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An Efficient Approach in Analyzing Linear Time-Varying Systems via Taylor Polynomials

(Taylor 다항식에 의한 선형 시변 시스템의 효과적인 해석)

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要 約

본 논문은 Taylor 다항식에 의해 선형 시변 시스템을 해석하는 한 효과적인 방법을 제안한다. Sparis와 Mouroutsos에 의한 방법은 구해야 할 상태 벡타가 닫혀진 형태(closed form)로 구해지지 않고 또한 사용하는 항이 증가할때 큰 차원의 선형 대수 방정식을 풀어야 하는 문제점을 가지고 있다. 반면에 본 논문에서 제안된 방법은 상태 벡타가 닫혀진 형태로 구해지며 선형 대수 방정식을 풀 필요가 없다.

Abstract

This paper presents an efficient method of analyzing linear time-varying systems via Taylor polynomials. While the approach suggested by Sparis and Mouroutsos gives an implicit form for unknown state vector and requires to solve a linear algebraic equation with large dimension when the number of terms increases, the method proposed in this paper shows an explicit form and has no need to solve any linear algebraic equation.

I. Introduction

Sparis and Mouroutsos [1] used the Taylor polynomials in analyzing a class of linear time-varying systems and obtained the feedback gains for the optimal control problems with quadratic performance index. Their motivations of adopting the Taylor polynomials rather than the Walsh functions [2]-[8] or Block pulse functions [9]-[20] are to achieve more accurate solution with small number of terms and to reduce the computational burden than the Walsh functions.

The final linear algebraic equations derived by Sparis and Mouroutsos for the analysis of linear time-varying systems is given as

$$H-(\underline{x}_0,\underline{0},\cdots,\underline{0})=A^*\begin{bmatrix}H_1\\H_2\\\vdots\\H_n\end{bmatrix}P_{(r\times r)}+\sum_{j=1}^{q}\beta_j\,G_j\,P_{(r\times r)} \quad (1)$$

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Where H and H_i, i=1,2, . . . , n are unknown matrices to be calculated and $P_{(r\times r)}$ is the operational matrix of integration for the Taylor polynomials. Also, for each j=1,2, . . . , 8, β_j is an $n\times r$ matrix each of whose rows is an $1\times r$ coefficient vector of the Taylor series expansion of the function in input matrix and G_j is a coefficient matrix corresponding to a coefficient vector of the Taylor series expansion of the input function. In eqn. (1), it should be observed that 1) eqn.(1) is not an explicit form for unknown matrices H and H_i, i=1,2, . . . , n, and 2) as the number of terms, r, increases, the dimension of eqn.(1) also becomes large. Thus it becomes very difficult to solve eqn.(1).

In this paper, we present another approach of analyzing a class of linear time-varying systems via Taylor polynomials in order to solve the problems included in their results. Our result is an explicit form for unknown state vector and does not require to solve any linear algebraic equation and thus does not include the procedure of matrix inverse which is generally difficult to compute.

II. Problem Formulation

When a function f(t) is expanded at the neighborhood of t=O as

$$f(t)=\sum_{i=0}^{\infty} f_i\,\phi_i(t),\quad t\in[0,1) \tag{2}$$

where

$$\phi_i(t)=t^i,\quad f_i=\frac{1}{i!}\left(\frac{d^i f(t)}{dt^i}\right)\Bigg|_{t=0} \tag{3}$$

$\phi_i(t)$, for i=0, 1,. . . , are called Taylor series basis functions [1]. Also a linear combination of finite number of Taylor series basis functions is called a Taylor polynomial. For an integer $m\geqslant 1$, let $\phi_{(m)}(t)$ be the m-vector function defined by

$$\phi_{(m)}^T(t)=[\phi_0(t),\ \phi_1(t),\cdots,\phi_{m-1}(t)],\quad t\in[0,1), \tag{4}$$

where the superscript T denotes the transpose.

Let us consider the following linear time-varying system

$$\begin{aligned}\dot{\underline{x}}(t)&=A(t)\ \underline{x}(t)+B(t)\ \underline{u}(t),\\ \underline{x}(0)&=\text{initial vector},\end{aligned} \tag{5}$$

where x(t) is an n-dimensional state vector, u(t) is an r-dimensional input vector, and A(t) and B(t) are $n\times n$ and $n\times r$ time-varying matrices, respectively. For each i,j=1,2, . . ., n, let $a_{ij}(t)$ be the entry in i-th row and j-th column of A(t) and for each i=1,2,. . ., n and k=1,2, . . ., r, let $b_{ik}(t)$ be the element in i-th row and j-th column of B(t). Assume that all the elements of A(t), B(t) and the input vector function are analytic in the time interval [0,1); then the expansions of those elements via m Taylor series basis functions are as follows:

$$a_{ij}(t)\cong[a_{ij0}\ \ a_{ij1}\cdots a_{ij,m-1}]\ \phi_{(m)}(t) \tag{6}$$

$$b_{ik}(t)\cong[b_{ik0}\ \ b_{ik1}\cdots b_{ik,m-1}]\ \phi_{(m)}(t) \tag{7}$$

and

$$u_k(t)\cong[u_{k0}\ \ u_{k1}\cdots u_{k,m-1}]\ \phi_{(m)}(t)\ . \tag{8}$$

Then the expansion of state variables $\underline{x}(t)$ via m Taylor series basis functions is

$$x_i(t)\cong[x_{i0}\ \ x_{i1}\cdots x_{i,m-1}]\ \phi_{(m)}(t)\,. \tag{9}$$

Our problem is to find a formula, which has an explicit form and does not require the inversion of large matrix, for the vectors $[X_{io}\ X_{i1}\cdots X_{i,m-1}]$. for i=1,2, . ., n, in terms of X(o), $[a_{ijo}\ a_{ij1}\cdots a_{ij,m-1}]$ for i,j=1,2,. . ., n, $[b_{iko}\ b_{ik1}\cdots b_{ik,m-1}]$ for i=1,2, . . , n and k=1,2, . ., r, and $[u_{ko}\cdot u_{k1}\cdots u_{k,m-1}]$ for k=1,2,. . ., r.

III. A Mathematical Preliminary

Let there be given the following two arbitrary Taylor polynomials each of which is a linear combination of m Taylor series basis functions;

$$\begin{aligned}p(t)&=p_0\phi_0(t)+p_1\phi_1(t)+\cdots+p_{m-1}\ \phi_{m-1}(t)\\ &\triangleq p^T\ \phi_{(m)}(t)\end{aligned} \tag{10}$$

and

$$\begin{aligned}q(t)&=q_0\phi_0(t)+q_1\phi_1(t)+\cdots+q_{m-1}\phi_{m-1}(t)\\ &\triangleq q^T\phi_{(m)}(t)\end{aligned} \tag{11}$$

where

$$p^T=[p_0\ \ p_1\cdots p_{m-1}] \tag{12}$$

$$q^T=[q_0\,q_1\cdots q_{m-1}]\,. \tag{13}$$

Then, it is easy to show that the multiplication of two functions p(t) and q(t) can be approximated as a linear combination of m Taylor series basis functions as follows:

$$p(t)\ q(t) \cong h^T \phi_{(m)}(t) \qquad (14)$$

where

$$h=\begin{bmatrix} p_0q_0 \\ p_0q_1+p_1q_0 \\ \vdots \\ p_0q_{i-1}+p_1q_{i-2}+\cdots+p_{i-1}q_0 \\ \vdots \\ p_0q_{m-1}+p_1q_{m-2}+\cdots+p_{m-1}q_0 \end{bmatrix} \leftarrow \text{i-th row} \qquad (15)$$

In the above, the terms which have higher order than $\phi_{m-1}(t)$ are neglected. The validity of this approximation [1] is confirmed by the fact that the considered time domain is $t \in [0, 1)$.

Now, the integration of the multiplication of two functions p(t) and q(t) cna be approximated as a linear combination of m Taylor series basis functions via the operational matrix for forward integration [1], P(mxm), as

$$\int_0^t p(s)uq(s)ds \cong \int_0^t h^T \phi_{(m)}(s)ds \cong h^T P_{(m\times m)} \phi_{(m)}(t) \qquad (16)$$

where

$$P_{(m\times m)}=\begin{bmatrix} 0 & 1 & 0 & 0 & 0\cdots 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0\cdots 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0\cdots 0 & 0 & 0 \\ & & & \cdots & & & \\ 0 & 0 & 0 & 0 & 0\cdots 0 & \frac{1}{m-2} & 0 \\ 0 & 0 & 0 & 0 & 0\cdots 0 & 0 & \frac{1}{m-1} \\ 0 & 0 & 0 & 0 & 0\cdots 0 & 0 & 0 \end{bmatrix} \qquad (17)$$

We can show that when P(t) is a nxn matrix and q(t) is a nx1 vector, $\int_0^t p(s)q(s)ds$ as can be similarly expressed.

Theorem 1.

Let there be given A(t) and $\underline{X}(t)$ as in eqn.(5). For each i,j=1,2,. . ., n, let $a_{ij}(t)$ be the entry in i-th row and j-th column of A(t). And let the expansions of $a_{ij}(t)$ and the each function of $\underline{x}(t)$ be expressed as follows:

$$a_{ij}(t) \cong [a_{ij0}\ a_{ij1} \cdots a_{ij,m-1}]\ \phi_{(m)}(t),\quad i,j=1,2,\cdots,n \qquad (18)$$

$$x_i(t) \cong [x_{i0}\ x_{i1} \cdots x_{i,m-1}]\ \phi_{(m)}(t),\quad i,=1,2,\cdots,n \qquad (19)$$

Also, let us define the nxn matrix A_w and nx1 vector $\underline{X}_{iv}$ as

$$A_w=\begin{bmatrix} a_{11w} & a_{12w} & \cdots & a_{1nw} \\ a_{21w} & a_{22w} & \cdots & a_{2nw} \\ \vdots & \vdots & & \vdots \\ a_{n1w} & a_{n2w} & \cdots & a_{nnw} \end{bmatrix} \qquad (20)$$

$$\underline{X}_w=\begin{bmatrix} x_{1w} \\ x_{2w} \\ \vdots \\ x_{nw} \end{bmatrix} \qquad (21)$$

where w=0,1, . . ., m-1.

Then,

$$\int_0^t A(s)\underline{X}(s)ds \cong [A_0\underline{X}_0,\ A_0\underline{X}_1+A_1\underline{X}_0, \cdots, A_0\underline{X}_{m-1}+A_1\underline{X}_{m-2}+\cdots +A_{m-1}\underline{X}_0]\ \underline{P}_{(m\times m)}\ \phi_{(m)}(t). \qquad (22)$$

Proof:

Since

$$A(t)\underline{X}(t)=\begin{bmatrix} a_{11}(t) & a_{12}(t) & \cdots & a_{1n}(t) \\ a_{21}(t) & a_{22}(t) & \cdots & a_{2n}(t) \\ \vdots & \vdots & & \vdots \\ a_{n1}(t) & a_{n2}(t) & \cdots & a_{nn}(t) \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}$$

$$=\begin{bmatrix} a_{11}(t)x_1(t)+a_{12}(t)x_2(t)+\cdots+a_{1n}(t)x_n(t) \\ a_{21}(t)x_1(t)+a_{22}(t)x_2(t)+\cdots+a_{2n}(t)x_n(t) \\ \vdots \\ a_{n1}(t)x_1(t)+a_{n2}(t)x_2(t)+\cdots+a_{nn}(t)x_n(t) \end{bmatrix} \qquad (23)$$

the integration of A(t) $\underline{X}$(t) is

$$\int_0^t A(s)\underline{X}(s)ds$$

$$=\begin{bmatrix} \int_0^t \{a_{11}(s)x_1(s)+a_{12}(s)x_2(s)+\cdots +a_{1n}(s)x_n(s)\}ds \\ \int_0^t \{a_{21}(s)x_1(s)+a_{22}(s)x_2(s)+\cdots +a_{2n}(s)x_n(s)\}ds \\ \vdots \\ \int_0^t \{a_{n1}(s)x_1(s)+a_{n2}(s)x_2(s)+\cdots +a_{nn}(s)x_n(s)\}ds \end{bmatrix} \qquad (24)$$

The first row in the eqn.(24) is

$$\int_0^t \{a_{11}(s)x_1(s)+a_{12}(s)x_2(s)+\cdots+a_{1n}(s)x_n(s)\}ds$$
$$=\int_0^t a_{11}(s)x_1(s)ds+\int_0^t a_{12}(s)x_2(s)ds+\cdots$$
$$+\int_0^t a_{1n}(s)x_n(s)ds. \tag{25}$$

By using the property of eqns.(14) and (16), we obtain

$$\int_0^t a_{11}(s)x_1(s)ds$$
$$\cong [a_{110}x_{10},\ a_{110}x_{11}+a_{111}x_{10},\ \cdots,\ a_{110}x_{1,m-1}+a_{111}x_{1,m-2}$$
$$+\cdots+a_{11,m-1}x_{10}]P_{(m\times m)}\phi_{(m)}(t)$$
$$\int_0^t a_{12}(s)x_2(s)ds$$
$$\cong [a_{120}x_{20},\ a_{120}x_{21}+a_{121}x_{20},\ \cdots,\ a_{120}x_{2,m-1}+a_{121}x_{2,m-2}$$
$$+\cdots+a_{12,m-1}x_{20}]P_{(m\times m)}\phi_{(m)}(t)$$
$$\vdots$$
$$\int_0^t a_{1n}(s)x_n(s)ds$$
$$\cong [a_{1n0}x_{n0},\ a_{1n0}x_{n1}+a_{1n1}x_{n0},\ \cdots,\ a_{1n0}x_{n,m-1}+a_{1n1}x_{n,m-2}$$
$$+\cdots+a_{1n,m-1}x_{n0}]P_{(m\times m)}\phi_{(m)}(t) \tag{26}$$

Therefore the first row in the eqn.(24) becomes

$$\int_0^t \{a_{11}(s)x_1(s)+a_{12}(s)x_2(s)+\cdots+a_{1n}(s)x_n(s)\}ds$$
$$\cong [\underline{a}_{10}\underline{x}_0,\ \underline{a}_{10}\underline{x}_1+\underline{a}_{11}\underline{x}_0,\ \cdots,\ \underline{a}_{10}\underline{x}_{m-1}+\underline{a}_{11}\underline{x}_{m-2}+\cdots$$
$$+\underline{a}_{1,m-1}\underline{x}_0]\underline{P}_{(m\times m)}\phi_{(m)}(t) \tag{27}$$

where

$$\underline{a}_{1w}=[a_{11w}\ a_{12w}\cdots a_{1nw}] \tag{28}$$

$$\underline{x}_w^T=[x_{1w}\ x_{2w}\cdots x_{nw}] \tag{29}$$

for w=0,1, . . , m-1.
Similarly, for the second row in the eqn.(24), we can obtain easily

$$\int_0^t \{a_{21}(s)x_1(s)+a_{22}(s)x_2(s)+\cdots+a_{2n}(s)x_n(s)\}ds$$
$$\cong [\underline{a}_{20}\underline{x}_0,\ \underline{a}_{20}\underline{x}_1+\underline{a}_{21}\underline{x}_0,\ \cdots,\ \underline{a}_{20}\underline{x}_{m-1}+\underline{a}_{21}\underline{x}_{m-2}+\cdots$$
$$\underline{a}_{2,m-1}\underline{x}_0]\underline{P}_{(m\times m)}\phi_{(m)}(t) \tag{30}$$

where

$$\underline{a}_{2w}=[a_{21w}\ a_{22w}\cdots a_{2nw}] \tag{31}$$

$$\underline{x}_w^T=[x_{1w}\ x_{2w}\cdots x_{nw}] \tag{32}$$

for w=0,1, . . ,, m-1, and so forth.
Finally, we find

$$\int_0^t A(s)X(s)ds$$
$$=\begin{bmatrix} \underline{a}_{10}\underline{X}_0 & \underline{a}_{10}\underline{X}_1+\underline{a}_{11}\underline{X}_0\cdots\underline{a}_{10}\underline{X}_{m-1}+\underline{a}_{11}\underline{X}_{m-2}+\cdots & +\underline{a}_{1,m-1}\underline{X}_0 \\ \underline{a}_{20}\underline{X}_0 & \underline{a}_{20}\underline{X}_1+\underline{a}_{21}\underline{X}_0\cdots\underline{a}_{20}\underline{X}_{m-1}+\underline{a}_{21}\underline{X}_{m-2}+\cdots & +\underline{a}_{2,m-1}\underline{X}_0 \\ & \vdots & \cdots \\ \underline{a}_{n0}\underline{X}_0 & \underline{a}_{n0}\underline{X}_1+\underline{a}_{n1}\underline{X}_0\cdots\underline{a}_{n0}\underline{X}_{m-1}+\underline{a}_{n1}\underline{X}_{m-2}+\cdots & +\underline{a}_{n,m-1}\underline{X}_0 \end{bmatrix}\underline{P}_{(m\times m)}\phi_{(m)}(t) \tag{33}$$

Since

$$\begin{bmatrix}\underline{a}_{1w}\\ \underline{a}_{2w}\\ \\ \underline{a}_{nw}\end{bmatrix}=\begin{bmatrix}a_{11w} & a_{12w} & \cdots a_{1nw}\\ a_{21w} & a_{22w} & \cdots a_{2nw}\\ n & \vdots & \\ a_{n1w} & a_{n2w} & \cdots a_{nnw}\end{bmatrix}=A_w \tag{34}$$

we obtain finally

$$\int_0^t A(s)X(s)ds$$
$$\cong [A_0\underline{X}_0\ \ A_0\underline{X}_1+A_1\underline{X}_0\cdots A_0\underline{X}_{m-1}+A_1\underline{X}_{m-2}+\cdots$$
$$+A_{m-1}\underline{X}_0]\underline{P}_{(m\times m)}\phi_{(m)}(t) \tag{35}$$

This completes the proof.

Q.E.D

IV. Main Result

We present the main result in the following.
Theorem 2.

For the linear time-varying system in eqn.(5), when the entries of A(t), B(t), and U(t) are expanded as in eqns.(6), (7), and (8), respectively, and the expansion of each function of x(t) via m Taylor series basis functions is

$$X_i(t)\cong[X_{i0}\ \ X_{i1}\cdots X_{i,m-1}]\phi_{(m)}(t), \tag{36}$$

the 1xm vectors $[X_{io}\ X_{i1}\ .\ .\ .\ X_{i,m-1}]$ for i=1,2, . . ., n can be obtained by

$$\underline{X}_0=\underline{X}(0)$$
$$\underline{X}_{V-1}-\underline{O}=\frac{A_0\underline{X}_{V-2}+A_1\underline{X}_{V-3}+\cdots+A_{V-2}\underline{X}_0}{V-1}+$$
$$\frac{B_0\underline{U}_{V-2}+B_1\underline{U}_{V-3}+\cdots+B_{V-2}\underline{U}_0}{V-1} \tag{37}$$

where v=2,3, . . . , m and

$$B_w=\begin{bmatrix} b_{11w} & b_{12w}\cdots b_{1rw} \\ b_{21w} & b_{22w}\cdots b_{2rw} \\ & \vdots \\ b_{n1w} & b_{n2w}\cdots b_{nrw}\end{bmatrix} \tag{38}$$

$$\underline{U}_w=\begin{bmatrix} u_{1w} \\ u_{2w} \\ \vdots \\ u_{rw}\end{bmatrix} \tag{39}$$

for w=0,1,. ., m-1.

Proof:

Integrating the eqn.(5) from 0 to t, we obtain

$$\underline{X}(t)-\underline{X}(o)=\int_0^t A(s)\underline{X}(s)ds+\int_0^t B(s)\underline{U}(s)ds \tag{40}$$

Expanding the state variables with m Taylor series basis functions

$$\underline{X}(t)=\begin{bmatrix} X_1(t) \\ X_2(t) \\ \vdots \\ X_n(t)\end{bmatrix}\cong$$

$$\begin{bmatrix} x_{10}\,\phi_0(t)+x_{11}\,\phi_1(t)+\cdots+x_{1,m-1}\,\phi_{m-1}(t) \\ x_{20}\,\phi_0(t)+x_{21}\,\phi_1(t)+\cdots+x_{2,m-1}\,\phi_{m-1}(t) \\ \vdots \\ x_{n0}\,\phi_0(t)+x_{n1}\,\phi_1(t)+\cdots+x_{n,m-1}\,\phi_{m-1}(t)\end{bmatrix}$$

$$=\underline{X}_0\,\phi_0(t)+\underline{X}_1\phi_1(t)+\cdots+\underline{X}_{m-1}\,\phi_{m-1}(t) \tag{41}$$

is obtained.

Substituting this into the eqn.(40) and using the Theorem 1, we find

$$[\underline{x}_0\,\underline{x}_1\cdots\underline{x}_{m-1}]\,\phi_{(m)}(t)-[\underline{x}(o)\ \underline{o}\cdots\underline{o}]\,\phi_{(m)}(t)$$
$$=[A_0\underline{X}_0\ \ A_0\underline{X}_1+A_1\underline{X}_0\cdots A_0\underline{X}_{m-1}+A_1\underline{X}_{m-2}+\cdots$$
$$+A_{m-1}\underline{X}_0]\,\underline{P}_{(m\times m)}\,\phi_{(m)}(t)$$
$$+[B_0\underline{U}_0\ \ B_0\underline{U}_1+B_1\underline{U}_0\cdots B_0\underline{U}_{m-1}+B_1\underline{U}_{m-2}+\cdots$$
$$+B_{m-1}\underline{U}_0]\,\underline{P}_{(m\times m)}\,\phi_{(m)}(t) \tag{42}$$

Since the eqn. (42) should be satisfied for all t in the time interval $t\in[0,1)$,

$$[\underline{X}_0\underline{X}_1\cdots\underline{X}_{m-1}]-[\underline{X}(0)\ \underline{0}\cdots\underline{0}]$$
$$=[A_0\underline{X}_0\ \ A_0\underline{X}_1+A_1\underline{X}_0\cdots A_0\underline{X}_{m-1}+A_1\underline{X}_{m-2}+\cdots$$
$$+A_{m-1}\underline{X}_0]\,\underline{P}_{(m\times m)}$$
$$+[B_0\underline{U}_0\ \ B_0\underline{U}_1+B_1\underline{U}_0\cdots B_0\underline{U}_{m-1}+B_1\underline{U}_{m-2}+\cdots$$
$$+B_{m-1}\underline{U}_0]\,\underline{P}_{(m\times m)} \tag{43}$$

is obtained.

Substituting eqn.(17) into eqn.(43), we find

$$[\underline{X}_0\underline{X}_1\cdots\underline{X}_{m-1}]-[\underline{X}(0)\ \underline{0}\cdots\underline{0}]$$
$$=[\underline{0}\ \ A_0\underline{X}_0\quad \frac{A_0\underline{X}_1+A_1\underline{X}_0}{2}\cdots$$
$$\frac{A_0\underline{X}_{m-2}+A_1\underline{X}_{m-3}+\cdots+A_{m-2}\underline{X}_0}{m-1}]$$
$$+[\underline{0}\ \ B_0\underline{U}_0\quad \frac{B_0\underline{U}_1+B_1\underline{U}_0}{2}\cdots$$
$$\frac{B_0\underline{U}_{m-2}+B_1\underline{U}_{m-3}+\cdots+B_{m-2}\underline{U}_0}{m-1}] \tag{44}$$

By equating each column in eqn.(44), we can obtain finally

$$\underline{X}_0-\underline{X}(0)=\underline{0}$$
$$\underline{X}_1-\underline{0}=A_0\underline{X}_0+B_0\underline{U}_0$$
$$\underline{X}_2-\underline{0}=\frac{A_0\underline{X}_1+A_1\underline{X}_0}{2}+\frac{B_0\underline{U}_1+B_1\underline{U}_0}{2}$$
$$\vdots$$
$$\underline{X}_{v-1}-\underline{0}=\frac{A_0\underline{X}_{v-2}+A_1\underline{X}_{v-3}+\cdots+A_{v-2}\underline{X}_0}{V-1}+$$
$$\frac{B_0\underline{U}_{v-2}+B_1\underline{U}_{v-3}+\cdots+B_{v-2}\underline{U}_0}{V-1}$$
$$\underline{X}_{m-1}-\underline{0}=\frac{A_0\underline{X}_{m-2}+A_1\underline{X}_{m-3}+\cdots+A_{m-2}\underline{X}_0}{m-1}+$$
$$\frac{B_0\underline{U}_{m-2}+B_1\underline{U}_{m-3}+\cdots+B_{m-2}\underline{U}_0}{m-1} \tag{45}$$

This completes the proof.

Q.E.D

V. An Example

Let us consider the following linear time-varying system [1]

$$\overset{\circ}{X}(t)=A(t)\ X(t)$$

where

$$A(t)=\begin{bmatrix}0 & 0\\ t & 0\end{bmatrix} \quad \text{and} \quad X(0)=\begin{bmatrix}x_1(0)\\ x_2(0)\end{bmatrix}=\begin{bmatrix}1\\ 1\end{bmatrix}.$$

When the number m of terms is 4,

$$a_{11}(t)=a_{12}(t)=a_{22}(t)=[0\ 0\ 0\ 0]\,\phi_{(4)}(t)$$
$$a_{21}(t)=t=[0\ 1\ 0\ 0]\,\phi_{(4)}(t)$$

Therefore, we find

$$A_0=A_2=A_3=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix} \quad \text{and} \quad A_1=\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}$$

The eqn.(45) yields

$$\underline{X}_0=\begin{bmatrix}1\\ 1\end{bmatrix}$$

$$\underline{X}_1=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\underline{X}_2=\frac{1}{2}\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix}=\begin{bmatrix}0\\ \frac{1}{2}\end{bmatrix}$$

$$\underline{X}_3=\begin{bmatrix}0\\ 0\end{bmatrix}$$

Thus, the solution of this system is

$$X(t)=\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}\cong\begin{bmatrix}1\\ 1\end{bmatrix}\phi_0(t)+\begin{bmatrix}0\\ \frac{1}{2}\end{bmatrix}\phi_2(t)$$
$$=\begin{bmatrix}1\\ 1+\frac{t^2}{2}\end{bmatrix}$$

The solution obtained by eqn.(45) is same as that of Sparis and Mouroutsos [1], while this approach is much simpler than that of [1].

VI. Concluding Remarks

In this paper, we have presented an efficient method of analyzing linear time-varying systems via Taylor polynomials. While the approach based upon the product and coefficient matrix proposed by Sparis and Mouroutsos yields an implicit form for unknown state vector and requires to solve a linear algebraic equation with large dimension when the number m of terms increases, the method proposed in this paper yields an explicit form for unknown state vector and has no need to solve any linear algebraic equation and further more does not include any inversion of matrix. Therefore it is obvious that the method suggested in this paper shows noticeable merits in form, computation and numerical stability over that of sparis and Mouroutsos [1].

References

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