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Heuristic Procedure on General n/m Job-Shop Scheduling Generation

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ABSTRACT

The general *n/m* job-shop problem is easy to state what is required, but it is extremely difficult to make any progress whatever toward a solution.

This paper was first to examine a heuristic procedure of general *n/m* scheduling generation, focused on the procedure of MWRK (Most Work Remaining) presented by Giffler and Thompson (1960) among others. Then modified procedure was proposed to obtain better solution in light of the key measure of performance compared with that of the literature presented by Baker (1974). The modified procedure then has been extended to other example problem to test the better results and to assure the properness of application.

1. INTRODUCTION

The general job-shop problem is fascinating challenge, many proficent people have considered the problem, and most of them have come away essentially empty-handed (1967). Since this frustration is not reported in the literature, the problem continues to attract investigators, who just can not believe that a problem so simply structured can be so difficult, until they have tried it. Jeremiah, Lalchandani, and schrage (1964) selected procedures for the set of "schedulable" operations as the following:

RANDOM: Select operation at random.

MOPNR : Select operation for the job with the largest number of operation remaining to be processed.

MWKR-P : Select operation for the job that has the most processing-time on operations subsequent to the "schadulable" operation.

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MWKR/P : Select operation for the job that has the grestest ratio between work remaining to be done and the processing-time of the "schedulable" operation.
SPT : Select operation which has the shortest processing-time.
MWKR : Select operation for the job that has the most work remaining.
LPT : Select operation which has the longest processing-time.
FCFS : Select operation which first-come, first-served.

Given a patial schedule for any job shop problem, a unique corresponding set of schedulable operations may be constructed.

let

PS_t = A partail schedule containing 't' scheduled operation.
S_t = The set of schedulable operation at stage 't', corresponding to a given 'PS_t'
σ_j = The earliest time at which operation $j \in S_t$ could be started.
ϕ_j = The earliest time at which operation $j \in S_t$ could be completed.

For a given active patial schedule, the potential start time σ_j is determined by the completion time of the direct processor of operation 'j' and the latest completion time on the machine required by operation 'j' The lager of these two quantities is 'σ_j'. The potential finish time 'ϕ_j' is simply '$\sigma_j = t_j$'. where 't_j' is the processing time of operation 'j'.

Baker [1] illustrated the calculation of example problem for Table 1 according to schedule generation algorithm with the priority dispatching rule first on minimum 'σ_j' and then produced by the MWRK heuristic procedure with SPT for tie breaking rule.

Table 1.

a. Processing time

Job	Operation 1	Operation 2	Operation 3
1	4	3	2
2	1	4	4
3	3	2	3
4	3	3	1

b. Routing

Job	Operation 1	Operation 2	Operation 3
1	1	2	3
2	2	1	3
3	3	2	1
4	2	3	1

Source: Baker, "Introduction to Sequencing and Scheduling," John Wiley & Sons, New York, (1974), p. 180.

The complete set of calculation was shown in Table 2 and the complete schedule was shown as Figure 1.

The results were as follows.

$$M = 14$$

$$\bar{F} = 1/n \sum_{j=1}^{4} F_j = \frac{14 + 12 + 11 + 12}{4} = 12.25$$

$$I = \sum_{i=1}^{3} \sum_{j=1}^{4} I_{ij} = 1 + 1 = 2$$

* p. 169, and p. 198 in (1964)

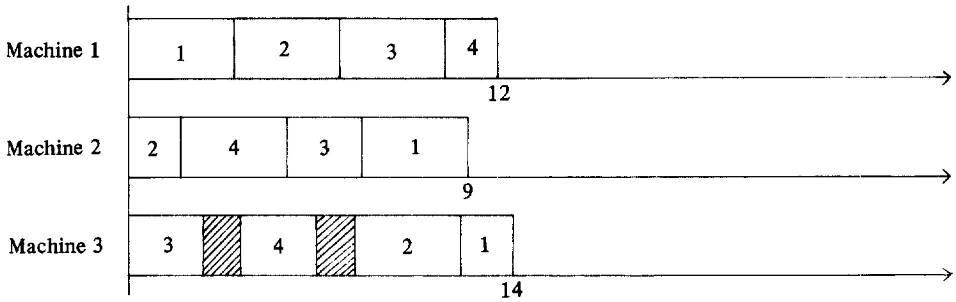

Figure 1. A complete schedule produced by the MWRK heuritic procedure.

Source: p. 201 Baker (1974), Introduction to Sequencing and Scheduling.

Table 2. Frequently in the tree of nondelay schedules than in the tree of active schedules.

Stage	(f_1, f_2, f_3)	$j \in S_{t-1}$	σ_j	MWKR Priority
1	(0, 0, 0)	111	0	9
		212	0	9*
		313	0	8
		412	0	7
2	(0, 1, 0)	111	0	9*
		221	1	–
		313	0	8
		412	1	–
3	(4, 1, 0)	122	4	–
		221	1	–
		313	0	8*
		412	1	–
4	(4, 1, 3)	122	4	–
		221	4	–
		322	3	–
		412	1	7*
5	(4, 4, 3)	122	4	5
		221	4	8*
		322	4	5
		423	4	4
6	(8, 4, 3)	122	4	5
		233	8	–
		322	4	5*
		423	4	4
7	(8, 6, 3)	122	6	–
		233	8	–
		331	8	–
		423	4	4*
8	(8, 6, 7)	122	6	5*
		233	8	–
		331	8	–
		431	8	–
9	(8, 9, 7)	133	9	–
		233	8	4*
		331	8	3
		431	8	1
10	(8, 9, 12)	133	12	–
		331	8	3*
		431	8	1
11	(11, 9, 12)	133	12	–
		431	11	1*
12	(12, 9, 12)	133	12	2*

Source: Baker Introduction to Sequencing and Scheduling. John Wiley & Sons, New York, (1974), p. 199.

2. Modified Procedure of the heuristic schedule Generation.

Modification (1).

If there is no any machine required for all jobs in the first operation, identify which jobs require the machine operation in the second stage (for example problem in Table 4, column 1 of routing indicates no machine 1 is required in the first operation. Column 2 of the routing indicates machine 1 is require for $j = [2,3]$ in the second operation), then assignment priority shall first be given to these jobs with SPT rule.

Note that the ground of this augmentation is to minimize maximum flow time. Johnson's presentation [5]:

$$F_{max} \geqq A_{[1]} + \sum_{i=1}^{n} B_{[i]}$$

where

$A_i = P_{i,1}$ the processing time (including setup, if any) of the first operation of the ith job.
$B_i = P_{i,2}$ the processing time (including setup, if any) of the second operation of the ith job.
$F_i =$ the time at which the ith job is completed.

Modification (2):

LPT is used as a tie breaker rather than SPT.

Modification (3).

examine whether there is a possible pairwise interchange between adjacent operations in order to improve solution.

According to the modified procedure the complete set of calculation for 4/3 job-shop problem of Table 1 is shown as Table 3, and the complete schedule produced by modified procedure is shown as Figure 3.

A comparison of the results are:

	M	$\bar{F}$	I
Bakeis procedure	14	12.25	2
Modified procedure	13	10.75	1

$M = 13$

$$\bar{F} = \frac{9 + 13 + 12 + 9}{4} = 10.75$$

$I = 1$

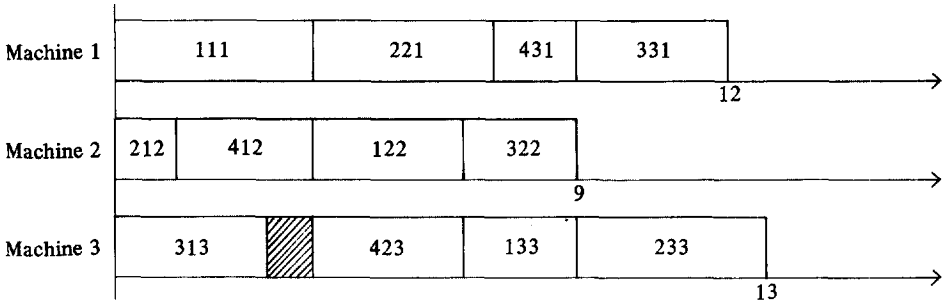

Figure 2. A complete schedule produced by modified procedure.

Table 3. (A set of calculation for 4/3 job-shop problem produced by modified procedure)

Stage	(f_1, f_2, f_3)	$j \in S_{t-1}$	t_j	σ_j	R_j	MWKR priority	Stage	(f_1, f_2, f_3)	$j \in S_{t-1}$	t_j	σ_j	R_j	MWRK priority
1	(0, 0, 0)	111	4	0	9	* LPT	7	(8, 7, 3)	133	2	7	(2)	
		212	1	0	9				233	4	8	(4)	
		313	3	0	8				322	2	7	(5)	
		412	3	0	7				423	3	4	4	*
2	(4, 0, 0)	122	3	4	(5)		8	(8, 7, 7)	133	2	7	(2)	
		212	1	0	9	*			233	4	8	(4)	
		313	3	0	8				332	2	7	5	*
		412	3	0	7				423	1	8	(1)	
3	(4, 1, 0)	122	3	4	(5)		9	(8, 9, 7)	133	2	7	2	*
		221	4	4	(8)				233	4	8	(4)	
		313	3	0	7	*			331	3	9	(1)	
		412	3	1	(7)				431	1	8	(1)	
4	(4, 1, 3)	122	3	4	(5)		10	(8, 9, 9)	233	4	9	(4)	
		221	4	4	(8)				331	3	9	(1)	
		322	2	3	(5)				431	1	8	1	*
		412	3	1	7	*	11	(9, 9, 9)	233	4	9	4	*
5	(4, 4, 3)	122	3	4	(5)				331	3	9	1	
		221	4	4	8	*	12	(9, 9, 13)	331	3	9	1	*
		332	2	4	(5)								
		423	3	4	(4)								
6	(8, 4, 3)	122	3	4	5	* LPT							
		233	4	8	(4)								
		322	2	4	5								
		423	3	4	(4)								

3. Application of modified procedure to other example problem.

Let consider other example of 5/3 job-shop problem by Bakers procedure and by modified procedure on Table 4.

Table 4. 5/4 job-shop problem example

Job	Operation 1	2	3	4
1	2	3	2	–
2	1	3	1	3
3	3	4	2	–
4	2	3	3	2
5	1	3	–	–

Job	Operation 1	2	3	4
1	2	3	1	–
2	2	1	2	3
3	3	1	2	–
4	2	3	1	2
5	3	2	–	–

Source: Conway, Maxwell, Miller, "Theory of Scheduling, Adison-Wesley Publishing Co. Reading Massachusets, (1967), p. 103.

Results produced by Baker's procedure (shown as Table 5 and Figure 3) versus by modified procedure (shown as Table 6 and Figure 4, and figure 5 which is improved by Pairwise interchange of the result of Figure 4) are the following.

A comparison of the results are:

	M	$\bar{F}$	I	
Bakers procedure	17	14.60	7	
Modified procedure	13	11.80	1	optimum solution

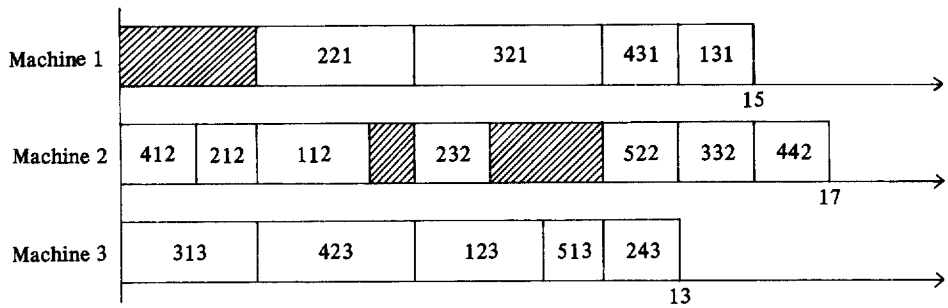

Figure 3. Complete schedule of example problem by Bakers procedure.

$M = 17, \; I = 3 + 1 + 3 = 7, \quad \bar{F} = \dfrac{15 + 13 + 15 + 17 + 13}{5} = 14.6$

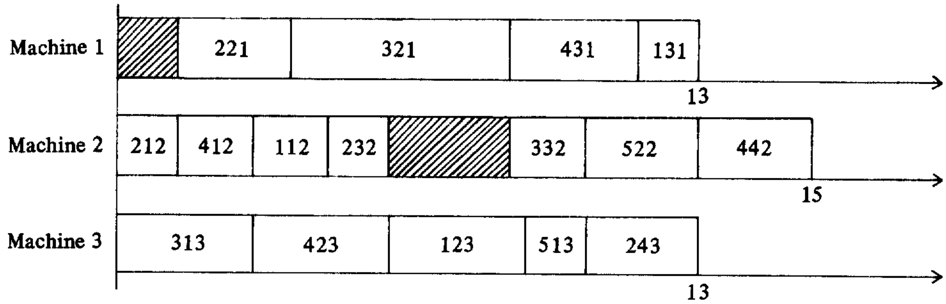

Figure 4. Schedule by modification (1) and (2), without pairwise interchange.

$M = 15, \; I = 3, \; \bar{F} = \dfrac{13 + 13 + 9 + 15 + 9}{5} = 11.8$

Pairwise interchange with 513 and 423 by modified procedure (3).

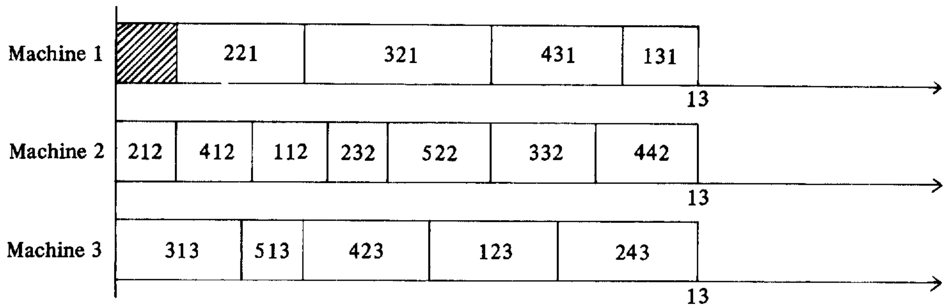

Figure 5. Complete schedule by modification (3), with pairwise interchange.

$M = 13$

$\bar{F} = \dfrac{13 + 13 + 11 + 13 + 9}{5} = 11.8$

$I = 1$

Table 5. Example problem by modified procedure

Stage	(f_1, f_2, f_3)	$j \in S_{t-1}$	t_j	σ_j	R_j	MWKR priority
1	(0, 0, 0)	112	2	0	7	
		212	1	0	8	
		313	3	0	9	
		412	2	0	10	*
		513	1	0	4	
2	(0, 1, 0)	112	2	2	(7)	
		212	1	2	(8)	
		313	3	0	9	*
		423	3	2	(8)	
		513	1	0	4	
3	(0, 1, 3)	112	2	2	7	
		212	1	2	8	*
		321	4	3	(6)	
		423	3	3	(8)	
		513	1	3	(4)	
4	(0, 3, 3)	112	2	3	7	
		221	3	3	7	
		321	4	3	6	
		423	3	3	8	*
		513	1	3	4	
5	(0, 3, 6)	112	2	3	7	* SPT
		221	3	3	7	
		321	4	3	6	
		431	3	6	(5)	
		513	1	6	(4)	
6	(0, 5, 6)	123	3	6	(5)	
		221	3	3	7	*
		321	4	3	6	
		431	3	6	(5)	
		513	1	6	(4)	
7	(6, 5, 6)	123	3	6	5	
		232	1	6	4	
		321	4	6	6	*
		431	3	6	5	
		513	1	6	4	
8	(10, 5, 6)	123	3	6	4	*
		232	1	6	4	
		332	2	10	(2)	
		431	3	10	(5)	
		513	1	6	4	
9	(10, 5, 9)	131	2	9	(2)	
		232	1	6	4	*
		332	2	10	(2)	
		431	3	10	(5)	
		513	1	9	(4)	
10	(10, 7, 9)	131	2	10	(2)	
		243	3	9	3	
		332	2	10	(2)	
		431	3	10	(5)	
		513	1	9	4	*
11	(10, 7, 10)	131	2	10	2	
		243	3	10	3	
		332	2	10	2	
		431	3	10	5	*
		522	3	10	3	
12	(13, 7, 10)	131	2	13	(2)	
		243	3	10	3	*
		332	2	10	2	
		442	2	13	(2)	
		522	3	10	3	
13	(13, 7, 13)	131	2	13	(2)	
		332	2	10	2	
		442	2	13	(2)	
		522	3	10	3	*
14	(13, 13, 13)	131	2	13	2	*
		332	2	13	2	
		442	2	13	2	
15	(15, 13, 13)	332	2	13	2	*
		442	2	13	2	
16	(15, 15, 13)	442	2	13	2	*

Table 6. Example problem by modified procedure

Stage	(f_1, f_2, f_3)	$j \in S_{t-1}$	t_j	σ_j	R_j	MWKR priority
1	(0, 0, 0)	112	2	0	(7)	
		212	1	0	8	*SPT for t_2 & t_3
		313	3	0	9	
		412	2	0	(10)	
		513	1	0	(4)	
2	(0, 1, 0)	112	2	0	7	
		221	3	1	(7)	
		313	3	0	9	*
		412	2	2	(10)	
		513	1	0	4	
3	(0, 1, 3)	112	2	1	7	
		221	3	1	7	
		321	4	3	(6)	
		423	2	1	10	*
		513	1	3	(4)	
4	(0, 3, 3)	112	2	3	(7)	
		221	3	1	7	*
		321	4	3	(6)	
		412	3	3	(8)	
		513	1	3	(3)	
5	(4, 3, 3)	112	2	3	7	
		232	1	4	(4)	
		321	4	4	(6)	
		423	3	3	8	*
		513	1	3		
6	(4, 3, 6)	112	2	3	7	*
		232	1	4	(4)	
		321	4	4	(6)	
		431	3	4	(5)	
		513	1	6	(4)	
7	(4, 5, 6)	123	3	6	(5)	
		232	1	5	(4)	
		321	4	4	6	*
		431	3	6	(5)	
		513	1	6	(4)	

Stage	(f_1, f_2, f_3)	$j \in S_{t-1}$	t_j	σ_j	R_j	MWRK priority
8	(8, 5, 6)	123	3	6	(5)	
		232	1	5	4	*
		332	2	8	(2)	
		431	3	8	(5)	
		513	1	6	(4)	
9	(8, 6, 6)	123	3	6	5	*
		243	3	6	3	
		332	2	8	(2)	
		431	3	8	(5)	
		513	1	6	4	
10	(8, 6, 9)	131	2	9	(2)	
		243	3	9	(3)	
		332	2	8	2	
		431	3	8	5	*LPT
		513	1	9	(4)	
11	(11, 6, 9)	131	2	11	(2)	
		243	3	9	(3)	
		332	2	8	2	*
		442	2	11	(2)	
		513	1	9	(4)	
12	(11, 10, 9)	131	2	11	(2)	
		243	3	9	3	
		442	2	11	(2)	
		513	1	9	4	*
13	(11, 10, 10)	131	2	11	(2)	
		243	3	10	3	*
		442	2	11	(2)	
		522	3	10	3	
14	(11, 10, 13)	131	2	11	(2)	
		442	2	11	(2)	
		522	3	10	3	*
15	(11, 13, 13)	131	2	11	2	*
		442	2	11	2	
16	(13, 13, 13)	442	2	11	2	*

4. Evalvation and conclusion

The comparison of the results produced by Baker's procedure and modified procedure for both illustrative 4/3 job-shop problem and Application of 5/3 job-shop example problem is summarized as Table 7.

Table 7. Summary results

problem	procedure	makespan	mean flow time	idle time
illustration 4/3 job-shop	Baker's modified	14 13	12.25 10.75	2 1
application 5/3 job-shop	Baker's modified	17 13	14.60 11.80	7 1

Modified procedure dominates key performance measure such as makespan, mean flow time, and total idle time of machines. One of the striking result is shown that SPT rule, which is deemed as the most powerful tools for minimizing mean flow time, is no loger maintained the crown of the tool of minimizing mean flow time as shown in this case of both illustrative and application of example problem. LPT priority took place SPT and produced better results in this case. Test of the example problem by modified procedure reached optimum solution and assured the properness of application in general *n/m* job-shop heuristic scheduling.

Therefore, conclusion is to propose that the modified procedure be used as an effective tool for general heuristic n/m job-shop scheduling generation.

REFERENCES

1. Baker, K. (1974), *"Introduction to Sequencing and Scheduling"*, John Wiley and Sons, Inc.
2. Bakhru, A.N. and Rao, M.R. (1964), *"An Experimental Investigation of Job-Shop Scheduling"*, Research Report, Department of Industrial Engineering, Cornell University.
3. Conway, R.W., Maxwell, W.L. and Louis W.Miller (1967), *"Theory of Scheduling"*, Addison-wesley Publishing Company.
4. Jeremiah, B., Lalchandani, A., and Schrage, L., (1964), *"Heuristic Rules Toward Optimal Scheduling"*, research report, Department of Industrial Engineering, Cornell University.
5. Johnson, S.M. (1954), *"Optimal Two and Three Stage Production Schedules with Setup times Included,"* Nav. Res. Log. Quart. 1, No. 1, Mar.
6. Giffler, B., and Thompson, G.L. (1960), *"Algorithm for solving production scheduling problems"* Operations Research, Vol. 8, No. 4, July.
7. Maxwell, W.L., *"Priority Dispatching and Assembly Operations in a Job-Shop,* "RM-5370-PR. RAND Corporation, Santa Monica, Cal.
8. Maxwell, L. (1970), *"On Sequencing in Jobs on one machine to minimize member of late Jobs."* Management Science, Vol. Sixteen, No. 5, January.