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Bayesian Estimation for the Left Truncated Exponential Lifetime Distribution with Inclusion and Exclusion of an Outlier*

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ABSTRACT

It is wellknown that the left truncated exponential distribution with positivity constraint on the location parameter is appropriate as a lifetime distribution model. In this paper, some Bayes estimators of the parameters and reliability for the left truncated exponential lifetime distribution when an unidentified-failure outlier is included and it is excluded in the exchangeable outlier model are proposed, and the performances of these proposed Bayes estimators are also discussed.

1. Introduction

We consider the two-parameter exponential distribution with positivity constraint on the location parameter defined by the probability density function(pdf),

$$f(x|\theta,\ \lambda)=\frac{1}{\lambda}\exp\left[-\frac{1}{\lambda}(x-\theta)\right],\quad x>\theta,\ \lambda>0, \tag{1.1}$$

where $\theta>0$ for the pdf to be left truncated.

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This model will be referred to as the left truncated exponential distribution(*LTED*), *LTE* (λ, θ). It is wellknown that the *LTED* is appropriate as a lifetime distribution model. For the *LTED*, the scale parameter λ is the reciprocal of the failure rate, the location parameter θ represents a guaranteed lifetime which is a period with no initial failures, and a reliability of a given system(or component) which is the probability that performs until a given time t is given by

$$R(t)=Pr(X>t)$$

$$=\exp\left(-\frac{t-\theta}{\lambda}\right), \quad t>\theta. \qquad (1.2)$$

The two-parameter exponential distribution is employed in the situations that the probability of failure up to a particular time θ is zero such as the bearing failures in reciprocating engines. Evans and Nigm(1980) emphasized that the use of the two--parameter exponential distribution with no positivity constraint on the location parameter as a lifetime distribution model is unrealistic and may lead to inefficient inferences and predictions because it is meaningless that the guaranteed lifetime is negative.

There are two different approaches with respect to the estimation of the parameters and reliability for a given system ; classical estimation on the one hand, Bayesian estimation on the other.

Several authors including Epstein and Sobel(1953), Basu(1964), Pugh(1963), Laurent (1963), Zacks and Even(1966), and Sinha(1972) investigated maximum likelihood(*ML*) estimators and minimum variance unbiased (*MVU*) estimators of the parameters and reliability for the one-parameter exponential distribution and the *LTED*. Also the Bayesian approaches to the estimation of the parameters and reliability for the one-parameter exponential distribution and the *LTED* were considered by many authors. For example, Bhattacharya(1967) proposed the various Bayes estimators of the parameters and reliability under the squared-error loss using some general classes of the prior distributions having finite supports for the one-parameter exponential distribution model. Varde(1969) compared the Bayes estimators under the two kinds of loss function and a conjugate prior distribution with the classical estimators of the reliability for the *LTED*. Sinha and Guttman(1976) proposed the Bayes estimators of the parameters and reliability under the squared-error loss and the general classes of the noninformative prior distributions, and derived the posterior bounds of the reliability for the one-parameter exponential distribution and the *LTED*. Pierce(1973) considered the Bayesian reliability estimation for the *LTED* assuming the prior distribution is a special case of the noninformative prior distribution used by Sinha and Guttman(1976). In recent year, Trader(1985) discussed the Bayes estimators of the parameters and reliablilty under the squared--error loss and truncated normal distribution as a conjugate prior distribution for the *LTED*.

Now, if we assume that there exists an outlier in the random lifetime observations for a given life testing experiment of n items, $(n-1)$ of them are distributed as $LTE(\lambda, \theta)$ and the remaining one of them (an outlier) is distributed as $LTE(\lambda/r, \theta)$ with the nuisance parameter r. The numerous surveys and discussions of outliers were provided by Barnett and Lewis (1978) and Beckman and Cook (1983).

The problem of estimating the scale parameter and reliability for the one-parameter exponential lifetime distribution under the presence of an outlier has been studied by many authors. Kale and Sinha (1971), Joshi (1972), Chikkagoudar and Kunchur (1980), and Rauhut (1982) proposed the estimators of the exponential scale parameter in the exchangeable-outlier model. Veale (1975), and Singh and Singh (1983) considered various estimators of the exponential scale parameter in the identified-outlier model. Recently, Gather (1986) studied some estimators of the exponential scale parameter under the assumption of the various outlier generating models. In the Bayesian viewpoint, Sinha (1972) discussed the behavior of the *MVUE* of the reliability for the one-parameter exponential lifetime distribution by a semi-Bayesian approach with a beta prior of the nuisance parameter in the exchangeable-outlier model. Also Sinha (1973) proposed some Bayes estimators of the parameter and reliability for the one-parameter exponential lifetime distribution in the exchangeable-outlier model.

The purpose of this paper is to propose some Bayes estimators of the parameters and reliability for the *LTED* when an unidentified-failure outlier is present and an unidentified-failure is excluded in the exchangeable-outlier model, and to compare the proposed Bayes estimators of parameters and reliability in the presence of an outlier with respect to the Bayes estimators of the parameters and reliability in the delition of an outlier each other in terms of the biases and mean squared errors (*MSE's*) by the Monte Carlo simulation.

2. Exchangeable-outlier model with inclusion of an outlier

Let $x_1, x_2, \cdots, x_{i-1}, x_{i+1}, \cdots, x_n$ be independent observations of $(n-1)$ failure times from $LTE(\lambda, \theta)$, whereas x_i be an observation of the unidentified-failure time as an outlier from $LTE(\lambda/r, \theta)$, $0<r<1$, under test without replacement. The index i is an observation of I with

$$Pr[I=i]=\frac{1}{n},$$

for all $i \in I=\{1, 2, \cdots, n\}$. If $x_1, x_2, \cdots, x_n$ are n independent observations of the failure times under the test, and $(n-1)$ of which come from $LTE(\lambda, \theta)$ and the remaining one x_i comes from $LTE(\lambda/r, \theta)$, $0<r<1$, with probability $1/n$, then x_i which comes from $LTE(\lambda/r, \theta)$ is called an unidentified-failure as an outlier for the *LTED*.

Now, if we assume that the nuisance parameter r on the scale parameter λ is variable, and that r has the beta prior distribution, denoted by $BE(p, q)$,

$$h_0(r)=\frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)}r^{p-1}(1-r)^{q-1}, \quad p,\ q>0, \tag{2.1}$$

then the likelihood function of λ, θ and r can be written as

$$L_1(\lambda,\ \theta,\ r|\underline{x}) \propto \frac{r}{\lambda^n} \exp\left[-\frac{n(\bar{x}-\theta)}{\lambda}\right] \sum_{i=1}^{n} \exp\left[\frac{(1-r)(x_i-\theta)}{\lambda}\right], \tag{2.2}$$

where $0<\theta<y=\min(x_1,\ x_2,\ \cdots,\ x_n)$, $\lambda>0$, $0<r<1$, and $\underline{x}=(x_1,\ x_2,\ \cdots,\ x_n)$.

By the Jeffreys' rule, if $|I(\underline{\theta})|$ is the determinant of the Fisher's information matrix, then the noninformative prior distribution of the parameter space $\underline{\Theta}$ is given by

$$g(\underline{\theta}) \propto |I(\underline{\theta})|^{\frac{1}{2}}.$$

Thus, for $LTE(\lambda,\ \theta)$, a general class of the noninformative prior distribution of $(\lambda,\ \theta)$ is

$$g_0(\lambda,\ \theta) \propto \frac{1}{\lambda^a},\quad a>0,\ \lambda>0,\ 0<\theta<y. \tag{2.3}$$

From the noninformative prior distribution in (2.3) and the likelihood function in (2.2), the joint posterior distribution of $(\lambda,\ \theta)$ given $\underline{x}$ is

$$g_1(\lambda,\ \theta|\underline{x}) = \frac{\int_0^1 L_1(\lambda,\ \theta,\ r|\underline{x})\, g_0(\lambda,\ \theta)\, h_0(r)\, dr}{\int_0^1\int_0^\infty\int_0^y L_1(\lambda,\ \theta,\ r|\underline{x})\, g_0(\lambda,\ \theta)\, h_0(r)\, d\theta d\lambda dr}$$

$$= \frac{1}{D_1 \Gamma(n+a-2)\lambda^{n+a}} \sum_{i=1}^{n} \exp\left[-\frac{n\bar{x}-x_i-(n-1)\theta}{\lambda}\right]$$

$$\cdot M\left(p+1,\ p+q+1\ ;\ -\frac{(x_i-\theta)}{\lambda}\right), \tag{2.4}$$

$\lambda>0$, $0<\theta<y$,
where

$$D_1 = n^{-1}\sum_{i=1}^{n}[W_{1i}(0,\ 0,\ 0,\ y) - W_{1i}(0,\ 0,\ 0,\ 0)],$$

$$W_{1i}(t,\ k,\ k',\ A) = [n\bar{x}+kt-(n+k)A]^{2+k'-a-n} H_{1i}(t,\ k,\ k',\ A),$$

$$H_{1i}(t,\ k,\ k',\ A) = F_1\left(q,\ 1,\ n+a-2-k'\ ;\ p+q+1\ ;\ \frac{1}{n+k},\ \frac{x_i-A}{n\bar{x}+kt-(n+k)A}\right).$$

$M(\alpha,\ \gamma\ ;\ z)$ is a confluent hypergeometric function in Kummer's form, and

$F_1(\alpha, \beta, \beta'; x, y)$ is a hypergeometric function in Picard's form of two variables [Erdélyi (1953)].

Integrating out λ and θ from (2.4), we have the marginal posterior distributions of λ and θ as

$$h_{11}(\lambda|\underline{x}) = \frac{1}{D_1\Gamma(n+a-2)n\lambda^{n+a-1}}\exp\left(-\frac{n\bar{x}}{\lambda}\right)$$

$$\cdot\sum_{i=1}^{n}\left\{\exp\left(\frac{ny}{\lambda}\right)\Phi_1\left(q,\ 1,\ p+q+1;\frac{1}{n},\ \frac{x_i-y}{\lambda}\right)\right.$$

$$\left.-\Phi_1\left(q,\ 1,\ p+q+1;\frac{1}{n},\ \frac{x_i}{\lambda}\right)\right\}, \tag{2.5}$$

$\lambda>0,$

and

$$h_{12}(\theta|\underline{x}) = D_1^{-1}(n+a-2)[n(\bar{x}-\theta)]^{1-a-n}$$

$$\cdot\sum_{i=1}^{n}{}_2F_1\left(n+a-1,\ q;p+q+1;\frac{x_i-\theta}{n\bar{x}-n\theta}\right), \tag{2.6}$$

$0<\theta<y,\ n+a>2,\ p,\ q>0,$

respectively, where $\Phi_1(\alpha, \beta, \gamma; x, y)$ is a degenerate hypergeometric function in Gauss' form of two variables, and ${}_2F_1(\alpha, \beta; \gamma; z)$ is a confluent hypergeometric function in Gauss' form [Erdélyi (1953)].

By means of (2.4), (2.5) and (2.6), we obtain the k-th monents of λ, θ and $R(t)$ as

$$E_1[\lambda^k|\underline{x}] = \int_0^\infty \lambda^k h_{11}(\lambda|\underline{x})\,d\lambda$$

$$= \frac{\Gamma(n+a-k-2)}{D_1\Gamma(n+a-2)n}\sum_{i=1}^{n}[W_{1i}(0,\ 0,\ k,\ y)-W_{1i}(0,\ 0,\ k,\ 0)], \tag{2.7}$$

$n+a-k>2$ for all k,

$$E_1[\theta^k|\underline{x}] = \int_0^y \theta^k h_{12}(\theta|\underline{x})\,d\theta$$

$$= \frac{\sum_{i=1}^{n} I_{1i}(k)}{D_1\Gamma(n+a-2)B(p+1,\ q)}\ , \tag{2.8}$$

$k \geq 0$, and

$$E_1[R(t)^k|\underline{x}] = \int_0^y \int_0^\infty \exp\left[-\frac{k(t-\theta)}{\lambda}\right] g_1(\lambda, \theta|\underline{x})\, d\lambda d\theta$$

$$= \frac{1}{D_1(n+k)} \sum_{i=1}^{n} [W_{1i}(t, k, 0, y) - W_{1i}(t, k, 0, 0)], \tag{2.9}$$

$t > y$,

respectively, where $W_{1i}(t, k, k', A)$ is given in (2.4), and

$$I_{1i}(k) = \int_0^1 r^p (1-r)^{q-1} \int_0^\infty \frac{1}{\lambda^{n+a}} \exp\left[-\frac{n\bar{x} - (1-r)x_i}{\lambda}\right]$$

$$\cdot \left[\int_0^y \theta^k \exp\left(\frac{n+r-1}{\lambda}\theta\right) d\theta\right] d\lambda dr,$$

$k \geq 0$.

Therefore, taking $k=1$ in (2.7), (2.8) and (2.9), we have the following theorem :

Theorem 2.1. Under the noninformative prior distribution of (λ, θ) in (2.3) and the beta prior distribution of r in (2.1), the Bayes estimators of λ, θ and $R(t)$ for the $LTE(\lambda, \theta)$ with an unidentified-failure outlier which is influenced of the beta nuisance variable r in the exchangeable-outlier model are given by

$$\lambda_1^{BE} = \frac{\sum_{i=1}^{n}[W_{1i}(0, 0, 1, y) - W_{1i}(0, 0, 1, 0)]}{(n+a-3)\sum_{i=1}^{n}[W_{1i}(0, 0, 0, y) - W_{1i}(0, 0, 0, 0)]}, \tag{2.10}$$

$n+a > 3$,

$$\theta_1^{BE} = \frac{\sum_{i=1}^{n}[y(n+a-3)W_{1i}(0, 0, 0, y) - W_{2i}(0, 0, 1, y) + W_{2i}(0, 0, 1, 0)]}{(n+a-3)\sum_{i=1}^{n}[W_{1i}(0, 0, 0, y) - W_{1i}(0, 0, 0, 0)]}, \tag{2.11}$$

$n+a > 3$, and

$$R_1^{BE}(t) = \frac{n\sum_{i=1}^{n}[W_{1i}(t, 1, 0, y) - W_{1i}(t, 1, 0, 0)]}{(n+1)\sum_{i=1}^{n}[W_{1i}(t, 0, 0, y) - W_{1i}(t, 0, 0, 0)]}, \tag{2.12}$$

$t > y$

respectively, where $W_{1i}(t, k, k', A)$ is given in (2.4),

$W_{2i}(t, k, k', A) = n^{-1}[n\bar{x} + kt - (n+k)A]^{2+k'+a-n} H_{2i}(t, k, k', A)$,

and $H_{2i}(t, k, k', A) = F_1\left(q, 2, n+a-2-k', p+q+1; \frac{1}{(n+k)}, \frac{x_i - A}{n\bar{x} + kt - (n-k)A}\right)$.

3. Exchangeable-outlier model with exclusion of an outlier

Now, we consider the model that an unidentified--failure outlier is excluded in the exchangeable-outlier model.

Let $x_1, x_2, \cdots, x_{i-1}, x_{i+1}, \cdots, x_n$ be independent observations of $(n-1)$ failure times from $LTE(\lambda, \theta)$, whereas x_i be a single outlying observation from $LTE(\lambda/r, \theta)$, $0<r<1$, with probability $1/n$. Then $x_1, x_2, \cdots, x_n$ are, on this model, exchangeable. Thus the likelihood function of λ and θ under the exchangeable-outlier model with exculsion of an unidentified-failure outlier is given by

$$L_2(\lambda, \theta|\underline{x}) = \frac{1}{n}\sum_{i=1}^{n} \prod_{\substack{j=1\\ j\neq i}} f(x_j|\theta, \lambda)$$

$$= \frac{1}{n\lambda^{n-1}} \exp\left[-\frac{n\bar{x}-(n-1)\theta}{\lambda}\right] \sum_{i=1}^{n} \exp\left(\frac{x_i}{\lambda}\right), \qquad (3.1)$$

where $0<\theta<y=\min(x_1, x_2, \cdots, x_n)$, $\lambda>0$, and $\underline{x}=(x_1, x_2, \cdots, x_n)$.

By means of (2.3) and (3.1), we obtain the joint posterior distribution of (λ, θ) given $\underline{x}$ as

$$g_2(\lambda, \theta|\underline{x}) = \frac{1}{D_2\Gamma(n+a-3)\lambda^{n+a-1}}$$

$$\cdot \sum_{i=1}^{n} \exp\left[-\frac{n\bar{x}-x_i-(n-1)\theta}{\lambda}\right], \qquad (3.2)$$

where $D_2 = (n-1)^{-1}\sum_{i=1}^{n} S_{2i}(0, 0, 0)$ and

$$S_{2i}(k, k', t) = [n\bar{x}+kt-x_i-(n-k-1)y]^{3+k'-a-n}$$

$$-[n\bar{x}+kt-x_i]^{3+k'-a-n}.$$

Integrating out λ and θ from (3.2), the marginal posterior distribuions of λ and θ are given by

$$h_{21}(\lambda|\underline{x}) = \frac{1}{D_2\Gamma(n+a-3)(n-1)\lambda^{n+a-2}}$$

$$\cdot \sum_{i=1}^{n}\left\{\exp\left[-\frac{n\bar{x}-x_i-(n-1)y}{\lambda}\right]-\exp\left[-\frac{n\bar{x}-x_i}{\lambda}\right]\right\}, \qquad (3.3)$$

$\lambda>0$,

$$h_{22}(\theta|\underline{x}) = \frac{(n+a-3)}{D_2}\sum_{i=1}^{n}[n\bar{x}-x_i-(n-1)\theta]^{2-a-n}, \quad 0<\theta<y, \tag{3.4}$$

respectively.

Then, we obtain the k-th moments of λ, θ and $R(t)$ as follows ;

$$E_2[\lambda^k|\underline{x}] = \frac{\Gamma(n+a-k-3)}{D_2\Gamma(n+a-3)(n-1)}\sum_{i=1}^{n}S_{2i}(0,\ k,\ 0), \tag{3.5}$$

$n+a-k>3$ for all k,

$$E_2[\theta^k|\underline{x}] = D_2^{-1}(n+a-3)\sum_{i=1}^{n}I_{2i}(k,\ n+a-2), \tag{3.6}$$

$k\geq 0$, and

$$E_2[R(t)^k|\underline{x}] = D_2^{-1}(n+k-1)^{-1}\sum_{i=1}^{n}S_{2i}(k,\ 0,\ t), \tag{3.7}$$

respectively, where $S_{2i}(k,\ k',\ t)$ is given in (3.2) and

$$I_{2i}(k,\ n+a-2) = \int_0^y \theta^k[n\bar{x}-x_i-(n-1)\theta]^{2-a-n}d\theta.$$

Therefore, taking $k=1$ in (3.5), (3.6) and (3.7), we obtain the following theorem :

Theorem 3.1. Under the squared-error loss function and the noninformative prior of $(\lambda,\ \theta)$, the Bayes estimators of λ, θ and $R(t)$ for the $LTE(\lambda,\ \theta)$ in the case that an unidentified-failure outlier is excluded from the complete sample are given by

$$\lambda_2^{BE} = \frac{\sum_{i=1}^{n}S_{2i}(0,\ 1,\ 0)}{(n+a-4)\sum_{i=1}^{n}S_{2i}(0,\ 0,\ 0)}, \quad n+a>4, \tag{3.8}$$

$$\theta_2^{BE} = \frac{\sum_{i=1}^{n}[(n-1)y(n+a-4)T_{2i}(0,\ 0,\ 0)-S_{2i}(0,\ 1,\ 0)]}{(n-1)(n+a-4)\sum_{i=1}^{n}S_{2i}(0,\ 0,\ 0)}, \tag{3.9}$$

$n+a>3$, and

$$R_2^{BE}(t) = \frac{(n-1)\sum_{i=1}^{n}S_{2i}(1,\ 0,\ t)}{n\sum_{i=1}^{n}S_{2i}(0,\ 0,\ 0)}, \quad t>y, \tag{3.10}$$

respectively, where $T_{2i}(k,\ k',\ t) = [n\bar{x}+kt-x_i-(n+k-1)y]^{3+k'-a-n}$.

4. Monte Carlo comparisons

In this section, we compare the proposed Bayes estimators of the parameters and reliability in the presence of an outlier with those in the deletion of an outlier each other in terms of the MSE performances and stabilities by the Monte Carlo simulation. Also, the efficiencies of the proposed Bayes estimators measured in term of the ratio of the MSE's in the deletion of an outlier relative to those in the presence of it.

We generate random samples from the $LTE(\lambda, \theta)$ and then we evaluate the Bayes estimators of the parameters and reliability on these samples. We evaluate the estimates of biases, MSE's and efficiencies of the Bayes estimators of the parameters and reliability based on 750 repetitions [Ahrens and Dieter(1974), Kennedy and Gentle(1980), and Rubinstein(1981)].

Simulations are performed on HP−1000/19 at the National Fisheries University of Pusan and the results of simulations appear in Table 1.

From the table, the results can be summarized as follows :

(1) For the λ^{BE} with the beta nuisance variable r, it is more efficient to exclude an outlier than to include it when $p<q$.

(2) For the θ^{BE} with the beta nuisance variable r, it is more efficient to include an outlier than to exclude it.

(3) For the $R^{BE}(t)$ with the beta nuisance variable r, it is more efficient to include an outlier than to exclude it when $p>q$.

Table 1. Comparisons of the Bayes estimators in the presence of an unidentified-failure outlier with respect to those in the deletion of an unidentified-failure outlier according to the beta nuisance variable r.

(1) $n=5,\ a=5,\ \lambda=2,\ \theta=1$

	$(p,\ q)$	λ		θ		$R(t)$			
		λ_1^{BE}	λ_2^{BE}	θ_1^{BE}	θ_2^{BE}	$R_1^{BE}(2.39)$	$R_2^{BE}(2.39)$	$R_1^{BE}(1.45)$	$R_2^{BE}(1.45)$
Bias	(2,20)	−0.0775	−0.0780	0.0162	0.0098	−0.0167	−0.0169	−0.0070	−0.0052
	(10,10)	−0.0762		0.0147		−0.0142		−0.0059	
	(20, 2)	−0.0758		0.0164		−0.0132		−0.0048	
MSE	(2,20)	0.0979	0.0966	0.0143	0.0146	0.0070	0.0061	0.0043	0.0042
	(10,10)	0.0952		0.0111		0.0063		0.0041	
	(20, 2)	0.0852		0.0128		0.0056		0.0033	
efficiency	(2,20)	0.987		1.021		0.871		0.977	
	(10,10)	1.015		1.315		0.953		1.024	
	(20, 2)	1.134		1.140		1.089		1.273	

(2) $n=7$, $a=5$, $\lambda=2$, $\theta=1$

	(p, q)	λ		θ		$R(t)$			
		λ_1^{BE}	λ_2^{BE}	θ_1^{BE}	θ_2^{BE}	$R_1^{BE}(2.39)$	$R_2^{BE}(2.39)$	$R_1^{BE}(1.45)$	$R_2^{BE}(1.45)$
Bias	(2 ,20)	−0.0669		0.0101		−0.0144		−0.0059	
	(10,10)	−0.0657	−0.0639	0.0095	−0.0116	−0.0130	−0.0139	−0.0043	−0.0046
	(20, 2)	−0.0634		0.0111		−0.0127		−0.0030	
MSE	(2 ,20)	0.0721		0.0077		0.0051		0.0030	
	(10,10)	0.0703	0.0714	0.0067	0.0082	0.0042	0.0046	0.0024	0.0024
	(20, 2)	0.0668		0.0073		0.0038		0.0021	
efficiency	(2 ,20)	0.990		1.065		0.902		0.800	
	(10,10)	1.016		1.224		1.095		1.000	
	(20, 2)	1.069		1.123		1.211		1.143	

(3) $n=11$, $a=5$, $\lambda=2$, $\theta=1$

	(p, q)	λ		θ		$R(t)$			
		λ_1^{BE}	λ_2^{BE}	θ_1^{BE}	θ_2^{BE}	$R_1^{BE}(2.39)$	$R_2^{BE}(2.39)$	$R_1^{BE}(1.45)$	$R_2^{BE}(1.45)$
Bias	(2 ,20)	−0.0393		0.0014		−0.0106		−0.0053	
	(10,10)	−0.0387	−0.0374	0.0015	0.0017	−0.0104	−0.0103	−0.0051	−0.0050
	(20, 2)	−0.0356		0.0014		−0.0091		−0.0050	
MSE	(2 ,20)	0.0421		0.0030		0.0030		0.0016	
	(10,10)	0.0418	0.0417	0.0022	0.0030	0.0025	0.0024	0.0014	0.0012
	(20, 2)	0.0409		0.0017		0.0021		0.0010	
efficiency	(2 ,20)	0.990		1.000		0.800		0.750	
	(10,10)	0.998		1.364		0.960		0.857	
	(20, 2)	1.020		1.765		1.143		1.200	

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