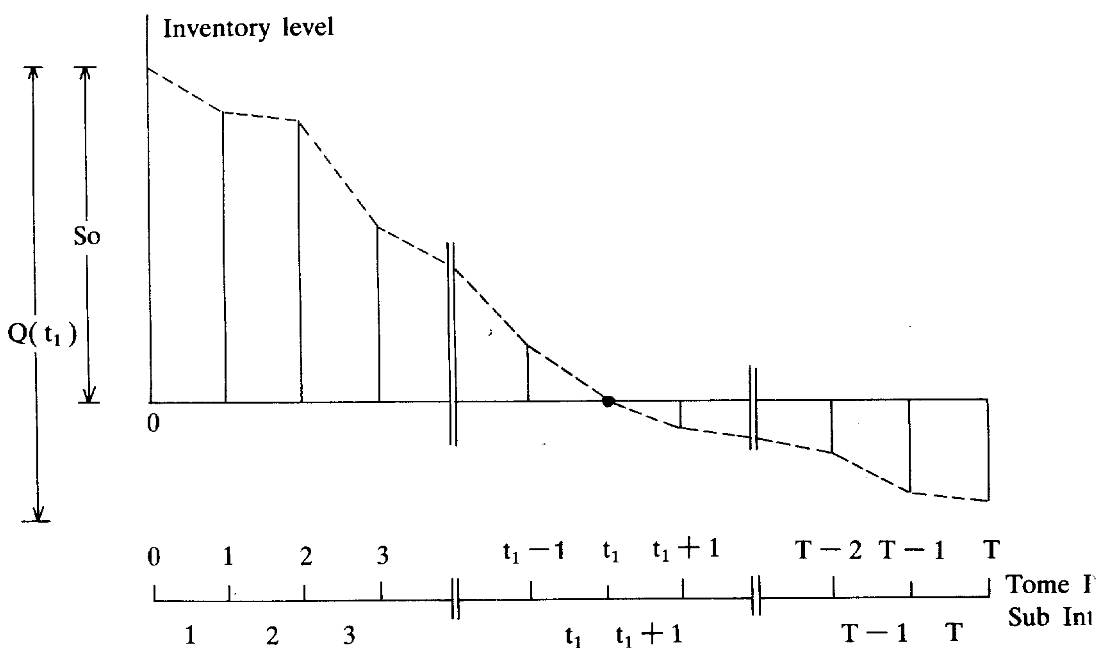

Fig.1. Back orders case

The total number of units that deteriorate during scheduling period $L(t_1)$ is θ (S_0 + S
+ ··· +, S_{t_1-1})

$$L(t_1) = \theta \sum_{i=1}^{t_1} \left(D_i \sum_{j=1}^{i} (1-\theta)^{-j} \right) \qquad ($$

The sum of average inentory levels in each sub-intervals is

$$I(t_1) = \frac{1}{2}(S_0 + 2S_1 + 2S_2 + \cdots + 2S_{t_1-1})$$

$$= \frac{1}{2}\sum_{j=1}^{t_1} D_j (1-\theta)^{-j} + \sum_{i=2}^{t_1} (D_i \sum_{j=1}^{i-1} (1-\theta)^{-j}) \tag{3}$$

and the sum of average shortages in each sub-intervals is

$$S(t_1) = \frac{1}{2}\sum_{j=t_1+1}^{T} D_j + \sum_{j=t_1+1}^{T} D_j (T-j). \tag{4}$$

From (1), the order quantity $Q(t_1)$ is given by

$$Q(t_1) = \sum_{j=1}^{t_1} D_j (1-\theta)^{-j} + \sum_{j=t_1+1}^{T} D_j. \tag{5}$$

Then using (2), (3), (4) and (5) the total average profit per unit time during scheduing period TAP (t_1) is given by

$$TAP(t_1) = \begin{cases} \frac{1}{T}\{C_v \sum_{j=1}^{T} D_j - C_h I(t_1) - C_s S(t_1) - (C - C_r a) Q(t_1)\}, & L(t_1) > a Q(t_1) \quad (6) \\ \frac{1}{T}\{C_v \sum_{j=1}^{T} D_j - C_h I(t_1) - C_S S(t_1) - CQ(t_1) + C_r L(t_1)\}, & L(t_1) \leq a Q(t_1) \quad (7) \end{cases}$$

where $aQ(t_1)$ represents the limit of returns for deteriorated items and $c_v \sum_{j=1}^{T} D_j$ is total sales during scheduling period.

Since t_1 is an integer, the optimal value of t_1 should satisfy the following conditions:

$$TAP(t_1^* + 1) - TAP(t_1^*) \leq 0 \tag{8}$$

$$TAP(t_1^* - 1) - TAP(t_1^*) \leq 0 \tag{9}$$

where t_1^* is the optimal value of t_1.

Using (6) in the conditions (8) and (9), we get

$$\frac{D_{t_1+1}}{T}\{-C_h \sum_{j=1}^{t_1} (1-\theta)^{-j} + (1-\theta)^{-(t_1+1)} (-\frac{C_h}{2} - C + C_r a)$$

$$+ \frac{C_s}{2} - C_s t_1 + C_s (T-1) + (C - C_r a)\} \leq 0 \tag{10}$$

and

$$\frac{D_{t_1}}{T}\{C_h \sum_{j=1}^{t_1-1} (1-\theta)^{-j} + (1-\theta)^{-t_1} (\frac{C_h}{2} + C - C_r a)$$

$$- \frac{C_s}{2} + C_s t_1 - C_s T - (C - C_r a)\} \leq 0. \tag{11}$$

Similary using (7) in the conditions (8) and (9), we get

$$\frac{D_{t_1+1}}{T} \{ (-C_h + C_r\theta) \sum_{j=1}^{t_1} (1-\theta)^{-j} + (1-\theta)^{-(t_1+1)} (-\frac{C_h}{2} - C + C_r\theta) + \frac{C_s}{2} - C_s t_1 + C_s(T-1) + C \} \leq 0 \quad (12)$$

and

$$\frac{D_{t_1}}{T} \{ (C_h - C_r\theta) \sum_{j=1}^{t_1-1} (1-\theta)^{-j} + (1-\theta)^{-t_1} (\frac{C_h}{2} + C - C_r\theta) - \frac{C_s}{2} + C_s t_1 - C_s T - C \} \leq 0. \quad (13)$$

Since all $D_j \geqq 0$, the conditions (10), (11), (12) and (13) are independent of D_j and hold good for any D_j.

The optimality conditions (10) and (11) are simplified to

$$M_1(t_1 - 1) \leq M_1 \leq M_1(t_1) \quad (14)$$

where

$$M_1 = \{ C_s + 2 (C - Cra) \} / 2$$

and

$$M_1(t_1) = C_h \{ (1-\theta)^{-t_1} - 1 \} / \theta + (1-\theta)^{-(t_1+1)} (\frac{C_h}{2} + C - C_r a) - C_s (T - t_1 - 1).$$

Similary the optimality conditions (12) and (13) are simplified to

$$M_2(t_1 - 1) \leq M_2 \leq M_2(t_1) \quad (15)$$

where

$$M_2 = (C_s + 2C) / 2$$

and

$$M_2(t_1) = (C_h + C_r\theta) \{ (1-\theta)^{-t_1} - 1 \} / \theta + (1-\theta)^{-(t_1+1)} (\frac{C_h}{2} + C - C_r\theta) - C_s (T - t_1 - 1).$$

2.2 Lost Sales Case

In this case, the system starts with the inventory level of S_0 and the demands occuring after the time t_1 becomes lost sales. Since the order quantity $Q(t_1)$ is equal to the initial inventory level,

$$Q(t_1) = S_0 = \sum_{j=1}^{t_1} D_j (1-\theta)^{-j}$$

The sum of shortages during scheduling period is

$$S(t_1) = \sum_{j=t_1+1}^{T} D_j$$

$L(t_1)$ and $I(t_1)$ are represented the same form as (2) and (3) respectively.

Then the total average profit per unit time during scheduing period TAP (t_1) is given by

$$TAP(t_1) = \begin{cases} \frac{1}{T} \{ Cv \sum_{j=1}^{t_1} D_j - C_h I(t_1) - C_s S(t_1) - (C - C_r a)\, Q(t_1) \}, & L(t_1) > aQ(t_1) \quad (16) \\ \frac{1}{T} \{ C_v \sum_{j=1}^{t_1} D_j - C_h I(t_1) - C_s S(t_1) - C\, Q(t_1) + C_r L(t_1) \}, & L(t_1) \leq aQ(t_1) \quad (17) \end{cases}$$

Using (16) in the conditions (8) and (9), we can obtain the simplified optimality conditions as follows :

$$M_1(t_1 - 1) \leq M_1 \leq M_1(t_1) \qquad (18)$$

where

$$M_1 = C_v + C_s$$

and

$$M_1(t_1) = C_h \{(1-\theta)^{-t_1} - 1\} / \theta + (\frac{C_h}{2} + C - C_r a)(1-\theta)^{-(t_1+1)}$$

Similary using (17) in the conditions (8) and (9), we get

$$M_2(t_1 - 1) \leq M_2 \leq M_2(t_1) \qquad (19)$$

where

$$M_2 = C_v + C_s$$

and

$$M_2(t_1) = (C_h - C_r\theta) \{(1-\theta)^{-t_1} - 1\} / \theta + (\frac{C_h}{2} + C - C_r\theta)(1-\theta)^{-(t_1+1)}$$

2.3 Additional Orders Case

In this case as shown in Fig. 2, the inventory level comes to zero at time t_1 and the additional order are allowed at the same time. At time t_2, the inventory level comes to zero again.

Let C_1 and C_2 be the cost per unit item and the cost per unit for additional order respectively. If $C_2 - C_v > C_s$, then we do not additional order at time t_1, because the loss per unit item is greater than the shortage cost per unit. Hence, there is the relationship among the cost terms.

$$C_r \leqq C_1 \leqq C_2 \leqq C_v + C_s$$

At the time t_2, the inventory level is zero,

i.e.

$$S_{t_2} = 0.$$

But

$$S_{t_2} = (1-\theta) S_{t_2-1} + D_{t_2}$$

i.e.

$$S_{t_2-1} = D_{t_2}(1-\theta)^{-1}.$$

Similary we can write

$$S_{t_2-2} = D_{t_2}(1-\theta)^{-2} + D_{t_2-1}(1-\theta)^{-1}$$

$$S_{t_2-3} = D_{t_2}(1-\theta)^{-3} + D_{t_2-1}(1-\theta)^{-2} + D_{t_2-2}(1-\theta)^{-1}$$

$$\vdots$$

$$S_{t_1+1} = D_{t_2}(1-\theta)^{-(t_2-t_1-1)} + D_{t_2-1}(1-\theta)^{-(t_2-t_1-2)} + \cdots\cdots + D_{t_1+2}(1-\theta)^{-1}$$

$$S_{t_1} = D_{t_1}(1-\theta)^{-(t_2-t_1)} + D_{t_2-1}(1-\theta)^{-(t_2-t_1-1)} + \cdots\cdots + D_{t_1+1}(1-\theta)^{-1}$$

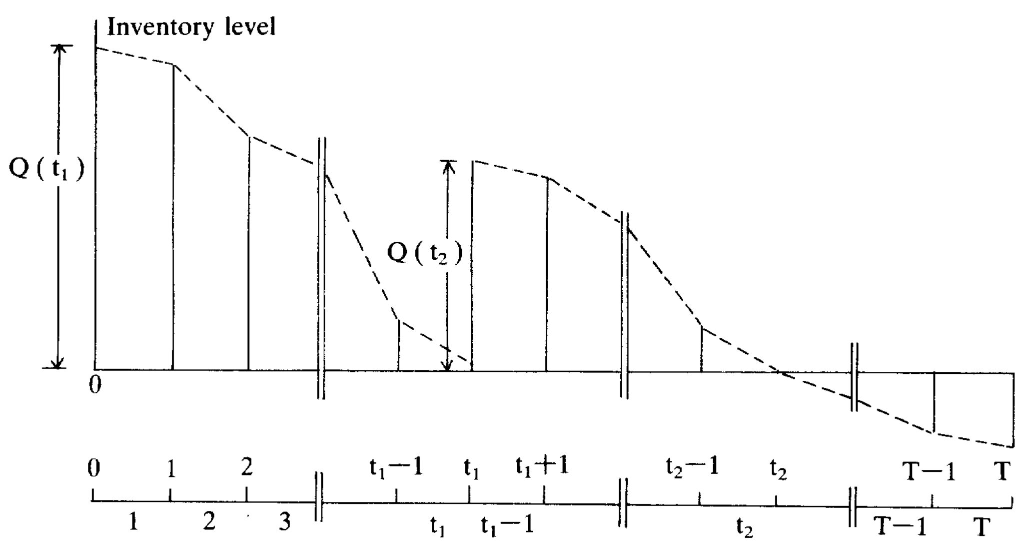

Fig.2. Additional Orders Case

The number of units that deteriorate until time t_1, $L(t_1)$, is given by (2) and the number of units that deteriorate between time t_1 and t_2, $L(t_2)$, is given by

$$L(t_2) = \theta\,(S_{t_1} + S_{t_1+1} \cdots\cdots + S_{t_2-1})$$

$$= \theta \sum_{i=t1+1}^{t2} (D_i \sum_{j=1}^{i-t_1} (1-\theta)^{-j})$$

The initial order quantity is

$$Q(t_1) = S_0 = \sum_{j=1}^{t_1} D_j (1-\theta)^{-j}$$

and the additional order quantity is

$$Q(t_2) = S_{t_1} = \sum_{j=t_1+1}^{t_2} D_j (1-\theta)^{-(j-t_1)}$$

The sum of average inventory levels in each sub-intervals until time t_1 is given by (3) and the sum of average inventrory levels in each sub-intervals between time t_1 and t_2, $I(t_2)$, becones

$$I(t_2) = \frac{1}{2}(S_{t_1} + 2S_{t_1+1} + \cdots\cdots + 2S_{t_2-1})$$

$$= \frac{1}{2} \sum_{j=t_1+1}^{t_2} D_j (1-\theta)^{-(j-t_1)} + \sum_{i=t_1+2}^{t_2} (D_i \sum_{j=1}^{i-t_1-1} (1-\theta)^{-j}).$$

Since the demands occuring after the time t_2 becomes lost sales, the total number of shortages during scheduling period is

$$S(t_2) = \sum_{j=t_2+1}^{1} D_j$$

In this case, we can obtain th following four types of total average profit per unit time $TAP(t_1, t_2)$ according to the limits of returns for deteriorated items.

1) $L(t_1) > aQ(t_1)$ and $L(t_2) > aQ(t_2)$

$$TAP(t_1, t_2) = \frac{1}{T} [C_v \sum_{j=1}^{t_2} D_j - C_h \{I(t_1) + I(t_2)\} - C_s S(t_2)$$
$$- C_1 Q(t_1) - C_2 Q(t_2) + C_r a \{Q(t_1) + Q(t_2)\}]$$

2) $L(t_1) > aQ(t_1)$ and $L(t_2) \leqq aQ(t_2)$

$$TAP(t_1, t_2) = \frac{1}{T} [C_v \sum_{j=1}^{t_2} D_j - C_h \{I(t_1) + I(t_2)\} - C_s S(t_2)$$
$$- C_1 Q(t_1) - C_2 Q(t_2) + C_r a \{aQ(t_1) + L(t_2)\}]$$

3) $L(t_1) \leqq aQ(t_1)$ and $L(t_2) > aQ(t_2)$

$$TAP(t_1, t_2) = \frac{1}{T}\Big[C_v \sum_{j=1}^{t_2} D_j - C_h \{I(t_1) + I(t_2)\} - C_s S(t_2)$$

$$- C_1 Q(t_1) - C_2 Q(t_2) + C_r \{L(t_1) + aQ(t_2)\}\Big]$$

4) $L(t_1) \leqq aQ(t_1)$ and $L(t_2) \geqq aQ(t_2)$

$$TAP(t_1, t_2) = \frac{1}{T}\Big[C_v \sum_{j=1}^{t_2} D_j - C_h \{I(t_1) + I(t_2)\} - C_s S(t_2)$$

$$- C_1 Q(t_1) - C_2 Q(t_2) + C_r \{L(t_1) + L(t_2)\}\Big]$$

3. Solution Procedure

3.1. Back Orders and Lost Sales Cases

For the Back orders case, using optimality conditions (14) and (15), we can derive the algorithm 1 to find a optimal order quantity and this procedure is described in detail in Fig. 3.

Algorithm 1. Solution Procedure for Back Orders Case

Step 1. Compute M_1 and $M_1(t)$, $t = 1, 2, \cdots$, and find t_1 that satisfies the condition (14).

Step 2. Compute $L(t_1)$ from (2) and $Q(t_1)$ from (5).

Step 3. If $L(t_1) > aQ(t_1)$, then stop.
Otherwise, go to step 4.

Step 4. Compute M_2 and $M_2(t)$, $t = 1, 2, \cdots$, and find t_1 that satisfies the condition (15).

Step 5. Compute $L(t_1)$ from (2) and $Q(t_1)$ from (5).

For the lost sales case, using optimality conditions (18) and (19) and the identical procedure with algorithm 1, we can obtain the optmal ordering policy.

3.2 Additional orders Case

In this case, it is very hard to determine the time t_1 and t_2 which maximize $TAP(t_1. t_2)$ because of the interaction of the cost terms according to the changes of t_1 and t_2.

Here, we propose the algorithm 2 to obtain the good ordering policies for this case.

Algorithm 2. Solution Procedure for Additional Orders Case

Step 1. Find t_1 and t_2 independently by the same manner as the lost sales case and compute $TAP(t_1, t_2)$.

Step 2. Store the value of t_1 and t_2 into t_1^0 and t_2^0 repectively.

Step 3. Move t_1 forward by unit time ($t_1 \leftarrow t_1 - 1$) and Compute $TAP(t_1, t_2)$

Step 4. If $TAP(t_1, t_2)$ increments, then go to step 3.

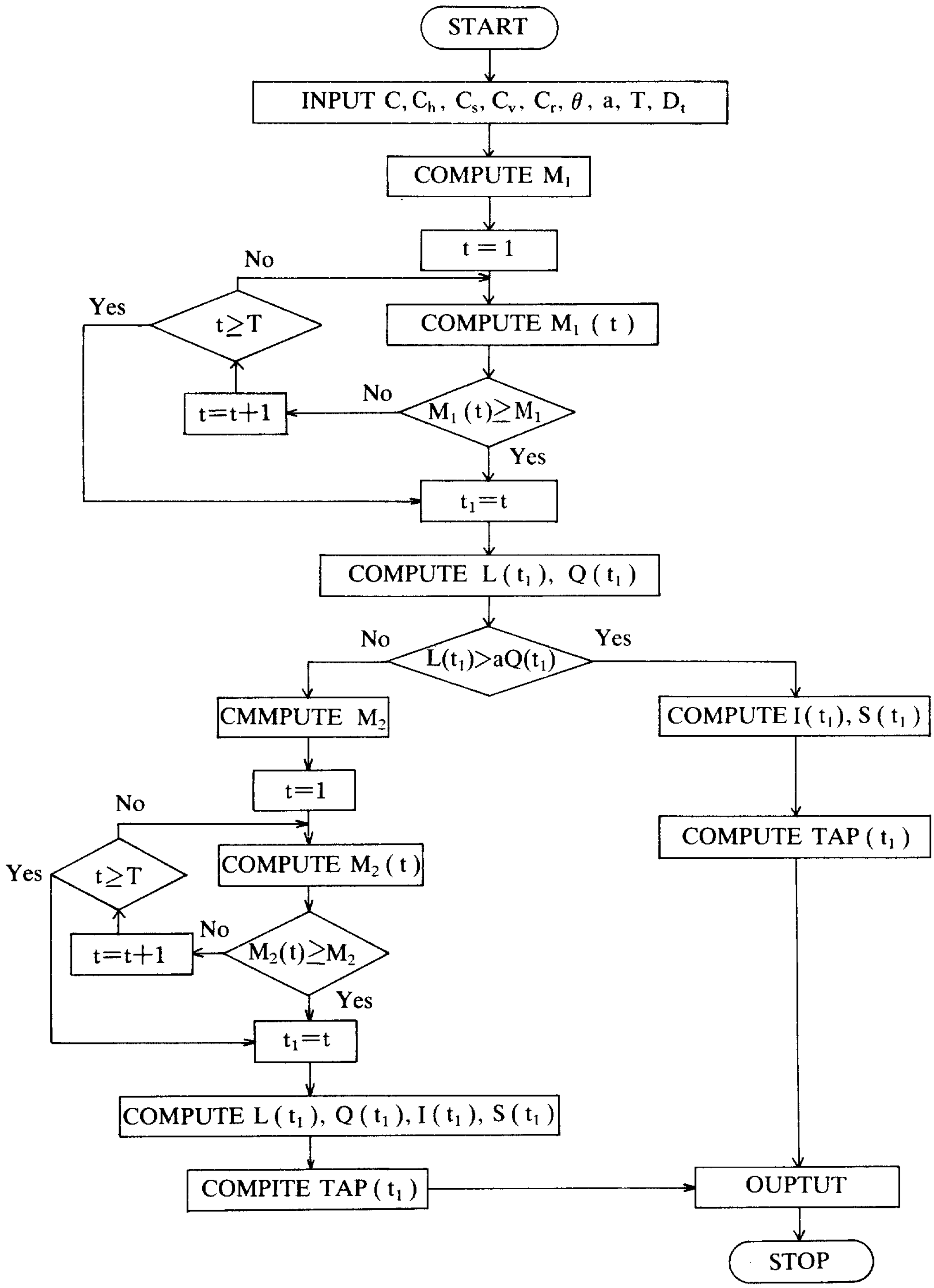

Fig.3. Solution Procedure for Back orders case

Otherwise, if step 2 is executed once only, then $t_1 \leftarrow t_1 + 1$, compute TAP t_1, t_2) and go to step 5, otherwise, $t_1 \leftarrow t_1 + 1$ compute TAP (t_1, t_2) and g to step 7.

Step 5. Move t_1 backward by unit time ($t_1 \leftarrow t_1 + 1$) and compute TAP (t_1, t_2).

Step 6. If TAP (t_1, t_2) increments, then go to step 5.
Otherwise, go to step 7.

Step 7. If $t_1 = t_1^0$, then stop.
Otherwise, go to step 8.

Step 8. Move t_2 forward by unit time ($t_2 \leftarrow t_2 - 1$) and comput TAP (t_1, t_2).

Step 9. If TAP (t_1, t_2) increments, then go to step 8.
Otherwise, if step 8 is executed once only, then $t_2 \leftarrow t_2 + 1$, compute TAP t_1, t_2) and go to step 10,
otherwise, $t_2 \leftarrow t_2 + 1$, compute TAP (t_1, t_2) and go to step 12.

Step 10. Move t_2 backward by unit time ($t_2 \leftarrow t_2 + 1$) and compute TAP (t_1, t_2)

Step 11. If TAP (t_1, t_2) increments, then go to step 10
Otherwise, go to step 12.

Step 12. If $t_2 = t_2^0$, then stop.
Otherwise, go to step 2.

4. Numerical Examples

To illstrate the computational scheme developed, some numerical examples are con dered for the back orders and aditional orders cases.

Example 1. Back Orders Cases

The input data used for this example is given below:

$D_i = 200$ units/day ($1 = 1, 2, \cdots, T$), $C = \$80$/unit,

$C_h = \$1$/unit/day, $C_s = \$9$/unit/day, $C_v = \$90$/unit

$C_r = \$60$/unit, $T = 12$ days, $\theta = 0.05$ and $a = 0.2$.

The value of $M_1 = 72.5$ and the values of $M_1(t)$ for $t = 1, 2, 3, \cdots, 7, \cdots$ are

$-13.05, 1.06, 15.43, 30.08, 45.03, 60.30, 75.89, \cdots\cdots$.

Here 72.5 lies between $M_1(6)$ and $M_1(7)$ so that $t_1 = 7$.

Using (2) and (5),

$L(7) = 328$ units and $aQ(7) = 546$.

Since this result does not satisfy (6), then we must continue to execute the next step. T value of $M_2 = 84.5$ and the values of $M_2(t)$ for $t = 1, 2, 3, \cdots, 9$, are

$-6.32, 5.07, 16.50, 28.05, 39.73, 51.56, 63.54, 75.67, 87.97, \cdots$.

Here 84.5 lies between $M_2(8)$ and $M_2(9)$ so that $t_1^* = 9$

Therefore,

$L(9) = 547$ units, $A(9) = 2947$ units and $TAP(9) = 851.06$

Example 2. Additioal Orders Case

The input data used for this example is given below:

$C_1 = \$80$/unit, $C_2 = \$90$/unit, $C_h = \$3$/unit/day,

$C_s = \$5$/unnit/day, $C_v = \$100$/unit, $C_r = \$70$/unit,

$\theta = 0.04$ and $a = 0.2$.

The demand pattern in scheduling period is shown in Table 1.

Table. 1 Demand Pattern

Time Unit	1	2	3	4	5	6	7	8	9	10	11	12
Demand	200	300	250	200	250	300	250	200	200	250	300	250

Table. 2 Changes of t_1, t_2 and TAP (t_1, t_2)

t_1	t_2	TAP (t_1, t_2)
6	12	562.616
5	12	247.483
7	12	720.759
8	12	776.224
9	12	671.924
8	12	776.224
8	11	777.985
8	10	670.016
8	11	777.985
7	11	818.081
6	11	759.480
7	11	818.081
7	10	820.198
7	9	730.223
7	10	820.198
6	10	876.268
5	10	784.275
6	10	867.268
6	9	878.030
6	8	806.052
6	9	878.030
5	9	881.597
4	9	646.641
5	9	881.597
5	8	883.009
5	7	811.029
5	8	883.009
4	8	724.499
6	8	806.052
5	8	883.009

Table 2 repersents the changes of t_1, t_2 and TAP (t_1, t_2) according to the algorithm 2 We can obtain the following results :

$t_1 = 5$, $t_2 = 8$. $L(t_1) = 159$, $L(t_2) = 60$, $Q(t_1) = 1359$,

$Q(t_2) = 810$ and $TAP(t_1, t_2) = 883.01$

5. Conclusion and Extensions

Many of the inventory system dealing with food items, food grains, chemicals, pertroleum product, etc. can be tackled by our model, in which the production or replenishment is measured per hour, per day, per week, etc.

In this paper we derived the solution procedures for obtaining optimal order quantities for cases of back orders and lost sales when the parital returns for deteriorated items are allowed. We also showed that the optimlity conditions for initial stock do not depend on the nature of demand, Futhermore for the case of additional orders, we proposed the efficient solution procedure to determine the additional ordering time and quantity.

One important but rather difficult extension might be consider the situation in which the order quantity Q is restricted to dicrete units. Another would be to consider te case of the probabilistic demand.

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