사면안정의 확률론적 해석

Probabilistic Analysis of the Stability of Soil Slopes

金 泳 壽*
Kim, Young Su

요 지

균질토 사면에서 파괴에 대한 확률론적모델이 제시되었다. 사면의 안전은 관례적인 안전율 보다는 파괴확률로써 측정된다. 사면파괴의 Safety Margin은 정규분포라 가정하였다. 어떤 균질한 흙층에 있어서 흙의 특성에 영향을 주는 불확실성의 원인은 본래의 공간적인 가변성, 불충분한 시료에서 오는 판단오차 그리고 실험오차등이 있다. 파괴면을 따라 존재하는 전단강도의 불확실성은 1차원 Random Field Models로 표현되었다. 파괴면의 양상은 대수나선 곡선을 따른다고 가정하였다. 파괴면과 그것을 따라 작용하는 힘의 통계적 특성을 유도하여 사면의 파괴확률을 계산하였다. 마지막으로 개발된 절차가 사면의 신뢰성 해석에 대한 하나의 예제 연구에 적용되었다.

Abstract

A probabilistic model for the failure in a homogeneous soil slope is presented. The Safety of the slope is measured through its probability of failure rather than the customary factor of safety. The safety margin of slope failure is assumed to follow a normal distribution. Sources of uncertainties affecting characterization of soil property in a homogeneous soil layer include inherent spatial variability., estimation error from insufficient samples, and measurement errors. Uncertainties of the shear strength along potential failure surface are expressed by one-dimensional random field models. The rupture surface, created at toe of a soil slope, has been considered to propagate towards the boundary along a path following an exponential (log-spiral) law. Having derived the statistical characteristics of the rupture surface and of the forces which act along it, the probability of failure of the slope was found. Finally the developed procedure has been applied in a case study to yield the reliability of a soil slope.

1. Introduction

Although much experience has been already accumulated about their design and performance, geotechnical engineers still face considerable uncertainties when they analyze their stability. These uncertainties reflect the slope's loading conditions, the variability of the material parameters, the

* 正會員·慶北大學校 工科大學 副教授

seasonal changes of the ground water table, the particular method used in the analysis, etc. Thus, efforts directed toward an approach which could yield a reliable measure for the safety of a slope must, of necessary, take into account these uncertainties.

The probabilistic study of soil slopes has been most widely performed in the field of geotechnical reliability in recent years[4,10,17,22,23]. This work provides an alternative to the conventional factor measure of safety of a slope: its probability of failure. As is usually the case, it will be assumed that failure of a slope is realized when its calculated available strength R is exceeded by the applied loading S; that is,

"Failure" $=[R<S]$

The probability of failure is then defined as

$P_f=P[R<S]$

In earlier reliability analysis of soil slopes, the shape of the failure surface has been assumed to be a circle and a $\phi=0$-condition has been considered in a probabilistic analysis, and uncertainties of soil shear strength has been assumed to be a probability distribution regardless of the length of the failure surface[11,18,19].

In this paper, uncertainties of the shear strength along potential failure surface are expressed by one-dimension random field models. The shape of the failure is assumed to be a log-spiral curve, and a drained condition is adopted to explain the notion of the present model.

2. Random Field Model of Soil Properties

On the base of measured point properties, it is necessary to infer the statistics of the spatial average property. The discrepancy in soil property between two points in a stratum is expected to increase as the two points become further apart. Random field modeling of the soil properties in a homogeneous stratum provides a mathematically tractable tool to bridge this gap[21]. Using a one-dimensional random field model[20], the mean and variance of the spatial average soil property overall length can be expressed as Fig. 1.

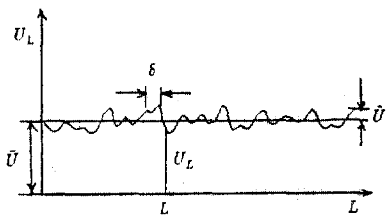

Fig. 1. One-dimensional Random Field Model.

$$\bar{U}_L=\bar{U}$$

$$\hat{U}_L^2=\left[\frac{2}{L}\int_0^L\left(1-\frac{\tau}{L}\right)\rho_u(\tau)d\tau\right]\hat{U}^2=\Gamma_u^2(L)\hat{U}^2 \quad (1)$$

where the model parameters $\bar{U}$, $\hat{U}$, and $\rho_u(\tau)$ are respectively the mean, standard deviation and correlation coeffcient between soil properities at distance τ apart and $\Gamma_u^2(L)$ is the variance function whose value range between 0 and 1.

3. Probability of Failure

The shape of the failure surface is assumed to be a log-spairal (Fig. 2) : the exact location of which depends on the ϕ-parameter of strength and two polar coordinates of its center (h_0, θ_0)[2,3].

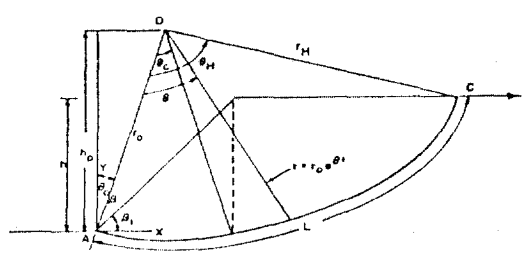

Fig. 2. Shape of Rupture Surface.

The statistacal values of h_0 and θ_0 are functions of the height h and angle β of the slope. In this paper, both h_0 and θ_0 are taken to be random variables following a general beta distribution. The mean values, variances and lower and upper limits of h_0 and θ_0 depend on the boundary geometry of the slope. The total length of the failure surface is expressed as a fuction of three factors (h_0, θ_0, t) and is equal to

$$L=\frac{h_0}{\cos\theta_0}(1+\frac{1}{t^2})^{1/2}(e^{\varrho_H t}-1) \qquad (2)$$

where $t=\tan\phi$. In Fig. 3 are shown the static forces which act on a differential element dL of the failure surface. If dx is the width of a slice, the weight dW is equal to

$$dW=\gamma_m(z-w)dx+\gamma_m'wdx \qquad (3)$$

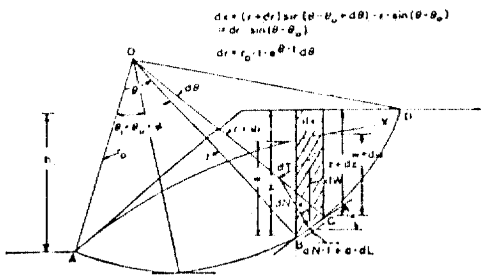

Fig. 3. Forces on a Differential Element Along the Rupture Surface.

where γ_m is the unit weight of the soil and γ_m' is its submerged unit weight, w is the height of the water above the failure surface, and z is the vertical distance of the failure surface from the slope boundary. The elevation of the ground water table is specified by the parameter R_u defined as

$$R_u=\frac{u}{\gamma_m z} \qquad (4)$$

where u is the pore water pressure at the failure surface. The normal and tangent components of the weigth dW are respectively

$$dN=dW\cos\epsilon$$

$$dT=dW\sin\epsilon \qquad (5)$$

where the angle of the failure surface with horizontal ϵ and equal to

$$\epsilon=\frac{\tan(\theta-\theta_0)-\tan\phi}{\tan(\theta-\theta_0)\tan\phi+1} \qquad (6)$$

The driving force dS and the resistance force dR acting along the element dL of the failure surface are respectively equal to

$$dS=dT \qquad (7)$$

$$dR=dNt+cdL$$

where t and c are the strength parameters of the soil. The overall driving force S and resisting force R are found through a numerical integration of Equation 7. The numerical values of the probability of failure of the soil slope are found by using Program MDNOR. (IMSL Routine Name). A flow that leads to the critical failure surface is shown in Fig. 4.

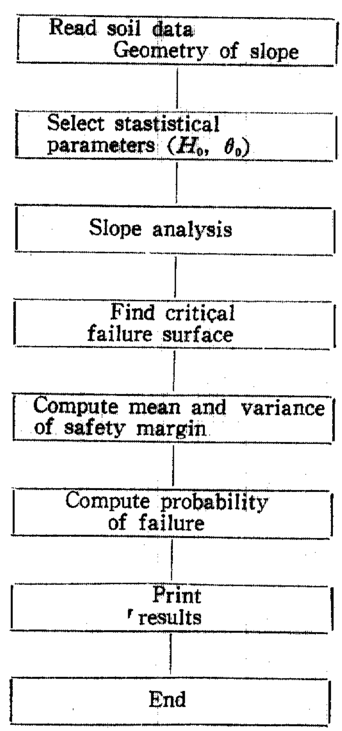

Fig. 4. Risk analysis of flopes: simplified flow chart.

It is assumed that the soil mass above the critical slip surfaces subdivided into a number of vertical slices(Fig. 3). The base of each slices is thus a segment of potential slip surface. The normal and tangential components of the weight of the i th slice W_i are

$$N_i = W_i \cos\epsilon_i$$
$$T_i = W_i \sin\epsilon_i \qquad (8)$$

The capacity, C_i (shearing resistance), and the demand D_i (shear force), along ith slice of the slip surface is given by

$$C_i = N_i \tan\bar{\phi}_p + \bar{c}_p L_i$$
$$D_i = W_i \sin\epsilon_i \qquad (9)$$

where $\bar{c}_p$ and $\bar{\phi}_p$ are respectively point mean of cohesion and friction angle. The safety margin SM_i of the$i$ th slice is

$$SM_i = C_i - D_i \qquad (10)$$

The safety maigin SM of the slope is

$$SM = \sum_{i=1}^{n} SM_i = \sum_{i=1}^{n} (C_i - D_i) \qquad (11)$$

The probability of failure is equal to

$$P_f = P[SM \leq 0] \qquad (12)$$

In order to dertermine this probability, the probability density function of the safety margin is assumed to be a normal distribution. Using one dimensional random field model, The mean and variance of safety margin is

$$\overline{SM} = \bar{c}_p L + N(\tan\bar{\phi}_p)$$
$$Var(SM) = L^2[\Gamma_p^2(L)\delta_p^2 + \Delta_p^2]\bar{c}_p^2 + N^2(\tan\bar{\phi}_p)^2[\Gamma_{p1}^2(L)\delta_{p1}^2 + \Delta_{p1}^2]\left(\frac{\partial \tan^2\phi_p}{\partial \phi_p^2}\right) \qquad (13)$$

where δ_p and Δ_p ae respectively point coefficient of variation and systematic modeling error of cohesion (peak shear strength). δ_{p1} and Δ_{p1} are respectively those of friction angle (peak shear strength). Γ's is one-dimensional variance function. In this paper, a triangular correlation function is used. The triangular correlation function that dercreases linearly from 1 to 0 as $|\tau|$ goes from 0 to δ:

$$\rho_u(\tau) = \begin{cases} 1 - \dfrac{|\tau|}{\delta}, & |\tau| \leq \delta, \\ 0 & |\tau| \geq \delta. \end{cases} \qquad (14)$$

Its variance function is

$$\Gamma_j^2 = \begin{cases} 1 - \dfrac{L_j}{3\delta} & L_j \leq \delta \\ \dfrac{\delta}{L_j}\left(1 - \dfrac{L_j}{3\delta}\right) & L_j > \delta \end{cases} \qquad (15)$$

where j stands for p *and* p_1.

$$P[SM \leq 0] = \int_{-\infty}^{0} f_{SM}(SM)\,dSM = F_{SM}(0) \qquad (16)$$

where f_{SM} and F_{SM} are the normal and cumulative normal distribution respectively.

4. Case Study and Discussion

An analysis with respect to probability of failure is performed on a statistically homogeneous soil slope with dimensions as shown in Fig. 5.

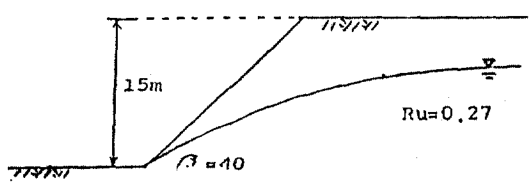

Fig. 5. Demensions of Soil Slope.

The center of the surface is expressed in polar coodinates by means of two random valiables, h_0 and θ_0 (Fig. 2). The mean values, variances and lower and upper limits of h_0 and θ_0 are givened in Table 1.

Table 1. Location of Center.

	h_0(m)	θ_0
mean	15	−1
standard deviation	5	9
maximum	30	−29
minimum	0	0
trial number	1000	1000

The total unit weight, submerged unit weight of soil, and pore pressure ratio are 19KN/m³, 9.2KN/m³, and 0.27 respectively.

The statistical values of point strength parameters are given in Table 2.

Table 2. Statistical Values of Strength Parameters.

Statistical Parameter	Soil Strength	
	Friction Angle of Soil (ϕ)	Cohesion (C)
Mean	20°	23KN/m²
Coefficient of Variation	0.1	0.4
Systematic error	0.08	0.3
Correlation Parameter (δ(m))	1 3 5 7 9 11	1 3 5 7 9 11

First, the critical failure surfave is determined using a one-dimensional homogeneous random field model of soil shear resistances and Program MDNOR(IMSL Routine Name). The analysis with respect to probability of failure is performed on the critical failure surface. Probabilities of failure are calculated with $\delta=1, 3, 5, 7, 9,$ and 11 respectivcly. It is clear from Table 3 that as δ increases probabilities of failure increase

Table 3. Overall Probability of Failure

	$\delta=1$	3	5	7	9	11
Probability of Failure	1.83×10^{-2}	2.62×10^{-2}	3.38×10^{-2}	4.10×10^{-2}	4.75×10	5.34×10^{-2}

5. Conclusion

A probabilistic model for the failure in a homogeneous soil slope is presented.

Uncertainties of the shear strength along potential failure surface are expressed by one-dimensional random field models.

It is expected that the proposed methodology for probabilistic characterization of soil properties would yield a more rational and realistic description of the in situ soil properties for further engineering analysis and design. Finally, in the case study it was found that the probability of failure of the slope increases as δ increases.

Acknowledgement

Thanks are due to the Science and Engineering Foundation for its financial support in the preparation of this paper.

Refernces

1. A-Grivas, D., "Seismic Analysis on Earth Slope," *Proceedings of the Specialty Conference on Probabilistic MechanicsStructural Reliability*, ASCE, Jan., 1979, pp. 338～342.
2. A-Grivas, D., "Probability of Failure of Soil Slope During Earthquakes," *Central American Conference of Earthquake Engineering*, Jan., 9～12, 1978, pp. 409～416.
3. Alonso, E.E., "Risk Analysis of Slopes and its Application to Slopes in Canadian Senstive Clays," *Geotechnique*, Vol. 26, No. 3, 1976, pp. 453～472.
4. Chowdhury, R.N., "Probabilistic Evaluation of Natural Slope Failures." *Proceedings of the Internal Conference on Engineering for Protection from Natural Disasters*, A.I.T., Bangkok, 1980, pp. 605～615.
5. Chowdhury, R.N., and A-Grivas, D., "Probabilistic model of Progressive Failure of Slopes," *Journal of the Geotechnical Engineering Division*, ASCE Vol. 108, No. GT6, 1982, pp. 803～819.
6. Tang, W.H., "*Principles of Probabilistic Characterizatioa of Soil Properties,*" in Peobabilistic Characterization of Soil Properties: Bridge Between Theory and Practice, ASCE, May 1984, pp. 74～89.
7. Tang, W.H., Yucemen, M.S., and Ang, A.H-s., "Probability-Based Short-term Design of Soil

Slopes," *Canadian Geotechnicai Journal*, 13, 1976, pp. 201~215.

8. Tang, W.H., Chowdhury, R.N., and Sidi, I., "Progressive Failure Probability of Soil Slope," *4th International Conference on Structure Safety and Reliability*, 1985, pp. III~363~373.
9. Vanmarcke, E.H., "Reliability of Earth Slopes," *Journal of the Geotechnical Engineering Division*, Vol. 103, No. GTII, Proc. Paper 13365, Nov., 1977, pp. 1247~1265.
10. Vanmarcke, E.H., "Random Fields: "*Analysis and Synthesis*," M.I.T. Press, Cambridge, Mass., 1983.
11. Wu, T.H., and Kraft, L.M., "Safety Analysis of Slope," *Jounal of the Soil Mechanics and Foundation Division*, Vol. 96, No. SM2, Proc. Peper 7174, Mar., 1970, pp. 609~631.
12. Yucemen, M.S., and Tang, W.H., "Long-Term Stability of Soil Slopes-A Reliability Approach," *Proceedings of the Second International Conference on Applications of Statistics and Probability in Soil and Structural Engineering*, 1975, pp. 1~12.

(接受：1988. 4. 21)