아스팔트 鋪裝道路의 引張强度에 대한 信賴度 考察

Evaluation of Reliability for the Tensile Strength of the Flexible Pavement System

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要　旨

層狀構造로 이루어진 아스팔트 鋪裝道路의 安全性은 交通荷重과 아스팔트 콘크리트의 層別强度와의 函數關係에 의하여 평가된다. 本 論文은 아스팔트 鋪裝道路의 信賴度를 考察하기 위하여 確率論的 方法을 적용시켰는데 이에 使用된 資料들은 現場 또는 文獻으로부터 수집하였다.

各層의 臨界引張强度를 目標信賴度에 根據하여 시뮬레이션 方法으로 算定하였는데 이 값을 실제로 表面에 損傷이 일어난 試料强度와 比較, 考察하였다.

確率論的方法으로 算定한 臨界强度는 現行의 道路條件에 適合한 값임을 알 수 있었다.

Abstract

The flexible highway pavement is a layered structure. The safety of the pavement is a function of the load induced by traffic and the layer strength of asphaltic concrete mixture. Therefore, the probabilistic approach was applied to the pavement system to evaluate the reliability. Monte Carlo simulation technique was used for the reliability study. Data used were colleted from the field or literature. A critical tensile strength for each layer was estimated based on a target reliability from the simulation. The critical strength was evaluated by comparing the strengths with the actual surface distress. The result shows that the critical strength estimated in the probabilistic approach is valid for the current highway condition.

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1. Introduction

The asphaltic concrete of flexible pavement is under stress condition due to axle loading applied to the pavement. The strength of the pavement should be strong enough to stress induced by the traffic loading. However, there has been no critical value of strength that can be applicable to each layer of mixture in the flexible pavement. This study is therefore intended to introduce an approach to determine an acceptable value of tensile strength for the flexible pavement using probabilistic concepts.

The pavement is a layered structure that consists of several layers(from subgrade to surface course) with different thickness. The layer strength is random variable and the stress under axle loading in the pavement is also random variable depending on traffic condition and layer geometry. Using the first random variable as the resistance of structural member and the second as the load effect, a probabilistic study is conducted to evaluate the reliability of the strength of the pavement in highway.

Data for each variable are collected from field and literatures. The field data used in this study are from Interstate Highway 85 in South Carolina, United States of America. The coring was conducted every half mile of each lane for 70 miles, except for the portion of portland-cement-concrete (PCC) pavement, resulting in a total of over 400 cores(3). Probability distribution and parameters for each variable were determined using appropriate statistical methods. Using the probability distributions together with subordinate factors, the layer stress and the layer strength were determined by simulation. Reliability of the pavement was evaluated by comparing the strength. The critical strength value of each layer was determined based on a certain level of probability of failure. As a practical consideration, the critical value was evaluated based on the field condition of the pavement.

To apply the probabilistic analysis concepts to the flexible pavement system, the following assumptions were made in this study. The failure of any layer in a flexible pavement system will cause a functional failure (serviceability failure) of the pavement structure. Among other factors, failure of the pavement layer is a function of the layer strength and the stress applied to the layer by vehicle loading. Therefore, failure of any layer was defined in this study to occur when the stress due to traffic loading exceeded the strength in the layer.

2. Basic Concepts for Reliability Evaluation

The first step toward evaluating the reliability of a structure is to decide the load and resistance parameters. If the load and resistance are random variables, X_i's, a functional relationship among them can be defined as a limit state eqation(4,5):

$$Z=g(X_1, X_2, \cdots\cdots, X_n) \qquad (1)$$

The failure surface can then be defined as $Z=0$ and failure occurs when $Z<0$. The probability of failure, P_f, is givin by integrating the joint probability density function (PDF) of X_i's, $f_x(x_1, x_2, \cdots\cdots, x_n)$, over the region where $g(\ )<0$.

$$P_f=\int\cdots\int f_x(x_1, x_2, \cdots\cdots, x_n)dx_1, \\ dx_2, \cdots\cdots, dx_n \qquad (2)$$

The reliability of a structure, P_s, is then described as

$$P_s=1-P_f \qquad (3)$$

The mean and variance of Z in equ. 1 can be evaluated by Taylor series as (2)

$$\bar{Z}\simeq g(\bar{x}_1, \bar{x}_2, \cdots\cdots, \bar{x}_n) \qquad (4)$$

and

$$\sigma_z^2 \simeq \left[\sum_{i=1}^{m}\sum_{j=1}^{m}\left(\frac{\partial g}{\partial X_i}\right)\left(\frac{\partial g}{\partial X_j}\right)\text{Cov}\ (X_i, X_j)\right] \tag{5}$$

Where, $\text{COV}(X_i, X_j)$ is covariance of X_i and X_j. Equations 4 and 5 are generalized forms for the mean and variance based on first-order, second-moment approximation.

A measure of safety can be evaluated by introducing a parameter β, called reliability index, as

$$\beta = \frac{Z}{\sigma_z} \tag{6}$$

Equ. 6 is a generalized expression of reliability index, and the failure region is the distance measured by σ_x and β from the mean, Z.

The monte Carlo simulation technique can be used to estimate the probabilistic characteristic of the relationship Z in equ. 1. The Monte Carlo simulation approach consists of drawing samples of the independent input variable according to their probability density function and then feeding them into the model g( ). The sample statistics thus obtained would give the probabilistic characteristics of the random variable $Z^{(15)}$. If the g( ) value evaluated is less than zero, a failure occurs.

Let N_f be the number of simulation cycle when g( ) is less than zero and N is the total number of simulation. Then, an estimate of the mean probability of failure could be expressed as

$$\overline{P}_f = \frac{N_f}{N} \tag{7}$$

The accuracy of equ. 7 can be estimated in terms of its variance, which depends on the number of simulation cycles N.

3. Variables for Reliability Study

There are many variables to be considered for reliability analysis of highway pavement system. However, the layers under base course, such as compacted subgrade, natural soil foundation and etc., as shown in Figure 1, are assumed in a sound condition and are not considered as a factors to be analyzed. Some portion of the surface is seal coat. Since the seal coat is a thin asphalt surface treatment used to waterproof and improve the texture of an asphalt wearing surface, it was not counted as a structural layer. The tack coat is a very light application of asphaltic materials to ensure a bond between the surface course and the binder course. The prime cost is an application of low-viscosity cutback to an untreated base prior to placement of the asphalt pavement (1). Thicknesses of tack coat and prime coat were negligible. A typical core that was taken from I-85 is illustrated in Figure 2.

Major variables used for the reliability study were the radial stresses and the

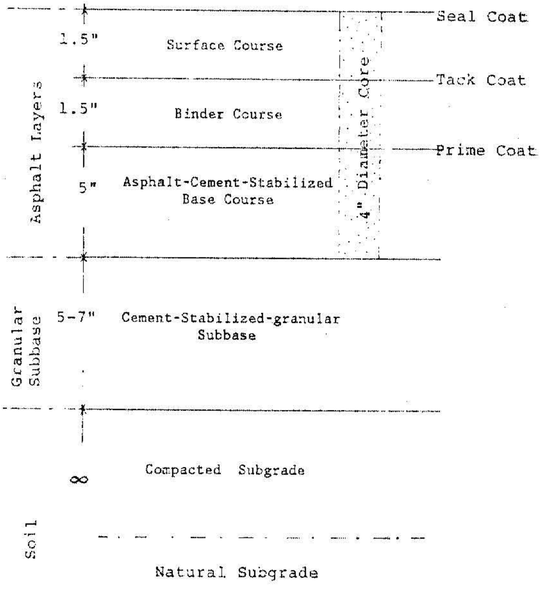

Fig. 1 Components of Pavement Structure in I-85, SC.

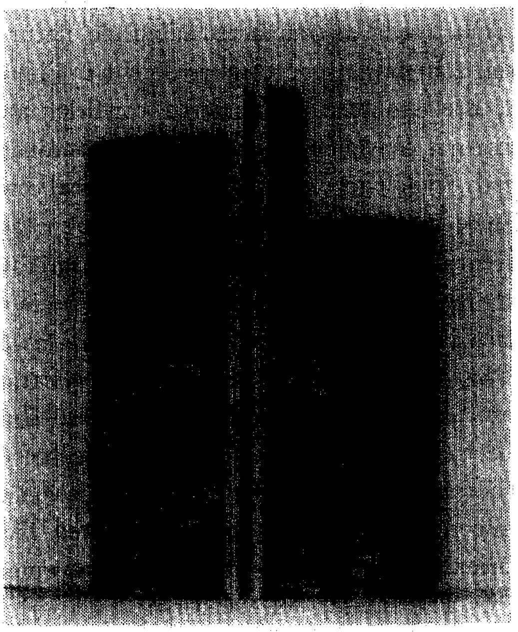

Fig. 2 Typical Cores Taken from I-85, SC.

tensile strengths of the top 3 bituminous layers. Since the radial stress is a function of the axle load and layer thickness, the axle loads on the given highway and the layer thickness were used for simulation of radial stress. Raw data for each variable were tested to choose its best-fit distribution model before goodness of kit test(17). Statistical parameters and probability distributions for each variable were then determined using a computer program for both Chi-Square and K-S goodness of fit tests at a 5% level of significance(18).

Data for layer thicknesses were measured from field cores(3). Since the layers under the base course are usually not asphaltic concrete mixture, as mentioned earlier, the thickness of layer one, two and three were used to determine probability distributions for layer thickness. The probability distributions for layer thickness determined for the three layer are shown in Figure 3.

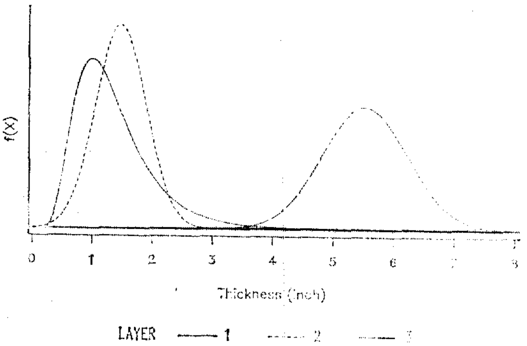

Fig. 3 PDF for Layer Thickness

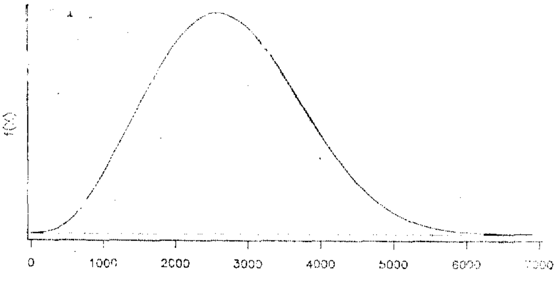

Fig. 4 PDF for Axle Load

Traffic data for axle loads was obtained from the state highway department and from truck weight data published by FHWA(10). Since the stress due to light vehicles such as passenger car is negligible compared to the stress due to heavy vehicles, only the probability distribution of axle loads for heavier vehicles was needed to be determined. The probability distribution for axle load used in this study is illustrated in Figure 4.

3.1 Radial Horizontal Stress

Radial stress at a certain layer is a function of axle load and layer thickness, both of which are random variables. Radial horizontal stress(RHS) in a pavement layer is calculated based on Boussinesq theory(21). According to Boussinesq one layer theory, stresses in the pavement is dependent on the magnitude of load, the depth and radial distance from the load point, and ***not dependent*** on the property of the transmitting medium(21). Since the layer properties (resilient modulus) are sometimes significantly different among layers, however, all the layers of the flexible pavement structure can not be regarded as homogeneous. Instead, considering some layers, which show identical characteristics, as one layer is more reasonable. According to Omer(12) who analyzed resilient modulus for highway pavement materials in this vicinity, there was no significant difference in resilient modulus between binder and surface course. The resilient modulus ratio of binder course to asphalt-cement-stabilized base course was also close to one(12). In other words, top 3 bituminous layers were identical. Therefore, Boussinesq equation can be applied to find a stress at any depth within the pavement portion(top 3 layers in this study).

The load at the surface of the flexible pavement is assumed to be distributed over a circular area of tire contact. The actual value of radius, a, of tire contact area depends upon the magnitude of axle load. However, since only heavy vehicles are treated in this study, one value of a(5 inches) was used for all heavy vehicles.

As the Boussinesq equations were originally developed, the pavement is considered as homogeneous, isotropic and elastic(21) as mentioned previously. The following Boussinesq equation was used to obtained radial horizontal stress.

$$S_r = p(2\mu A + C + (1-2\mu)F) \qquad (8)$$

where, S_r ; radial horizontal stress(RHS)

p ; pressure at tire-pavement contact

μ ; poisson's ratio

A,C and F ; one-layer elastic function values.

The value for p can be obtained by dividing the weight on a wheel by contact area. An appropriate value for Poisson's ratio can be obtained from experiment or from the literature. Since poisson's ratio was not obtained from the cored specimens in the project, a value of 0.35, which is widely accepted for asphaltic concrete, was used. The values of A,C and F are tabulated in reference as functions of depth and offest distance in radii(z/a and r/a)(21). The radial stresses at the bottom of each layer were determined by simulation. Probability distributions for the simulated radial stresses are depicted in Figure 5.

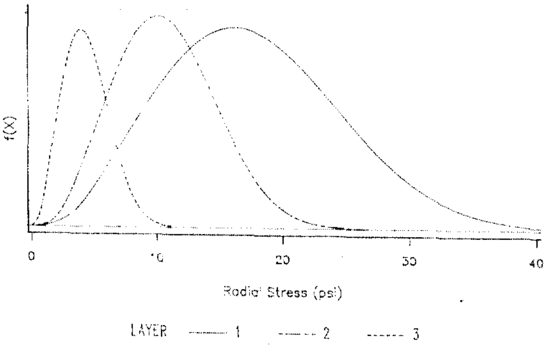

Fig. 5 PDF for Radial Stress

Maximum radial stress due to a single wheel occurs at a point along the vertical line beneath the geometric center of the tire imprint (load point). Maximum radial stress due to dual wheels occurs either at a point beneath the load point or at a point beneath the point halfway between the two tires. At a point beneath the midpoint of the two tires of dual wheels, radial stress is duplicated by the loads of the two wheels. Radial stress at this point is sometimes greater than the stress at a point vertically beneath the load point when the depth is greater than the clear distance between the two tires. Therefore, the absolute values of radial stress at the two points were compared and the larger was used as the maximum radial stress. Many trucks are equipped with tandom axles. Since the clear distance between tandom gears is at least 48 inches[19], however, no stress duplication occurs between tandom wheels at the asphaltic concrete layers, total depth of which is generally less than half the clear distance.

3.2 Tensile strength

Tensile Strength of the pavement layer was used for layer strength (resistance) in reliability evaluation. Tensile strength was measured for cored specimens from I-85 by the indirect tensile strength (ITS) test. The indirect tensile strength test is one type of strength test used for stabilized materials. This test involves loading a cylindrical specimen with a compressive load the diameter of the specimen. This results in a relatively uniform tensile stress acting perpendicular to and along the diametral plane of the applied load, resulting in a splitting failure generally occurring along along the diametral plane[21]. This failure mechanism is therefore assumed to be the same as the failure pattern occurring by radial stress in the pavement layer under the traffic loading.

The indirect tension test was conducted on the cored specimens after slicing the cores by layer in the laboratory. The testing room condition was at 25°C and also the specimens to be tested was kept at the same temperature. Load was applied vertically, using a Marshall testing machine at a rate of 2 inches per minute, through 0.5-inch-wide curved metal strips on the top and bottom of the specimen[3]. Since ITS is 15% greater than tensile strength, an estimate of the tensile strength was determined by dividing ITS by 1.15. This result was used as the layer strength. Probability distributions of ITS for each layer are shown in Figure 6.

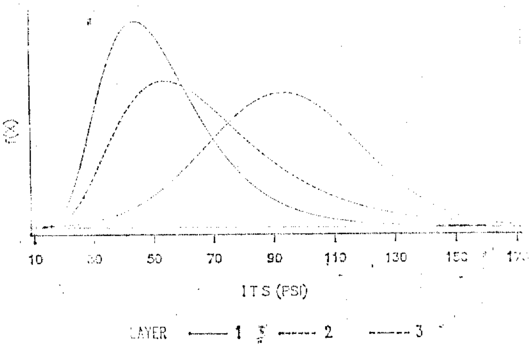

Fig. 6 PDF for Layer Strength

4. Reliability Analyses of the Layer Strength

The probability distribution for indirect tensile strengths was used for simulation of the layer strength (Resistance, R), and the probability distributions for layer thickness and axle for simulation of the radial stresses (load effect, L) on the layer. Table 1 shows the probability distribution for each variable and corresponding parameters.

The reliability for the strength of the layer was analyzed by comparing the layer strength and the radial horizontal stress at the bottom of the layer. Limit state function for this two variable problem is $Z=R-L=0$. The values for probability of failure (P_f) for each layer were Shown in Table 2. The first column of P_f value in Table 2 were obtained by 5,000 cycles of Monte Carlo simulation(15, 18). since both R and L do not folllow normal distribution, second column of P_f value by the equation, $\beta=(R-S)/\sqrt{\sigma_R^2+\sigma_L^2}$, for normal variates[4,7] showed much difference compared with those by Monte Carlo simulation (where, R and σ_R were obtained by dividing μ and σ of ITS by 1.15 as explained in Section 3.2 of this paper, and S and σ_L are μ and σ of RHS, respectively). Sometimes, a method for tr-

Table 1. Probability Distributions and Their Parameters

Variable	Distribution	Parameter
Layer Strength(ITS)		
Surface Course	Lognormal	$\mu=53.40$ psi $\sigma=19.49$ psi
Binder Course	Lognormal	$\mu=70.14$ psi $\sigma=33.62$ psi
Base Course	Normal	$\mu=96.42$ psi $\sigma=29.47$ psi
Axle Load	Weibull*	a=0 b=2784.7 lbs c=26.9
Layer Thickness		
Surface Course	Lognormal	$\mu=1.29$ inch $\sigma=0.24$ inch
Binder Course	Normal	$\mu=1.47$ inch $\sigma=0.30$ inch
Base Course	Normal	$\mu=5.52$inches $\sigma=0.74$ inch
Radial Stress		
Surface Course	Weibull*	a= 0.61 psi b=17.70 psi c= 2.42
Binder Course	Weibull*	a= 0.39 psi b=10.71 psi c= 2.59
Base Course	Weibull*	a= 0.06 psi b= 4.50 psi c= 2.34

* a=location factor, b=scale factor, c=shape factor (20)

Table 2. Statistical Parameters and Failure Probability of Each Layer

Layer	ITS		RST		P_f*	
	μ	σ	μ	σ	(1)	(2)
Surface Course	53.40	19.49	16.31	6.9	0.0210	0.0495
Binder Course	70.14	33.62	9.90	3.94	0.0012	0.0415
Base Course	90.42	29.47	4.04	1.81	0.0008	0.0019

* (1) by simulation of 5,000 times,
(2) by normal approximation.

ansforming non-normal variables to normal is used to find reliability of the system with non-normal variates. Since the method is still an approximate approach, however, it is not easy to see the accuracy level of the results obtained thereby. This problem is theirefore solved by Monte Carlo method because 1st, probability distributions for each variable are known (no assumption) and 2nd, high speed computer facility was available.

In general, when the probability of all variables are known, Monte Carlo method gives more accurate P_f than any other approximate solutions as long as the number of cycle is valid(6). The level of reliability (accuracy) in the results of Monte Carlo method is up to the number of cycle. In calculating P_f for layer 1 using Monte Carlo method, fluctuation in the value of P_f was leveled off approximately at 4000 to 5000 cycles. Fewer numbers of cycle than for layer 1 were required to get consistent results for layer 2 and 3.

Required number of cycle can also be determined using an equation by Harr(6), $N=(C(1-C)(h_{\alpha/2})^2/\varepsilon^2)^m$, where C is the probability of being correct in N trials, ε is allowable error in estimating C, α is level of significance and actually the same as ε, $h_{\alpha/2}=\Phi^{-1}(1-\alpha/2)$ where $\Phi(\ )$ is standard normal function, and m is the number of variables used in determining C. For example, let us see the valid number of simulation for layer 1. If a 95% of reliability is reqired in determining P_f, then, $\alpha=\varepsilon=0.05$, $C(=P_f)=0.021$(Table 2), $h_{\alpha/2}=\Phi^{-1}(0.975)=1.96$ in standard normal table, and $m=2$(resistance and load effect). $N=(0.021(1-0.021)1.96^2/0.05^2)^2=998$. Therefore, approximately 1000 times of simulation will give 95% of reliability in its result in this particular case.

From the simulation results, it is shown that the reliability of the surface course is lower than any other layers. The radial stress in the surface course, which is located in shallow depth and has direct contact with high-pressure tires, is highest among 3 layers(as shown in Table 1). This high stress has induced a failure of mixture bonding force in the surface course faster than in other layers. Direct contacts of tires with surface course also give more damages on the surface course than lower layers(14). Those stresss mechanisms may cause the strength of surface course to be weakend faster than any other layers. This type of failure (functional failure) of pavement structure. In this study, the serviceability failure should be differently interpreted from the structural failure that bridges or buildings might experience, collapse or catastrophic events.

A graphical illustration, for example, for P_f of layer 1 is shown in Figure 7. Change of reliability($1-P_f$) by change of ITS for each pavement course are shown in Figure 8. A critical value of indirect tensile strength for the surface course for a certain level of reliability was then obtained for the given traffic condition. Since it is known that P_f of functional failure for most of the engineering problems is 10^{-2}(7), a critical value of ITS for each pavement course can be obtained by the computer simulation. Critical values by 5000 cycles of simulation are approximately 65 psi and below 40psi for layer 1, layer 2 and layer 3, respectively, as can be seen in ITS for the reliability of 0.99 in Figure 8. If the variance of the distribution is assumed the same, a probability distribution for tensile strength with an acceptable P_f can then be determined as shown in Figure 7 (where $P_f=0.01$).

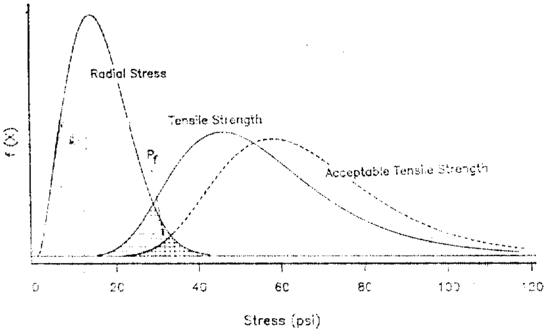

Fig. 7 Probability of Failure for Layer 1

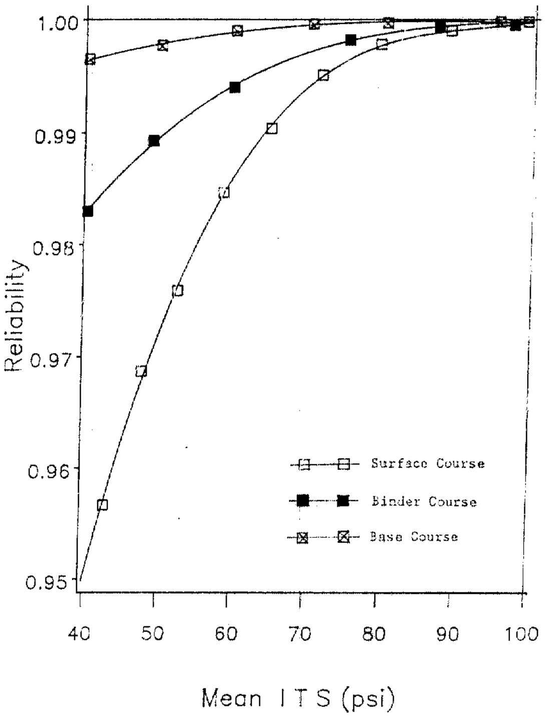

Fig. 8 Reliabilities of ITS

According to historical data about this section of highway, the highway was constructed early 1960s and recycled in surface approximately 5 years ago. The base course is still strong enough to sustain the applied load. The strength of binder course is also acceptable with its current condition. However, the surface course has been reached a serviceability failure stage by the reliability analysis. The surface course, which has served for 5 years, is considered losing its serviceability under current traffic condition, leading to a functional failure of the pavement structural system.

5. Evaluation of Critical Strength Based on Surface Distress

Pavement strength was related to the pavement condition in the field. Among many types of distress mechanism in the flexible pavement, surface rutting was found to be one of the most significant distress mechanisms that were correlated with low strength pavements. On the other hand, most of the cores from sites where cracks had developed on the surface showed vertical and/or horizontal cracks throughout the cores. Most of them fell apart while being removed from the pavement. Even when the cores were removed without breaking, they were usually so weak that ITS values could not be measured(3). Therefore, ITS values used in this study did not include values from cracked cores.

Since rutting is caused by consolidation or lateral movement of the materials due to traffic load, rutting results in a permanent deformation in one or more of the pavement layers or in the subgrade. This deformation causes a loss of strength in the mixture, leading to major structural failure of the pavement(9,13). The correlation of rutting with the ITS values was statistically analyzed using the General Linear Model(GLM) procedure in the Statistical Analysis System (SAS)(11,16). F-tests at the $\alpha=0.01$ level of significance were conducted in the analyses of variances(ANOVA) for rut depth. A significant differences in ITS values was found in the ANOVA.

The average ITS for the mixture from the sites that were free from rutting was 81 psi and 95 psi for layer 1 and 2,, respectively as Figure 9 shows. The average ITS of the same layer decreased as the surface rutting increased. It was however found that values of ITS for base course was not depend on the significance of rutting on the surface. Therefore, they were not analyzed in this study. For the mixtures from the minor-rutted (1/4 inch or less) sites(9), the average ITS for surface course was below the critical level that was determined by reliability study, but that for binder course was above the level. For the mixture from the medium-severity-level rutting(1/2—1 inch)(9), the strength for surface course was below 45 psi and that for binder course was reduced to below critical level(50 psi). The reduction of ITS of the surface course to below critical level came when the rut depth was increasing from 0 and 1/4 inch. Since a rut depth less than 1/4 inch is an allowable distress condition(1, 9), the critical strength(65 psi) for surface

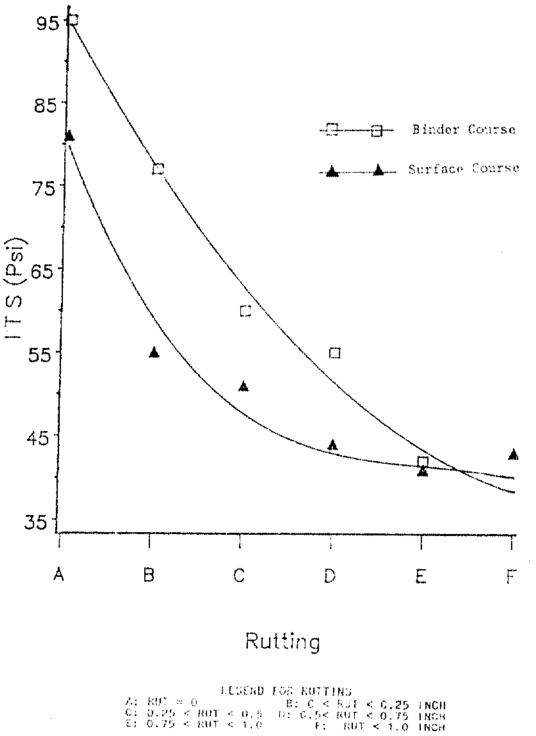

Fig. 9 Strength vs. Rutting

course seems to be a reasonable level.

The strength of the surface would be below critical level if the pavement showed a slight depth of rutting on the surface. In other words, as the strength of surface course is reduced to below critical level, a distress (rutting) seems to be occur on the surface of pavement as a sign of the failure. Consequently, the surface course will be failed first among all pavement layers. The failure of the surface course leads to a failure of serviceability of the pavement, initiating an entire failure of the structural system of the pavement. Therefore, critical strength for the surface course is more important indices for pavement stability than the critical strengths for the other layers. To maintain the pavement in a sound condition in the field, therefore, monitoring rutting is one of the important measures. Because, if the ITS of mixture in surface course stays above critical level, the pavement will remain in safe(distress free) condition.

6. Summary and Conclusion

It was shown that the probabilistic approach for structural analysis could be applied to the pavement structure in this study. The tensile strength of pavement layer was compared with the radial stress induced by traffic loading. A functional failure was defined when the radial stress exceed the tensile strength was measured from the cored specimens and radial strength was simulated based on the traffic distribution and layer thicknesses. Data of each of those variables were statistically tested to find a best-fit PDF using Chi-Square and K-S tests. Based on the probability of failure, 10^{-2}, for functional failure, an acceptable tensile strength of 65 psi for surface course and 50 psi for the binder course were estimated at 25°C test condition. The field evaluation based on the rut depth and ITS correlation showed that there was almost no rutting in the surface of pavement when the value of ITS was approximately above 65 psi and 50 psi in the surface course and the binder course, respectively.

Since the strength of binder course and base course were well over the estimated critical strength in this study, the strength of the surface layer seems to be the most important factor in indentifying functional safety of pavement. Since the gross weight of heavy traffic has been consistently increased in this section of highway for the past decade, somewhat higher value of critical ITS might be expected in the future of service life. However, according to the current strength and field conditions, the values estimated and suggested in this study is valid in the current stage.

The values are based on the assumptions made with respect to the simple loading of traffic. The accuracy of the value is dependent on the correctness of the assumptions. A number of study can be carried to obtain a more appropriate value if traffic studies were conducted for accurate traffic loading. While it has been recognized that estimating the performance of a pavement is very complex problem, the procedure presented in this paper is one of the possible methodologies that can be applied to any section of highway and traffic condition.

감사의 말

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