The Effects of Thermal Front on Sound Propagation in Shallow Seas of Korea.

한국 천해에서 수온전선이 수중음향전파에 미치는 효과

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ABSTRACT

The thermal front over the shallow coastal seas of Korea during the winter season provides very unique acoustic media such that wave equation is easily separable and the solutions turn out to be very simple and well known. In steady of using the WKB method to solve the radial equation the mode technique have been applied to obtain the solution. The radial propagation is rather weakly influenced by the presence of the thermal front that causes the horizontal variations of the sound speed. The physical description of the sound propagation is also presented in terms of ray tracing.

요 약

한국 남서 해안에 겨울철 발생하는 수온전선으로 매우 특이한 음향매체가 존재한다. 이러한 조건하에서의 음파방정식은 변수분리가 용이하고 그 해 역시 간단하며 이는 이미 잘 알려진 수학적인 해이다. 수평 성분의 방정식 해를 구하기 위하여 WKB 방법 대신 mode 방법을 사용하였으며 구한 해의 특성은 수온전선이 존재함으로 야기된 수중음속의 수평변화로 인한 영향은 주파수에 따라 그 크기가 달라진다. 이를 설명하기 위해 ray tracing을 이용, 그 물리적인 의미를 부여하였다.

I. INTRODUCTION

Acoustic transmission in the ocean, particularly in shallow seas is profoundly affected by the dependence of the speed of sound on depth and range. Since in the shallow seas the speed of sound is mostly affected by water temperature when horizontal variations of temperature within a narrow region exist (called thermal front) a change in the speed of sound should also occur over the same region. Thus the medium becomes range-dependent and this leads to very complicated problems in solving the acoustic transmi-

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ssion(1). The main reason for the difficulties of solving the problem is due to inseparability of the wave equation since the wave number (or the sound speed) which is function of both range and depth can not be separated.

For weakly range dependent media the coupled mode theory was proposed(2, 3) to approximate the range dependence by attempting a partial separation of variables. In the wave equation the range dependence of the medium enters in two ways. One is through the range dependence of the boundary conditions which involve the variations in either the depth of water or the geoacoustic parameters underneath the bottom. And the other is through the range dependence of the wave number.

Unlike the oceanic thermal front that exists over the broad area in deep water the thermal front in shallow water exists seasonally over relatively flat bottom. In addition the coastal water mass is well mixed vertically such that a constant gradient of the vertical sound profiles at any location could be expected.

Therefore the purpose of this study is to find an approximate method to solve the wave equation by separating the wave number following the range dependent profile of the sound speed in the shallow environment. The acoustic characteristics of the shallow thermal front will be described in terms of its vertical and horizontal variations of the speed of sound.

And based on the profiles observed along the southwest coast of Korea the profiles will be put into a profile function that could describe the observed speed of sound thereby the separation of the wave number is attemped. Finally the asymptotic behavior of the transmission will be discussed.

II. ACOUSTIC PROPERTIES OF THE THERMAL FRONT

According to Wilson(4) the sound speed at sea is given in equation (1)

$$c = 1449.2 + 4.623T - 0.0546T^2 + 1.391(S-35) + 0.016z \tag{1}$$

where c = speed (m/sec)
T = temperature (℃)
S = salinity (‰)
z = depth (m).

In shallow water where the maximum depth is less than 200m and the salinity change is rather small (<1‰) then the major effect comes from the temperature in estimation of the speed of sound.

The term "thermal front" has been defined in such a way that when two water masses of different temperatures meet then it produces a rather strong temperature gradient over a narrow region or a thermal front. The southern coastal sea of Korea is shallow in depth and it is strongly influence by seasonal change in atmospheric conditions. In particular, the cold and dry winter monsoon and strong tidal currents mix the whole column of the shallow water mass while maintaining the horizontal temperature gradient vertically. Therefore, the cold coastal water which has been vertically mixed and the warm and saline open-sea water produce the thermal front over the shallow coastal area where they meet(5). A very weak temperature gradient also exists during the summer season however its magnitude compared with the winter season is negligible in terms of the sound speed gradient in horizontal direction.

The observed sound speed profiles at some locations in the southern coastal area are presented in the figures 1 through 3. These have been obtained from the 20 years average of the monthly distributions of both temperature and salinity fields. Over that particular range the speed of sound is almost homogeneous in vertical and increases rather slowly toward the open sea until the frontal location emerges. It is also shown that the bottom topography over which the speed change occurs is relatively flat and covered with mud or clay as major consistuent.

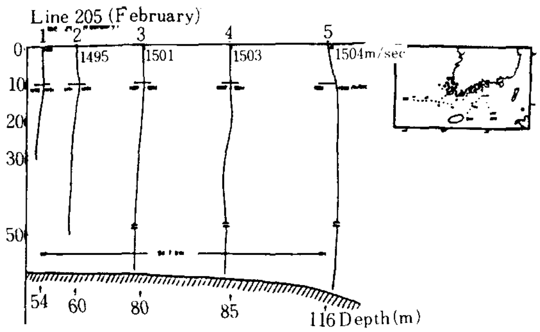

Fig. 1. The vertical profiles of sound at each station indicated on the inlet.

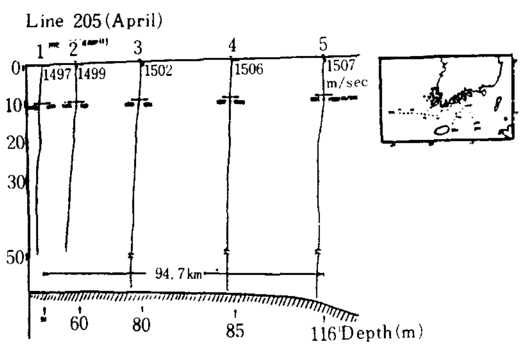

Fig. 2. The vertical profiles of sound at each station indicated on the inlet.

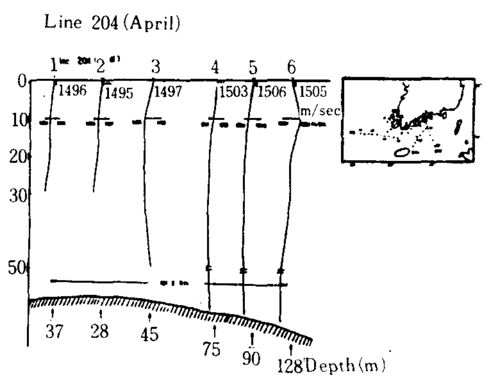

Fig. 3. The vertical profiles of sound at each station indicated on the inlet.

The horizontal gradient of the speed, dc/dr, across the coastal area from shallow to deep water is in the order of 10^{-4} per second. With this size of variation of the medium the adiabatic approximation may be applied to compute acoustic field since the effects of mode-mode coupling become small and may be negligible(6). In figure 4 the sound speed distribution along the line normal to the front shows that the front exists almost parallel with the isopleths of the depth. However, the acoustic properties of the coastal water is rather unique in that the possible difficulties involving the separation of the wave number may not exist. And it will provide better chance to look into the mechanism of sound propagation with simplified acoustic medium.

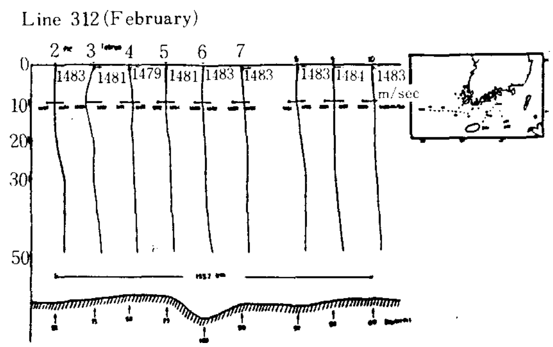

Fig. 4. The vertical profiles of sound at each station indicated on the inlet.

III. SIMPLE SOLUTION

Since the speed of sound, c, over the coastal sea is locally constant and varies only with horizontal distance it can be simply put into a range dependent function. And it is given as

$$\frac{1}{c^2(r)} = pr + q \tag{2}$$

where p, q are constants and r is horizontal distance. The equation (2) is selected as a profile function since it represents the observed sound speed profiles and furthermore it is more convenient for mode techniques.

In stead of using the equation (2), we could use c(r) as a linear function of range, but the exact solutions of the wave equation are linear combinations of Bessel functions whose order is a function of frequency, and they are not easy to use (7). Since $c = c(r)$, the wave number, $k = \frac{\omega}{c}$, where w is angular frequency, is also function of range only, this enables us to separate the wave equation without difficulty.

The time-independent wave equation in cylindrical coordinate system can now be separated into the followings;

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} + k^2(r) - \kappa^2\right] R(r) = 0 \qquad (3)$$

$$\left[\frac{d^2}{dz^2} + \kappa^2\right] \phi(z) = 0 \qquad (4)$$

where κ is the separation constant and $R(r)$, $\phi(z)$ are radial and depth dependent functions respectively. The sound can propagate from the low speed region toward the high speed or vice versa. It is assumed however that the top and the bottom are perfectly reflecting boundaries without loss of generality.

Then the depth function $\phi(z)$ can be easily obtained.

$$\phi_n = \sqrt{\frac{2}{H}} \sin \kappa_n z \qquad (5)$$

$$\kappa_n = (n - \frac{1}{2}) \frac{\pi}{H}, \quad n = 1, 2, 3$$

κ_n is the eigen values and H is the local depth of water where the equation is held. The radial equation (3) is not easy to solve, for the range dependence of k determines the type of Bessel functions as mentioned before. A more convenient differential equation is obtained from (3) by substituting

$$R(r) = \frac{F(r)}{\sqrt{r}} \qquad (6)$$

The reduced wave equation, then gives

$$\frac{d^2}{dr^2} F(r) + \left[k^2(r) - \kappa^2 + \frac{1}{4r^2}\right] F(r) = 0 \qquad (7)$$

For large r the last term in the bracket can be negligible and the WKB solution is possible provided that

$$\frac{1}{Q}\frac{d}{dr} l_n Q \ll 1 \qquad (8)$$

where $Q = (k^2 - \kappa^2)^{1/2}$

Then the solution becomes

$$R(r) = \frac{F(r)}{\sqrt{r}} = \frac{C_0}{\sqrt{r}} \frac{1}{Q} \exp \int iQdr \qquad (9)$$

where C_0 is constant.

The solution implies that the sound wave propagates with cylindrical spreading and its behavior depends on the sign of $(k^2 - \kappa^2)$. Since the WKB approximation is good for high frequency we now return to the mode techniques with least approximation.

For the profile given by equation (2), equation (7) can be rewritten

$$\frac{d^2F}{dr^2} + (\alpha^2 + \frac{1}{4r^2}) F = 0 \qquad (10)$$

where

$$\alpha^2 = \frac{\omega^2}{c^2} - K^2$$

$$= [\omega^2 (pr + q) - K^2] \qquad (11)$$

By neglecting the (1/4r) term for large r and introducing a new independent variable

$$S = \int^r \alpha\, dr$$

$$= \frac{2}{3p\omega^2} \alpha^3 \qquad (12)$$

Equation (10) becomes the Stokes equation (8)

$$\frac{d^2F}{ds^2} + sF = 0 \qquad (13)$$

The solution of the Stokes equation is well known and it gives us

$$F = \alpha [A J_{1/3}(s) + B J_{-1/3}(s)] \tag{14}$$

where $J_{\pm 1/3}$, Bessel functions of $\pm 1/3$ order and A, B are constants.

The property of the Bessel function is such that for large S, the asymptotic form for $J_{\pm 1/3}(s)$ are

$$J_{\pm 1/3}(s) \simeq \sqrt{\frac{2}{\pi s}} \cos[s - (\frac{1}{4} \pm \frac{1}{6}) \pi] \tag{15}$$

to give

$$F \simeq \frac{3A\sqrt{p}}{\sqrt{\pi \alpha}} \cos (s - \frac{\pi}{4}) \tag{16}$$

from the condition, A=B that must be satisfied for $F \to 0$ as $r \to \infty$.

The radial function R(r) thus becomes

$$R(r) \simeq \frac{3A\sqrt{p}}{\sqrt{\pi r \alpha}} \cos (s - \frac{\pi}{4}) \tag{17}$$

So far we have assumed that $\alpha^2 > 0$ from equation (11). In fact $\alpha^2 = 0$ corresponds to a turning point of equation (10), without the (1/4r) term.

Since the value of x can be obtained from the depth equation and it turns out to be constant values for each mode number, α can be determined by equation (11) and thus depends on the speed profile c(r). In order to have $\alpha^2 < 0$, x must exceed the value of ω / c or x must be very large. In other words $\alpha^2 < 0$ means the case of high modes in the depth function and it is not significant with regards to its contribution to the transmission of the sound.

For $\alpha^2 < 0$ and $\sigma \gg 1$

$$F(r) \simeq \frac{3A}{2} \frac{\sqrt{p}}{\sqrt{\pi \alpha r}} e^{\sigma} \tag{18}$$

where $\sigma = \frac{2}{3 p \omega^2} \alpha'^3$

$$\alpha^2 = -\alpha'^2$$

F(r) in (18) decays exponentially as r increases.

Thus for the observed sound field over the coastal area where the speed increases slowly toward the open sea, equation (17) may represent the radial dependence of the sound field. Incidently it is very much like the one we have obtained by using the WKB method in equation (19).

However equation (17) have been obtained without consideration of the frequency or the wave length limit. But the characteristics of the media arised from the presence of the thermal front in shallow water has provided a very simplified solutions such as (17).

IV. DISCUSSION AND CONCLUSION

When the thermal front exists over the shallow coastal seas the sound speed varies in such a unique way that it increases slowly over the flat bottom with almost isospeed profile in vertical. This type of propagation media is very common over the southwest coast of Korea in winter season.

Therefore, without difficulties in separation of variable in terms of the range dependence of the wave number simple wave equation has been obtained. By using the mode techniques, the radial dependent functions have been easily obtained for the present case of medium. The depth equation was futher simplified because of the flat and homogeneous bottom.

The overall physical picture of the sound propagation may be described by the figure 5. For any sound rays that propagates outward with some angles from the horizontal plane will be bent so that the ray angles becomes steeper and number of interactions with boundary will increase. Figure 6 shows the computed examples of the radial dependence of R(r) in both cases of range-dependent and independent medium for 100 Hz source for the first few modes. The profile of the sound speed used for the computation in figure 6 was determined with $p = -8\times10^{-14}\sec^2/m^3$ and $q = 4.46\times10^{-7}\sec^2/m^2$. Since α_n are range-dependent and it decreases with r the term in the denominator of equation (17) becomes smaller thereby gives slow decreasing of the radial function. This means that the resultant spreading loss would be small compared with the one corresponding to the case of range-independent medium where the horizontal wave numbers are constant with respect to the horizontal range. Comparison with the purely cylindrical loss (Fig. 7) shows that for slow variation of sound speed it looks more likely following the homogeneous or isovelocity medium over the perfectly reflecting boundary. Thus the overall effects of the thermal front existing over the flat coastal water may not significantly affect the low frequency sound propagation but as the frequency increases α^2 could be negative and the sound decays rapidly with range.

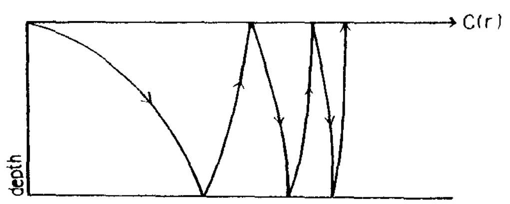

Fig. 5. Ray path showing the steeping of the reflection angles as sound propagates toward increasing C(r).

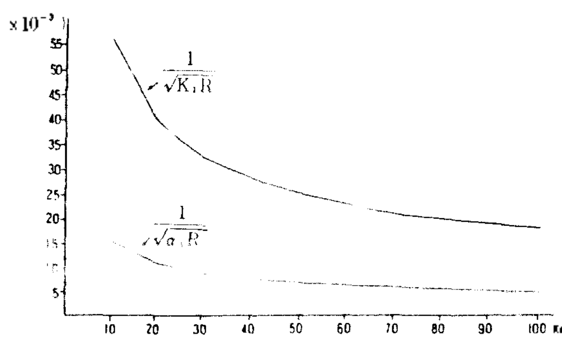

Fig. 6(a). Behavior of the radial functions of range-dependent and range-independent medium (Mode 1).

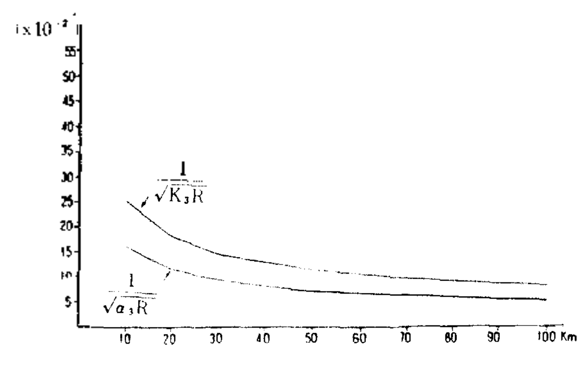

Fig. 6(b). Behavior of the radial functions of range-dependent and range-independent medium (Mode 3).

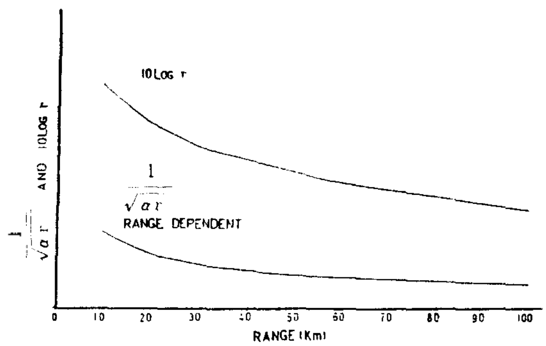

Fig. 7. Comparison of range-dependence between 10 log r and $1/\sqrt{\alpha r}$

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