Journal of the KSQC
Vol. 16, No.2 1988

Design Problem of Automated Warehouse Systems Subject to Minimum Cost and Maximum Throughput

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ABSTRACT

This study is concerned with a design algorithm to minimize the investment as well as maximize the throughput in automated warehouse system.

A simulation model is designed and a solution methodology is proposed. The experiments are conducted for the cases with 100, 90, 80, 70, 60, 50 and 0 % dual command policies in terms of the important factors such as the crane velocity, the height of system and the rack utilization.

The results indicate that the throughput is slightly decreased when the ratio of dual command is decreased and the other characteristics however are not affected. The result also shows that the optimal rack should be designed for a crane to take the same amount of travel time for horizontal and vertical movement.

1. Introduction

In developed countries Automated Storage/Retrieval System (AS/RS) is widely used in warehousing and often found in manufacturing. Recently, through the use of computer control, AS/RS has been integrated into manufacturing and distribution processes.

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Automated Storage/Retrieval Systems are of strong current interest due to such benefits as lower building and land cost, labor savings, reduced inventory levels and lower incidence of misplacement or theft.

Maximal benefits of automated warehouse systems are dependent on the optimal design of the system(e.g. the number of stacker cranes, their horizontal and vertical speeds, the length and height of the storage racks, etc.) and the optimal scheduling of the system. This research is emphasized on optimal design of automated warehouse system.

The literature on automated warehouse systems contains few articles on design optimization. Hausman, Schwarz and Graves(1977 and 1978), and Bozer and White(1984) are concerned with the layout of a warehouse by minimizing travel time of handling equipment.

Roll and Rosenblatt(1984) find optimal layout designs of warehouses by minimizing warehouse construction and handling costs.

Ashayeri, Gelders and Wassenhove(1985) present a microcomputer based optimization model to minimize investment costs and operating costs. Park and Park(1988) develope an overall warehouse storage system costs model to determine the maximum inventory levels accumulated in the receiving, storage and shipping areas.

This study developes a model that allows the determination of the major design characteristics of the automated warehouse to minimize the investment as well as maximize the throughput.

2. Model Development for Optimal Automated Warehouse Design

2.1 Assumptions and Notations

The following assumptions and notations are made through this study :

- Each pallet holds only one part number or item type.
- All storage locations are the same size.
- The S/R machine operates either on single or dual command basis.
- The S/R machine travels simultaneously in the horizontal and vertical directions. In calculating the travel time, constant velocities are used for horizontal and vertical travel.
- Random storage assignment is used.
- The rack is considered to be a continuous rectangular pick face where the I/O point is located at the lower left-hand corner.
- The arrival rate and length of stay(LOS), as well as the S/R machine speed in the horizontal and vertical directions, are known.
- Pick-up and deposit(P/D) times associated with load handling are constant and known.
- Sh=horizontal travel speed(m/min)
- Sv=vertical travel speed(m/min)
- X_1=system width(m)

- X_2 = system length(m)
- L = rack length(m)
- H = rack height(m)
- $t_h = L/Sh$: time to reach the end of the rack(min)
- $t_v = H/Sv$: time to reach the top of the rack(min)
- $T = \text{Max}(t_h, t_v)$ [min] : denormalizing factor
- $b = \text{Min}(t_h, t_v)/T$ [dimensionless], $0 \leq b \leq 1$: shape factor
- $U_r = 100 * \lambda * \mu / N_r$: rack utilization
- $E(SC)$ = expected travel time under single command
- $E(DC)$ = expected travel time for a complete dual command.

2.2 Mathematical Model

The total cost function can then be stated as follows :

$$\text{Min} \ C_1 N + C_2 V + \beta C_3 N + C_4 X_1 X_2 + C_5 X_1 X_2 \tag{1}$$

Subject to

$$X_1 = (3 A_2 + A_3) * N \tag{2}$$

$$X_2 = \frac{(A_4 + A_5) * V}{2 A_6 N} + A_7 \tag{3}$$

$$Nr = 100 * \lambda * \mu / Ur \tag{4}$$

$$X_1 <= A_8 \tag{5}$$

$$X_2 <= A_9 \tag{6}$$

$$X_1, \ X_2 >= 0 \tag{7}$$

The objective function and constraints has the following characteristics.
- Crane cost : $C_1 N$ where C_1 represent the individual crane cost.
- Rack(Steel) structure cost : $C_2 V$ where V is the volume expressed in number of pallets and C_2 is the rack cost/pallet.

$$C_2 = R_0 + R_1 * (H/A_1 - n_0)$$

where
R_0 : base rack cost/pallet at number n_0
R_1 : incremental rack cost/pallet
n_0 : nominal number of pallet height(reference point)

- Input/Output buffer costs : $\beta C_3 N$ where C_3 is the cost of one position and $(\beta/2)$ is the number of input or output positions that are provided between each stacker crane and conveyor loop.
- Land cost : $C_4 X_1 X_2$ where X_1 and X_2 represent system width and length respectively. C_4 is the land price per square meter.
- Building cost : $C_5 X_1 X_2$ where C_5 is the building cost.

$$C_5 = B_0 + B_1 * (H - h_0)$$

where
  B_0 : base building cost/sq. meter at height h_0
  B_1 : incremental building cost/sq. meter
  h_0 : nominal building height (reference point)
- System width :

$$X_1 = (3 A_2 + A_3) * N$$

where
  A_2 : width of a pallet
  A_3 : clearance area for one aisle unit
- System length :

$$X_2 = \frac{(A_4 + A_5) * V}{2 A_6 N} + A_7$$

where $(A_4 + A_5)$ equals bay width, *i. e.* pallet length + clearance.
  The parameter A_6 expresses the number of unit load height cumulated.
  The parameter A_7 is the clearance area for the cranes.
- Site restriction on system width or length :

$$X_1 <= A_8, \quad X_2 <= A_9$$

- Rack utilization :

$$Ur = 100 * \lambda * \mu / Nr$$

where λ is pallet arrival rate, μ is mean LOS and Nr is number of storage locations in rack.

The throughput per unit time is

$$\text{THROUGHPUT} = [\{(60/\text{SCTIME}) * \alpha\} + \{(60/\text{DCTIME}) * \delta\} * 2] * 0.85. \quad (8)$$

where

SCTIME : single command cycle time(α : single command ratio)
DCTIME : dual command cycle time(δ : dual command ratio)

3. Solution Methodology

3.1 Determination of the Expected Travel Time [Bozer and White, 1984]

The expected single and dual cycle time represents the following equations.

$$E(SC) = 1/3 * b^2 + 1 \tag{9}$$

$$E(DC) = 4/3 + 1/2 * b^2 - 1/30 * b^3 \tag{10}$$

3.2 The Solution Procedure

A concise flow chart of the proposed procedure is shown in figure 1.

3.3 Numerical Example

The numerical example discussed here is in the design of a warehouse for palletized procedure in a distribution area. A fully Automated Storage/Retrieval System is selected in this example.

Randomized storage is used. That is, any point within the pick face is equally likely to be selected for storage or retrieval. The S/R machine operates either on a single or dual command basis.

In addition, the following input data are used.

- pallet arrival rate(λ) = 40
- rack utilization(Ur) = 90(%)
- length of stay(μ) = 40(hour)
- minimum throughput = 40(pallet/hour)
- horizontal travel speed(Sh) = 100(m/min)
- vertical travel speed(Sv) = 18(m/min)
- crane cost = ₩72,000,000
- dual command ratio = 50(%)
- life time of the project = 10(years)

The solution is illustrated in table 1.

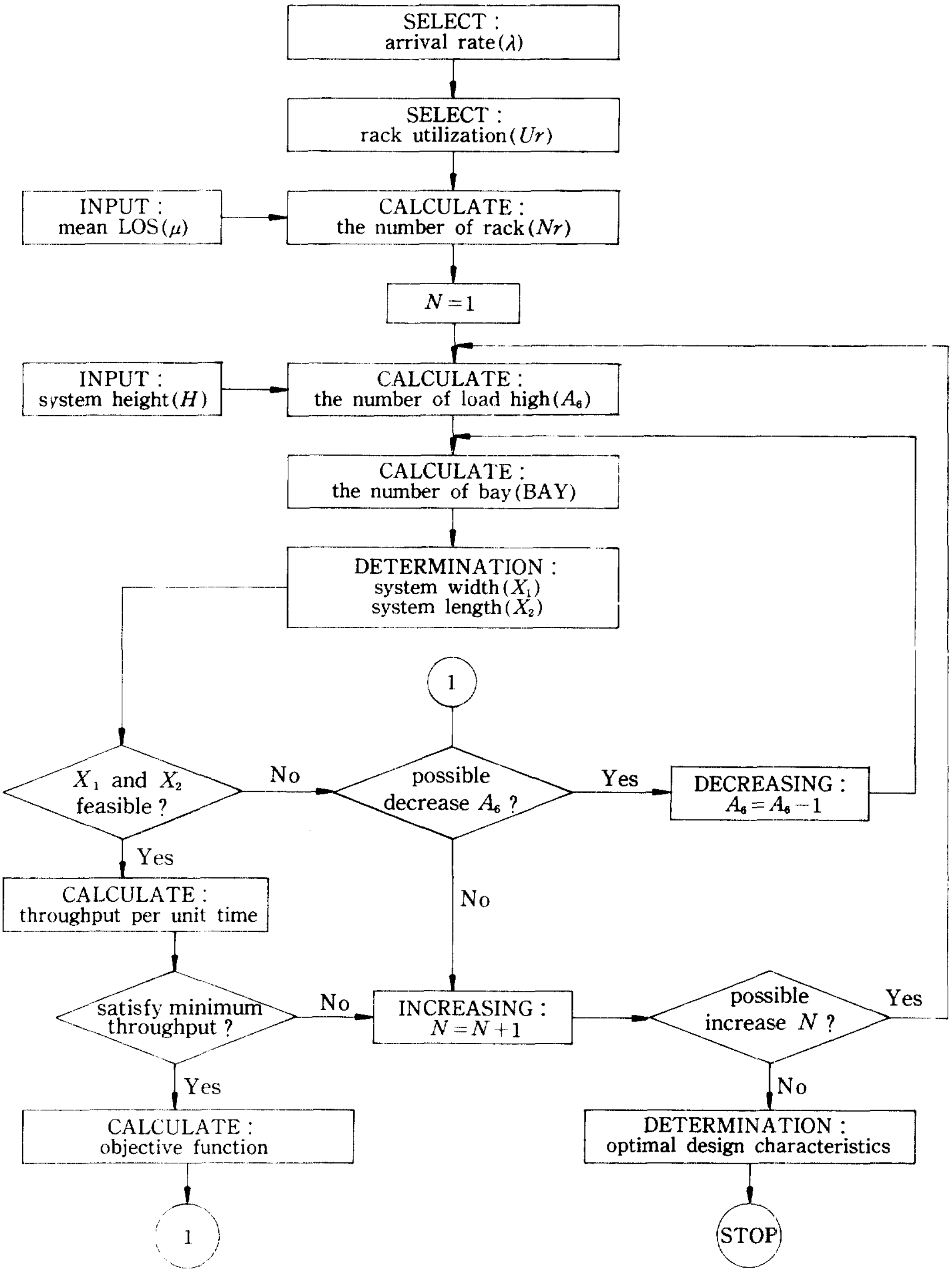

Figure 1. A flow chart of the computational procedure

Table 1. Results for numerical example

No. of Crane	No. of Rack	System width	System length	*No. of Load High	*No. of Bay	Through-put	Total cost
2	3212	7.62	100.17	11	73	56	489294700
2	3240	7.62	110.49	10	81	52	499557900
3	3234	11.43	69.21	11	49	56	566774500
3	3240	11.43	75.66	10	54	60	574695400
3	3240	11.43	83.40	9	60	64	581786300
3	3216	11.43	92.43	8	67	60	588593500
3	3234	11.43	105.33	7	77	54	605922900
4	3256	15.24	53.73	11	37	56	644254300
4	3280	15.24	58.89	10	41	60	654145000
4	3240	15.24	64.05	9	45	64	656882700
4	3264	15.24	71.79	8	51	70	669073100
4	3248	15.24	80.82	7	58	66	682606100
4	3216	15.24	92.43	6	67	60	697023000
4	3240	15.24	110.49	5	81	52	729467500
5	3300	19.05	44.70	11	30	56	724056000
5	3300	19.05	48.57	10	33	60	731438600
5	3240	19.05	52.44	9	36	64	731979000
5	3280	19.05	58.89	8	41	70	745936400
5	3220	19.05	65.34	7	46	76	754343600
5	3240	19.05	75.66	6	54	69	774979500
5	3250	19.05	89.85	5	65	61	805766200
5	3240	19.05	110.49	4	81	52	846163900
6	3300	22.86	38.25	11	25	56	799214100
6	3240	22.86	40.83	10	27	60	800107900
6	3240	22.86	44.70	9	30	64	807075400
6	3264	22.86	49.86	8	34	70	819183500
6	3276	22.86	56.31	7	39	76	835972500
6	3240	22.86	64.05	6	45	77	849973000
6	3240	22.86	75.66	5	54	69	879413200
6	3216	22.86	92.43	4	67	60	917603100

Note : No. of Load high=the height of rack/the height of one pallet
No. of Bay=the length of rack/the length of one pallet

If the system does not require increase of throughput during the project time, the warehouse with 2 cranes is selected. The overall required land is 763 square meters. The number of racks in a horizontal direction is 73 racks and the number of racks in a vertical direction is 11 layers.

If the system annually requires 5% increase of throughput for the ten years in future, the warehouse with 3 or 4 cranes is needed. For the minimum investment cost, the warehouse with

3 cranes is selected.

This system requires the following rack arrangement. The number of racks in a horizontal direction is 60 racks and the number of racks in a vertical direction is 9 racks. The throughput is 64 pallets and the total investment cost is ₩581,786,300 in this case.

3.4 Results of Experimentation

In this section the experiments are conducted for the cases with 100, 90, 80, 70, 60, 50 and 0% dual commands in terms of the important factors such as the crane velocity, the height of system and the rack utilization.

Table 2. Results for 27 cases

Case \ Ratio	100 (%)	90 (%)	80 (%)	70 (%)	60 (%)	50 (%)	0 (%)
1	3/43/6268	3/41/6268	4/49/6839	4/46/6839	4/44/6839	4/42/6839	–
2	3/47/5726	3/45/5726	3/43/5726	3/41/5726	4/45/6260	4/42/6260	–
3	3/52/5237	3/50/5237	3/48/5237	3/45/5237	3/43/5237	3/41/5237	–
4	3/63/6916	3/60/6916	3/58/6916	3/55/6916	3/53/6916	3/50/6916	4/46/7703
5	3/68/6374	3/66/6374	3/63/6374	3/60/6374	3/57/6374	3/55/6374	3/41/6374
6	3/74/5885	3/71/5885	3/68/5885	3/66/5885	3/63/5885	3/60/5885	3/45/5885
7	3/75/7348	3/72/7348	3/69/7348	3/66/7348	3/63/7348	3/60/7348	3/46/7348
8	3/81/6806	3/78/6806	3/75/6806	3/72/6806	3/69/6806	3/66/6806	3/50/6806
9	3/87/6317	3/84/6317	3/81/6317	3/78/6317	3/74/6317	3/71/6317	3/55/6317
10	3/47/6179	3/45/6179	–	–	–	–	–
11	2/42/4878	2/40/4878	–	–	–	–	–
12	2/45/4563	2/43/4563	–	–	–	–	–
13	3/73/6717	3/70/6717	3/67/6717	3/64/6717	3/61/6717	3/59/6717	3/44/6717
14	2/65/5310	2/62/5310	2/60/5310	2/57/5310	2/54/5310	2/52/5310	3/45/6199
15	2/70/4892	2/67/4892	2/64/4892	2/62/4892	2/59/4892	2/56/4892	2/42/4892
16	3/90/7074	3/87/7074	3/84/7074	3/80/7074	3/77/7074	3/74/7074	3/58/7074
17	2/77/5598	2/74/5598	2/71/5598	2/68/5598	2/65/5598	2/62/5598	2/47/5598
18	2/83/5180	2/80/5180	2/77/5180	2/74/5180	2/70/5180	2/67/5180	2/52/5180
19	–	–	–	–	–	–	–
20	–	–	–	–	–	–	–
21	–	–	–	–	–	–	–
22	2/63/5808	2/60/5808	2/58/5808	2/55/5808	2/53/5808	2/50/5808	–
23	2/66/5308	2/63/5308	2/61/5308	2/58/5308	2/55/5308	2/53/5308	–
24	2/70/4892	2/67/4892	2/64/4892	2/62/4892	2/59/4892	2/56/4892	–
25	2/82/6023	2/79/6023	2/76/6023	2/73/6023	2/70/6023	2/67/6023	2/51/6023
26	2/85/5533	2/82/5533	2/79/5533	2/75/5533	2/72/5533	2/69/5533	2/53/5533
27	2/87/5105	2/84/5105	2/81/5105	2/78/5105	2/74/5105	2/71/5105	2/55/5105

Note : minimum crane number/throughput/total cost*10^5

Table 2 shows that the minimum number of cranes satisfying the minimum throughput, the maximum throughput, and cost which are resulted from the 27 cases. The results indicate that the minimum number of cranes and cost are the same in many cases and when the ratio of dual command is decreased, the throughput is slightly decreased.

Also, the analysis for the 27 cases shows that the best conditions vertical and horizontal velocity of crane and the height of system are 30 m/min, 130 m/min and 25 m, respectively.

Table 3 shows that for the same number of cranes the height of rack maximized throughput is the same for the cases with 100, 90, 80, 70, 60, 50 and 0% dual commands.

Table 3. Results for 4 cranes with 100, 90, 80, 70, 60, 50 and 0% dual command

Ratio	No. of Rack	No. of Load High	No. of Bay	Throughput	Total cost
100%	3264	8	51	85*	669073100
90%	3264	8	51	82*	669073100
80%	3264	8	51	79*	669073100
70%	3264	8	51	76*	669073100
60%	3264	8	51	73*	669073100
50%	3264	8	51	70*	669073100
0%	3264	8	51	54*	669073100

Table 4. Sensitivity analysis for the number of cranes

No. of Crane / Ratio	2		3		4		5		6	
	*TR	*Cost	TR	Cost	TR	Cost	TR	Cost	TR	Cost
100%	70	4892	80	5817	85	6690	92	7543	94	8499
90%	67	4892	76	5817	82	6690	89	7543	91	8499
80%	64	4892	73	5817	79	6690	85	7543	87	8499
70%	62	4892	70	5817	76	6690	82	7543	84	8499
60%	59	4892	67	5817	73	6690	79	7543	81	8499
50%	56	4892	64	5817	70	6690	76	7543	77	8499
0%	42	4892	49	5817	54	6690	59	7543	61	8499

Note : TR = Throughput, Cost = Total Investment Cost $*10^5$

3.5 Sensitivity Analysis

To demonstrate the use of the model, sensitivity analysis was conducted in terms of the number of cranes and the number of load high.

The results indicate that the throughput is increased when number of cranes is increased and when the number of load high is increased the throughput is not always increased.

Table 4 and 5 show the results from 15 th case and figure 1, 2, 3, and 4 depict the results from the case with 50% dual command.

Fig. 2 shows throughput as a function of number of cranes. Fig. 3 shows total investment cost as a function of number of cranes. Fig. 4 shows throughput as a function of the number of load high. Fig. 5 shows total investment cost as a function of the number of load high.

Input data used in this sensitivity analysis are the same data that numerical example employed.

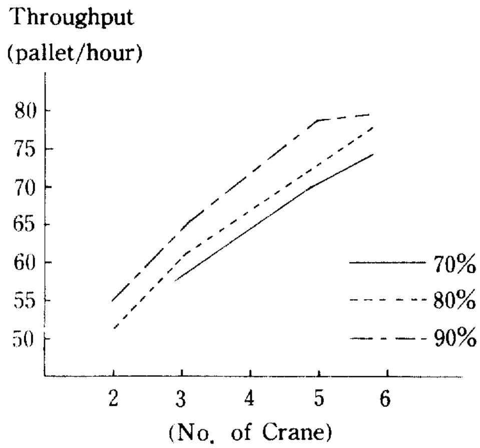

Figure 2. Throughput VS. No. of Crane

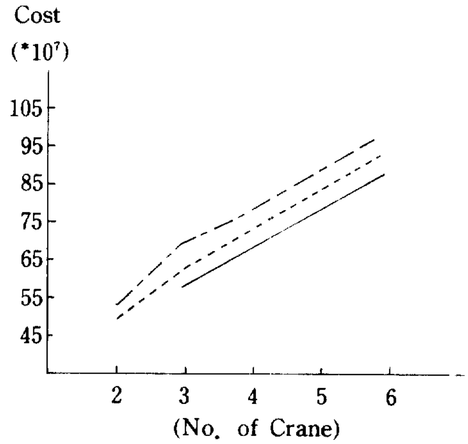

Figure 3. Total Cost VS. No. of Crane

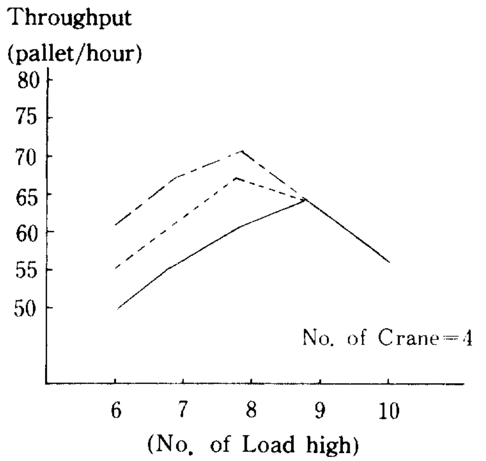

Figure 4. Throughput VS. No. of Load High

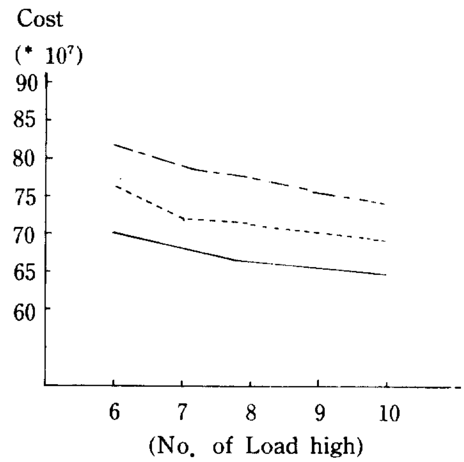

Figure 5. Total Cost VS. No. of Load High

Table 5. Sensitivity analysis for the number of Load High

No. of Load High / Ratio	6		7		8		9		10	
	TR	Cost	TR	Cost	TR	Cost	TR	Cost	TR	Cost
100%	74	6970	81	6826	85*	6690	80	6568	69	6541
90%	71	6970	78	6826	82*	6690	76	6568	72	6541
80%	68	6970	75	6826	79*	6690	73	6568	69	6541
70%	66	6970	72	6826	76*	6690	70	6568	66	6541
60%	63	6970	69	6826	73*	6690	67	6568	63	6541
50%	60	6970	66	6826	70*	6690	64	6568	60	6541
0%	45	6970	51	6826	54*	6690	49	6568	45	6541

Note : * = Maximum Throughput

4. Conclusion

The modeling approach presented in this study has attempted to determine number of cranes, crane velocity and the arrangement of the rack to minimize the investment as well as maximize the throughput.

A solution methodology is proposed and programmed in FORTRAN. This interactive program allows for extensive sensitivity analysis on important parameters such as length of stay (LOS), pallet arrival rate, rack utilization and height of the warehouse.

Though 100% dual command is desirable as shown in Table 2, a series of different ratios is tested because 100% dual operation is impossible and single operation is likely to occur in reality.

The results indicate that the throughput is slightly decreased when the ratio of dual command is decreased and the other characteristics however are not affected. The result also shows that the optimal rack should be designed for a crane to take the same amount of travel time for horizontal and vertical movement.

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