Properties and Performance of Generalized Wilcoxon Filters

일반화된 WILCOXON여파기의 성질과 성능

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ABSTRACT

In order to overcome the disadvantages of linear filters in certain cases of practical interest, a class of nonlinear filters (rank filters) are constructed based on a class of robust estimates, the rank estimates. A subclass of these filters, the limited-degree extended-averaging Wilcoxon filters, is then described as an interesting example of the rank filters with desirable characteristics. The properties of these filters are discussed and the performance of these filters are analyzed for ideal edges and narrow pulses.

요 약

몇몇 실제적인 경우에 나타나는 선형 여파기의 단점을 보완하기 위해, robust 추정량의 하나인 순서 추정량에 근본을 둔 비선형 여파기를 구현하였다. 특히 Wilcoxon여파기와 확장 Wilcoxon여파기의 특성을 조사하였고, 이상적인 edge 입력 및 좁은 pulse입력에 대한 이들 여파기의 성능을 분석 검토하였다.

I. INTRODUCTION

Linear filters have been widely used for suppressing additive Gaussian noise in a stream of noisy input data composed of desired signals and noise in many signal processing schemes. The linear filters, however, give poor performance characteristics in certain situations of practical interest. For example, they smear out edges and narrow pulses in the original signal, resulting in blurred edges in an image, and they are also very poor in suppressing impulsive (heavy-tailed) noise.

In order to overcome these disadvantages of linear filters, nonlinear techniques have been proposed and shown to be effective in such situations [1-4]. Median filters, for example, have strongly nonlinear characteristics, being able to reject quite effectively impulsive noise

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components while preserving edges in the original signal. Examples of applications of median filters can also be found in nonlinear speech processing and image processing [5,6]. Their deterministic as well as statistical properties have also been investigated in [7-9]. Efficient realizations for real-time median filtering and VLSI implementations of median filters have been considered quite recently in [10,11].

A median filter, however, does not in general allow the user a sufficient degree of control over its characteristics. Furthermore, median filters do not have the averaging operation that is particularly appropriate in reducing additive Gaussian noise components in noisy data; thus they may perform poorly in Gaussian noise. Therefore, for a better overall performance when the signal has both edges and details and the noise has both Gaussian and impulsive components, it is desirable to implement a filtering scheme with an algorithm that has both nonlinear and linear (averaging) characteristics.

Among the typical examples developed for this purpose are cascades of median and linear filters, the order-statistic (OS) filters or the L-filters [2,3], the M-filters [3], the K-nearest neighbor (KNN) filters [12], the symmetric nearest neighbor (SNN) mean filters, the symmetric nearest neighbor median filters [13] and the linear median hybrid (LMH) filters [14].

It is noteworthy that the L- and M- filters are based on classes of robust estimates [15,16], the L- and M- estimates, respectively. Since these two classes of robust estimates from statistical theory have successfully been exploited in the area of signal processing, it is quite natural to seek similar applications of the third major class of robust estimates, the rank estimates (R-estimates) [17,18], of statistical theory. In this paper a new class of nonlinear discrete-time filters for edge-preservation, detail retention and Gaussian and impulsive noise reduction is considered as an application of a class of rank-estimates in signal restoration.

II. THE RANK FILTERS

Our objective here is to define a new class of filters based on R-estimates, for use in restoration of signals containing edges and fine details, in addition to smoothly-varying portions, which are observed in additive noise containing impulses as well as Gaussian noise components.

One special form of finite impulse response (FIR) linear filtering is obtained when the output y_k is the arithmetic average of N values in a processing window of size N. The median filter may be viewed as its more robust counterpart which is quite effective in impulsive environments but which is not as good for additive Gaussian noise smoothing. The concept of robustness is particularly appropriate in signal processing when the noise has impulsive components (or outliers), since robustness implies insensitivity to a slight deviation (a small number of impulses) from a nominal assumption (usually of Gaussian noise). Such a robust scheme is also effective for edge preservation since near the start or the end of an edge the data in a window appears to contain a few impulses.

The main motivation of our investigation in this paper arises from the fact that there exist other classes of robust estimates in statistics which may function very well in our filtering problems. We will concentrate our attention on the class of R-estimates because this class of robust estimates is the third main class after L- and M-estimates. R-estimates were originally derived from nonparametric hypothesis testing theory (which gives us, amongst other possibilities, rank tests for testing shifts or location changes).

A general two-sample statistic for testing location shift between two samples may be defined as [15]

$$S_{m,n} = \sum_{i=1}^{m} a\left(\frac{R_i}{m+n+1}\right) \qquad (1)$$

where m and n are the sizes of the two samples,

$a(.)$ is a nondecreasing score function defined on (0,1) with $a(t) = -a(1-t)$, $0<t<1$ and R_i is the rank of the i-th observation from the sample of size m in the pooled sample of size $m+n$. (That is, R_i is the number of observations less than or equal to the i-th observation.)

If the locations of the two samples are the same, the ranks of the observations from the first sample (a sample of size m) are equally likely to be any set of m of the $m+n$ integers $\{1, 2, \cdots, m+n\}$. (Here a parameter θ is called a location parameter if the distribution has the form $f(x-\theta)$ for a mathematically specified distribution $f(.)$ On the other hand, if the location of the first sample is greater than that of the second sample, the ranks of the observations from the first sample will tend to have larger values, resulting in a larger value for $S_{m,n}$ on the average. Therefore by comparing $S_{m,n}$ to a threshold that can be set *a priori* or varied adaptively (to satisfy, for example, a false-alarm probability criterion), a test for location difference for the two samples is obtained.

Since only one sample of size N is available in each window in our problem of signal restoration, another set has to be defined from each of the original samples of size N before further processing in which such a rank statistic is used. For this purpose mirror-imaging of the sample in each window about a candidate location value (estimate) may be employed.

Let y_k be the candidate location estimate for the sample of size N centered on the K-th time index. We may center the original sample $(x_{k-n}, \cdots, x_{k+n})$ to become $(x_{k-n}-y_k, \cdots, x_{k+n}-y_k)$, and then take its mirror image $(y_k-x_{k-n}, \cdots, y_k-x_{k+n})$ as the second sample. Note that y_k will be the output of an R-filter of window size $N=2n+1$ at time index k, for a discrete-time input sequence $\{x_k\}$. The output y_k is that rank estimate that is calculated to satisfy the implicit equation

$$S_{N,N} = 0. \tag{2}$$

In essence, Equation (2) implies that an estimate y_k of x_k should be found so that the resulting ranks of elements of each of the two samples defined above, from the pooled sample, have statistically similar values. In other words, if y_k is an estimate of x_k the centered and mirror-imaged or reflected samples will be located at the same point.

Note that sometimes it is impossible to achieve an exact zero in Equation (2), because of the discreteness of the expression $S_{N,N}$. In this case we attempt to make $S_{N,N}$ as close to zero as possible. The discrete function $a(.)$ plays an important role in the R-filter, determining its characteristics. By choosing appropriate forms for $a(.)$, a number of special R-filters can be defined with characteristics different from one filter to another.

The essence of the operation of an R-filter is that what affects the output of the R-filter more is not the actual values of the data but their relative ranks in each processing window at any time index. Thus very large values such as produced by impulsive components will have much less of an effect on the output than they would have in linear filters that process actual values of the data.

III. THE WILCOXON FILTER AND ITS GENERALIZATIONS

In this section, our attention will be restricted to a subclass of R-filters for which it is possible to derive useful and practical properties and explicit structures.

The Wilcoxon Filter

Some particular scores are well-known in statistical theory for their useful properties. For example, the normal scores function [17] is $a(t) = \Phi^{-1}(t)$ with Φ^{-1} being the inverse of the standard normal distribution function. This results in asymptotically optimum Gaussian noise suppression. Another choice of interest, for its simplicity, is a limiter-type score function;

that is, $a(t)=1$ for $t>0.5$, 0 for $t=0.5$ and -1 for $t<0.5$ with the ranks of identical values defined to be the averages of their ranks. This particular choice leads to the median filter which is optimum for smoothing of noise with a double-exponential probability density function (pdf), which is a very heavy-tailed pdf.

The linear score function, $a(t)=t-0.5$, lies between the above two score functions and leads to the Wilcoxon filter [19]. In this case, it can be shown [15,19] that (2) can explictly be solved go give

$$y_k = \text{median}\left\{ \frac{x_p + x_q}{2}, \ \text{for} \ (p, q) \in H \right\}, \qquad (3)$$

which is the Hodges-Lehmann estimate [20]. Here $H=U=\{(p,q): k-n \leq p, q \leq k+n\}$. If $V=\{(p,q): k-n \leq p \leq q \leq k+n\}$ or $\widetilde{V}=\{(p,q): k-n \leq p < q \leq k+n\}$ is used for H in (3) the resulting filter can be shown [15] to be asymptotically equivalent to the filter using $H=U$ in (3). The estimate (3) represents a data-dependent L-filter with time-varying coefficients which are determined by the data in each window, at most two of which are nonzero. This is because at any time index k, if we denote the i-th order statistic as $x[i]$, y_k will be $\frac{x[r]+x[s]}{2}$ for some r and s, where r and s depend on the data in each window. Note also that (3) represents a generalization of median filters with an inherent averaging operation.

As has been shown in [19], (3) can be transformed when $H=V$ to

$$\sum_{p=k-n}^{k+n} \text{sign}(x_p - y_k) R(|x_p - y_k|) = 0, \qquad (4)$$

where $R(|x_p - y_k|)$ is the rank of $|x_p - y_k|$ in the set $\{|x_i - y_k|, i=k-n, \cdots, k, \cdots, k+n\}$. It is clear from (4) that the Wilcoxon filter may also be interpreted as a time-varying M-filter.

The Limited-Degree Extended-Averaging Wilcoxon (LEW) Filter

Even though the Wilcoxon filter has both linear (averaging) and nonlinear (median operation) characteristics, the edge-preserving and noise reduction properties of this filter are not remarkable. One reason for this mediocre edge performance of the Wilcoxon filter is that every possible pair of values in each window is averaged; for example, a value on one side of an edge is averaged not only with a value on the same side but also with one on the other side, which results in smearing of the edge. A reason for the relatively mediocre Gaussian noise reduction characteristic is that the output of each window is an average of only two values, independent of the actual size of the processing window.

The above considerations do offer some ideas for improving the performance characteristics of the Wilcoxon filter. In particular, it would appear to be quite reasonable to modify the Wilcoxon filter by (1) limiting the maximum distance between time indices used in the pair-wise averaging in each window to be less than some value D, and (2) replacing the pairwise average with a more general averaging of M terms Even though M can theoretically take on any integer value $M \geq 1$ and D can take on any integer value $1 \leq D \leq N$ it is quite obvious from the above observations that values greater than or equal to 2 should be used for M and D to get reasonable performance characteristics.

This modification will produce the limited-degree extended-averaging Wilcoxon (LEW) filters with the filter output

$$y_k = \text{median}\left\{ \frac{1}{M}\sum_{i=1}^{M} x_{m_i}, \ (m_1, m_2, \cdots, m_M) \in V_M, \ \max|m_i - m_j| < D \right\}, \qquad (5)$$

where

$$V_M = \{(m_1, m_2, \cdots, m_M): k-n \leq m_1 \leq m_2 \leq \cdots \leq m_M \leq k+n\}. \qquad (6)$$

The parameters M and D will be called the order and the degree of the filter, respectively. As the value of D becomes smaller, the LEW filter will be closer in performance to the median filter; on the other hand, as the values of D and M grow closer to N, the LEW filter will act more like a linear filter.

It is expected from the modifications above that the LEW filters have better overall performance characteristics than the original Wilcoxon filter. It is noteworthy that the performance characteristics of the LEW filters can be controlled by a set of parameters (D, M); if more smoothing is needed, larger values of D and M can be used; if a strong nonlinear characteristic is required because an original signal has many edges or narrow pulses, small values of D and M may be used.

If $\tilde{V}_M = \{(m_1, m_2, \cdots, m_M) : k-n \leqq m_1 < m_2 < \cdots < m_M \leqq k+n\}$ is used for the subscripts $(m_1, m_2, \cdots, m_M)$ instead of the set of subscripts $V_M = \{(m_1, m_2, \cdots, m_M) : k-n \leqq m_1 \leqq m_2 \leqq \cdots \leqq m_M \leqq k+n\}$ in (5), in which case we must have $M \leqq D$, the computational burden will be reduced.

The LEW filter has the following properties of interest:

1. The LEW filter is a scale and translation invariant filter: that is, if we denote the output sequence $\{y_j\}$ of an LEW filter for an input sequence $\{x_j\}$ as $\{y_j\} = F(\{x_j\})$, we have

$$F(a\{x_j\}+b\{1\}) = F(\{ax_j+b\}) = aF(\{x_j\})+b\{1\}, \tag{7}$$

   where $\{1\}$ is the sequence of constant value of 1, and a and b are any real constants.
2. For a linearly increasing (or decreasing) input sequence, that is for $x_j = j,\ j = 0, 1, 2, \cdots$ we have $y_j = j$.
3. If D = 1 or M = 1 the LEW filter is the median filter.
4. If D = N and M = 2 the LEW filter is the Wilcoxon filter.
5. If D = M = N the LEW filter is almost the running mean filter. (If $\tilde{V}_M$ is used for the subscripts $(m_1, m_2, \cdots, m_M)$ in (5), we would have exactly the running mean filter under the same condition.)
6. The LEW filter is a data-dependent L-filter with at most M out of N coefficients having nonzero values. These coefficients can take values only from the set

$$\left\{0, \frac{1}{M}, \frac{2}{M}, \cdots, \frac{M-1}{M}, 1\right\}.$$

IV. A DUAL CLASS OF FILTERS

It is noteworthy that the filters we have considered in the previous section are nontrivial subclasses of a more general class of filters whose filter transformation $T_1(X)$ can be expressed as

$$T_1(X) = \text{median}\{f(X)\} \tag{8}$$

where f is a vector linear operation mapping R^N to R^P, the median operator is a nonlinear function mapping R^P to R and X is the vector of N observations in a window of size N. For example, with f the averaging operation and $P = 1$ we get the running mean filters; with f the identity function, T_1 represents the median filters.

In particular, the LMH filter whose output is the median of means of disjoint subsets of the observations in a window can naturally be considered as a special case of this class, when the P components of $f(X)$ are averages of contiguous observations in disjoint subsets of X. It is clear that the choice of f in (8) which yields the Wilcoxon filter is

$$f(X) = \left\{ \frac{X_i + X_j}{2},\ 1 < i < j < N \right\}.$$

Let us now consider a dual class of nonlinear filters which can be constructed by an interchange of the linear and nonlinear operations in (8); the resulting class of nonlinear filters is defined by the transformations

$$T_2(X) = \text{mean}\{g(X)\} \tag{9}$$

where g is generally a nonlinear vector function mapping R^N to R^P. The P components of $g(X)$ may in many case of interest, be interpreted as each arising from some nonlinear transformation of a subset of the N observations. If g is the full median operation and $P=1$ then T_2 represents the median filters; if g is the identity function then T_2 represents the running mean filters. By choosing other functions or operations for g, more interesting filters such as the L-filters, the KNN filters and the SNN-mean filters can be derived from this general class of filters. It should also be noted that the nonlinear mean filters, which have recently been considered and analyzed in [21], are of this type.

A particularly interesting nonlinear filter class giving filters dual to the LEW filters is obtained with each of the P components of $g(X)$ being median $\{X_{m_1}, X_{m_2}, \cdots, X_{m_M}\}$ for P=H (N,M) or P=C (N,M) possible combinations of subscripts $(m_1, m_2, \cdots m_M)$ in V_M or $\tilde{V}_M$, respectively $C(k,j) = \frac{k!}{(k-j)!\,j!}$ and H (k, j) is the number of combinations, allowing duplication, of j elements out of k possible elements; it is equal to $C(k+j-1, j)$. In this case $T_2(X)$ represents a class of L-filters with nonequal symmetric coefficients [22]. More explicity, the filter transformation becomes, for M odd and $\tilde{V}_M$ in the definition of the filter,

$$T_2(X) = \frac{(N-M)!\,M!}{N!} \sum_{i=(M+1)/2}^{N-(M-1)/2} C\left(i-1, \frac{M-1}{2}\right) C\left(N-i, \frac{M-1}{2}\right) X[i] \tag{10}$$

where $X[i]$ is the i-th smallest component in X. This filter transformation becomes, for example,

$$T_2(X) = \frac{1}{21}\{6X[3] + 9X[4] + 6X[5]\}, \tag{11}$$

when $N=7$ and $M=5$. It can also be shown that the filters with $M=2l$ and $M=2l-1$ are exactly the same for given window size N [22].

Note that the above two dual classes of filters are not disjoint. Any given filter may be considered to belong more naturally to one of these two classes.

V. ANALYSIS OF FILTER PERFORMANCE

In this section, the performance of the LEW filter will be considered for an ideal edge input and inputs with narrow pulses of various widths.

Performance for an Ideal Edge

An ideal edge may be defined as a noise-free step from a constant value to another constant value. Without loss of generality, we will assume here that the step is from 0 to 1. Let us consider an input with an ideal edge which can be represented by the sequence {...0 0 0 0 1 1 1 1 ...}. It is quite straightforward to obtain the outputs of different LEW (M,D) filters for this input. Some results for window sizes $N=5$, $N=7$ and $N=9$ are given in Table 1(a) and (b). More generally, it can be proved [22] that the LEW(2,2) filters of any window size $N=2n+1 \geq 3$ preserve an ideal edge as does the median filter, while the Wilcoxon filter, which is the LEW(2,N) filter, does not preserve ideal edges.

In fact it is possible to obtain a general result on the ideal edge performance of the LEW(2,D) filters for any window of size $N \geq 3$; the result is that as D increases for fixed N the extent of smearing of the ideal edge remains constant with two values only around the edge being smeared up to a certain value of D; beyond this the edge begins to get smeared more, with more output values of 1/2 around the ideal edge position. Although we do not provide analytic performance characterization of LEW filters with $M>2$, it can be expected that more smearing of edges will occur with increasing M, as seen in Table 1(b).

Performance for Narrow Pulses

For narrow pulses of heights 1 and various widths $W>2$, the outputs of several filters of window size $N=7$ are given in Table 2. Table 2 clearly shows that the LEW(2,2) and LEW(3,3) filters retain narrow pulses better than the median and Wilcoxon filters of the same window size 7. Table 2 also shows the tradeoff between narrow pulse retention and output pulse integrity; in order to get reasonable narrow pulse retention characteristics, a small amount of smearing and loss of amplitudes is unavoidable.

If we consider a pulse of width $W=1$ (which is an impulsive noise component in practice), for which the input sequence is {..0 0 1 0 0..}, the outputs of the above three filters will be exactly the same, the all-zero sequence {..0 0 0 0 0..}, which implies good impulsive noise rejection properties of the filters.

Though the above observations have been made from results for a fixed window size, an analysis of the narrow-pulse retention characteristics of the LEW(2,2) filter for any window of size $N=2n+1\geq 3$ shows that the LEW(2,2) filter retains pulses of width $W\geq n+1$ and it also retains pulses of width $W\geq n-1$ with half the amplitude [22]. It should be noted that the median filter with the same window size will also retain pulses of width $W\geq n+1$, but will remove pulses of with $W<n$.

It is also possible to explain the pulse response of the Wilcoxon filters. For the Wilcoxon filter the response depends not on the pattern of the ones and zeroes for a binary input, but only on the numbers of ones and zeroes inside a window. Thus the ideal edge performance of the Wilcoxon filter can be used to obtain its ideal pulse performance. For example, when $N=7$ and $W=2$, the Wilcoxon filter will not be able to retain this pulse because at any time there are at most two ones thus producing a zero as the output. From its edge performance we conclude that when $N=7$ and $W=3$, the output of the Wilcoxon filter will be a smeared and reduced pulse $\{\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}\}$ and for a pulse of width $W=4$, the output of the Wilcoxon filter with window size 7 will again be a smeared and reduced pulse $\{\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}\}$ this is in agreement with the results in Table 2.

Table 1. Outputs of LEW filters of different window sizes for an ideal edge input.

(a) $M=2$

LEW (5, 2, 2) Filter	·· 0 0 0 0 1 1 1 1 ··
LEW (5, 2, 3) Filter*	·· 0 0 0 $\frac{1}{4}$ $\frac{3}{4}$ 1 1 1 ··
LEW (5, 2, 4) Filter*	·· 0 0 0 $\frac{1}{2}$ $\frac{1}{2}$ 1 1 1 ··
LEW (5, 2, 5) Filter	·· 0 0 0 $\frac{1}{2}$ $\frac{1}{2}$ 1 1 1 ··
LEW (7, 2, 2) Filter	·· 0 0 0 0 1 1 1 1 ··
LEW (7, 2, 3) Filter*	·· 0 0 0 $\frac{1}{4}$ $\frac{3}{4}$ 1 1 1 ··
LEW (7, 2, 4) Filter*	·· 0 0 0 $\frac{1}{2}$ $\frac{1}{2}$ 1 1 1 ··
LEW (7, 2, 7) Filter*	·· 0 0 0 $\frac{1}{2}$ $\frac{1}{2}$ 1 1 1 ··
LEW (9, 2, 9) Filter	·· 0 0 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 1 ··

(b) $M=3$

LEW (5, 3, 3) Filter*	·· 0 0 0 $\frac{1}{3}$ $\frac{2}{3}$ 1 1 1 ··
LEW (5, 3, 4) Filter*	·· 0 0 0 $\frac{1}{3}$ $\frac{2}{3}$ 1 1 1 ··
LEW (5, 3, 5) Filter	·· 0 0 0 $\frac{1}{3}$ $\frac{2}{3}$ 1 1 1 ··
LEW (7, 3, 3) Filter*	·· 0 0 0 $\frac{1}{3}$ $\frac{2}{3}$ 1 1 1 ··
LEW (7, 3, 4) Filter*	·· 0 0 0 $\frac{1}{3}$ $\frac{2}{3}$ 1 1 1 ··
LEW (7, 3, 5) Filter	·· 0 0 0 $\frac{1}{3}$ $\frac{2}{3}$ 1 1 1 ··
LEW (7, 3, 6) Filter	·· 0 0 $\frac{1}{3}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{2}{3}$ 1 1 ··
LEW (7, 3, 7) Filter*	·· 0 0 $\frac{1}{3}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{2}{3}$ 1 1 ··

* median of an even number of values is taken to be halfway between the two middle values.

Table 2. Outputs of LEW filters for narrow pulse input.

(a) $W=2$, $N=7$

Input	·· 0 0 0 0 1 1 0 0 0 0
Median Filter	·· 0 0 0 0 0 0 0 0 0 0 ··
LEW (2, 2) Filter	·· 0 0 0 0 0 0 0 0 0 0 ··
LEW (2, 7) Filter	·· 0 0 0 0 0 0 0 0 0 0 ··
LEW (3, 3) Filter	·· 0 0 0 0 $\frac{1}{3}$ $\frac{1}{3}$ 0 0 0 0 ··

(b) W=3, N=7

Input	··0 0 0 0 1 1 1 0 0 0 0··
Median Filter	··0 0 0 0 0 0 0 0 0 0 0··
LEW (2, 2) Filter	··0 0 0 0 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 0 0 0 0··
LEW (2, 7) Filter	··0 0 0 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 0 0 0··
LEW (3, 3) Filter	··0 0 0 0 $\frac{1}{2}$ $\frac{2}{3}$ $\frac{1}{2}$ 0 0 0 0··

(c) W=4, N=7

Input	··0 0 0 0 1 1 1 1 0 0 0 0··
Median Filter	··0 0 0 0 1 1 1 1 0 0 0 0··
LEW (2, 2) Filter	··0 0 0 0 1 1 1 1 0 0 0 0··
LEW (2, 7) Filter	··0 0 0 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 0 0 0··
LEW (3, 3) Filter	··0 0 0 $\frac{1}{3}$ $\frac{2}{3}$ $\frac{2}{3}$ $\frac{2}{3}$ $\frac{2}{3}$ $\frac{1}{3}$ 0 0 0··

VI. SUMMARY

In summary, a class of nonlinear filters has been introduced that has a basis in the class of rank estimates of statistical theory, exploiting the robustness of the estimates. Some useful filters that are effective for impulsive noise rejection, edge preservation and detail retention as well as Gaussian noise reduction have been considered. This class of filters also includes several common filters such as the mean, median and Wilcoxon filters as special cases. A discussion of classes of general nonlinear filters and the interrelation of L-, M- and R-filters was given. Analyses of performances of the filters were investigated and compared in various situations.

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