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# Analysis of Burnable Poison Effect on Power Distribution using Power Sensitivity Coefficient Concept

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# 출력민감도 계수개념을 이용한 가연성 독붕봉이 출력분포에 미치는 영향의 분석

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## Abstract

The low leakage loading pattern has features as the placement of some fresh fuel assemblies in the core interior to reduce the neutron fluence on the pressure vessel and to enhance the neutron economics. But as fresh fuel assemblies are loaded in the core interior, the local power tends to exceed safety limit due to the high reactivity of the fresh assemblies. Therefore, a large number of burnable poisons must be utilized in a low leakage scheme to suppress the high assembly power as well as the excess reactivity.

In this study the effects of burnable poisons are treated as a perturbation on the power distribution, and the 'Power Sensitivity Coefficient' concept is adopted. An application study is performed for cycle 1 of the Korea Nuclear Unit-7 (KNU-7) to justify the usefulness of the reverse depletion method coupled with the above concept. To obtain the optimal burnable poision distribution at the given burnup step, the linear programming technique is adopted. The result shows maximum 4.5% error in the amount of burnable poisons between the calculated and the reference values.

It is concluded that the design methodology which consists of the reverse depletion, the power sensitivity coefficient concept, and the linear programming technique can be used to find the optimal burnable poison distribution.

## 요 약

저누출 장전 모형은 새 연료를 안에서부터 넣는 in-out 형태를 취하여 격납 용기의 fluence를 줄이고 중성자 경제성을 높이고자 하는 것으로, 이 경우에는 노심내의 전체적인 중성자 경제성은 좋아지지만 노심 중앙부에서의 새연료의 과다 반응도 때문에 안전성 여유를 줄이게 되므로 많은 수의 가연성 독붕봉을 사용하여 첨두 인자를 조절해야만 한다.

본 논문에서는 가연성 독붕봉 연소에 따른 출력 변화를 섭동으로 취급하며, 이를 출력 민

감도 계수(Power Sensitivity Coefficient)로 표시한다. 최적화된 가연성 독봉봉의 분포를 구하기 위하여 알고 있는 주기말상태로부터, 노심 내의 출력과 과다 반응도를 제어하면서 주기초로 추적해 나가는 역연소법(Reverse Depletion Method)의 도입에 대한 타당성을 출력민감도 계수개념과 선형 계획법을 이용하여 원자력 7호기 제1주기에 응용하여 검증했으며, 가연성 독봉봉의 추정량과 실제량과의 차이는 최대 4.5%의 오차를 보였다.

## I. Introduction

In traditional PWR out-in loading scheme fresh fuel assemblies are inserted at the periphery of the reactor core, especially for the higher burnup. A higher periphery power results in increased neutron leakage as well as a higher fast neutron fluence on the pressure vessel, which introduces an economic penalty.

The low leakage loading pattern has features as the placement of some fresh assemblies in the core interior to reduce the fluence on the pressure vessel and to enhance the neutron economics. But as fresh assemblies are loaded in the core interior, the local power tends to exceed safety limit due to the high assembly reactivity of the fresh assemblies. Therefore, a large number of burnable poisons must be used to suppress the power-peaking at the in-board (inner core region) fresh fuel position as well as to insure the negative moderator temperature coefficient.

In this study, the optimization problem at each burnup step to determine the burnable poison distribution is formulated linearly and it is solved using the linear programming technique. The optimization problem is solved step by step from the EOC to the BOC reversely. This is the so-called reverse depletion method[1] and the reason to use this is as follows.

At EOC, there are relatively small amount of the burnable poisons and an acceptable core state without burnable poisons can be figured easily. It can give an optimal fuel loading pattern without burnable poisons. But the fuel loading pattern search itself is not included in this study and we start with a known pattern without burnable poisons. Therefore, it is necessary to relate the burnable poison with the power distribution. Of course the power distribution with a known burnable poison distribution can be calculated using the standard core analysis computer code. But in our case, as the burnable poison distribution itself is the variable to be optimized, a more simple tool is necessary. The power sensitivity coefficient (PSC) concept is introduced as the tool in this work.

The purpose of this study is to show the validity of the reverse depletion method coupled with the power sensitivity coefficient for the optimal burnable poison assignment problem.

## II. Power sensitivity coefficient and Reverse Depletion Method

### 2.1. Power Sensitivity Cefficient

Mathematically, the optimal burnable poison(BP) assignment can be formulated as the following optimization process. At each burnup step, the objective function is to minimize the difference between the calculated and the target power. It is defined as

$$J = \sum_{i=1}^{N} | P_i - P^*_i | \qquad (1)$$

and the imposed consraints are

$$0 \leqq BP_j \leqq BP_j^{max} \qquad j=1,...,K \qquad (2)$$

and

$$0 \leqq P \leqq PP, \qquad i=1,...,N \qquad (3)$$

where

i = spatial node index,

j = index of the node which BP can be assigned,

$P_i$ = relative power at node i which would be

formulated as a function of the BP distribution later,

$P_i^*$ = target power at node i,

PP = power peaking limit,

$BP_j^{max}$ = maximum number of BP rods at node j,

N = number of total fuel assemblies in the core, and

K = number of assemblies with BPs.

To determine the BP distribution and the objective function, it is necessary to connect the power and the BP distribution. It is easy to get the power distribution with a known BP distribution through the standard core calculation. In this manner, however, excessive number of core calculations is required to search for the optimal BP distribution. Therefore, a more simple tool is necessary to predict the power distribution from the known BP distribution. The power sensitivity coefficient concept is introduced as the tool to connect the BP and the power distributions. For this purpose, the power at node i is calculated from the amount of BPs at node, j, j=1,...,K, as follows

$$P_i^n = P_i^{n+1} + \sum_{j=1}^{K} \left(\frac{\partial P_i}{\partial BP_j}\right)^n \cdot \Delta BP_j^n \qquad (4)$$

where

n = burnup index,

$\Delta BP_j^n$ = the increment of the BP rods at node j between (n)-th and (n+1)-th burnup steps,

$P_i^n$ = resulting power at node i due to BP at node j at (n)-th burnup step, and

$P_i^{n+1}$ = resulting power at node i due to BP at node j at (n+1)-th burnup step.

The first order derivative term, $\partial P_i / \partial BP_j$, used in Eq. (4) represents the change of power at node i due to the depletion of burnable poisons at node j. Therefore, it is called the power sensitivity coefficient (PSC). The original PSC is defined in a same burnup step.[2] In this work, however, the power sensitivy cofficient (PSC) is defined as

$$\left(\frac{\partial P_i}{\partial BP_i}\right)^n = \left(\frac{P_i^n - P_i^{n+1}}{BP_i^n - BP_i^{n+1}}\right) \qquad (5)$$

where

$P_i^n$ = reference power at node i at (n)-th burnup step,

$BP_j^n$ = reference BP at node j at (n)-th burnup step.

If the PSCs are available, the relative power can be calculated easily from the Eq. (4) without core calculation.

### 2.2. Reverse Depletion Method

A major innovation of the reverse depletion method is that the assembly assignment can be performed at EOC core state. Achieved the EOC core state that the assembly pattern is figured in the absence of all control poisons, the burnable and soluble poison requirements are solved to control the power and core excess reactivity while depleting the cycle in reverse from the EOC exposure distribution to the BOC.

The burnable poison assignment algorithm developed in this study, used in conjunction with the standard core soluble boron search available in PWR analysis code such as KIDD (KAERI improved Diffusion Depletion) code,[3] provides the burnable poison requirements for controlling the power distribution. Each in-board BP position 'j' is identified and is assigned a variable, $\Delta BP_j$, to be used in the burnable poison search. Once optimal BPs are obtained at the burnup step, a 'negetive' step is conducted and the above process is repeated. After stepping throughout the burnup cycle in reverse, a burnable poison trajectory is provided for each fuel assembly, i.e. the number of BP rods which should be loaded at node j at BOC is the summation of all $\Delta BP$s for node j

In this study, we assume that the target power distribution for the (n)-th burnup step optimization is the distribution at the same step which is obtained from the normal KIDD depletion calculation. With the above assumption, the optimized BP distribution at (n)-th burnup step from the optimization calculation with data on (n+1)-th step can be compared with the reference BP at

(n)-th burnup step which is already known from the KIDD calculation.

### 2.3. Linear Programming Approach

For the mathematical senses, the constraint equations, Eqs. (2), (3) and (4), are linear but the objective function, Eq. (1), is not linear. If the objective function is linearized, then a linear programming technique could be applied. To arrive at the desired form, the independent variable $d_i$ is introduced. It is defined as

$$d_i = | P_i - P_i^* | \qquad i=1,...,N \tag{6}$$

which requires the constraints

$$d_i \geq P_i - P_i^* \tag{7}$$

and

$$-d_i \leq P_i - P_i^*. \tag{8}$$

Then the objective function can be rewritten in terms of $d_i$:

$$J^n = \sum_{i=1}^{N} d_i^n \tag{9}$$

Introducing Eq. (4) into Eqs. (7) and (8), the following constraints will be satisfied:

$$d_i^n \geq (P_i^{n+1} - P_i^*) + \sum_{j=1}^{K} \left(\frac{\partial P_i}{\partial BP_j}\right)^n \cdot \Delta BP_j^n \tag{10}$$

and

$$-d_i^n \leq (P_i^{n+1} - P_i^*) + \sum_{j=1}^{K} \left(\frac{\partial P_i}{\partial BP_j}\right)^n \cdot \Delta BP_j^n \tag{11}$$

In the linear programming method, the search variable $\Delta BP_j^n$ is required to be nonnegative.

The resulting linear programming problem consists of the following equations:

$$\text{Maximize } J^n = -\sum_{i=1}^{N} d_i^n \tag{12}$$

subject to

$$\sum_{j=1}^{K} \left(\frac{\partial P_i}{\partial BP_j}\right)^n \cdot \Delta BP_j^n - d_i^n \leq (P_i^* - P_i^{n+1}), \tag{13}$$

$$-\sum_{j=1}^{K} \left(\frac{\partial P_i}{\partial BP_j}\right)^n \cdot \Delta BP_j^n - d_i^n \leq -(P_i^* - P_i^{n+1}), \tag{14}$$

$$\sum_{j=1}^{K} \left(\frac{\partial P_i}{\partial BP_j}\right)^n \cdot \Delta BP_j^n \leq (PP - P_i^{n+1}), \tag{15}$$

$$\Delta BP_j^n \leq \Delta BP_j^{max} \qquad j=1,...,K \tag{16}$$

and

$$\Delta BP_j^n \geq 0,\ d_i^n \geq 0. \tag{17}$$

The subroutine ZX3LP in the International Mathematical and Statistical Library(IMSL) is used to solve the optimization problem. The subroutine solves the linear programming problem via the revised simplex method.[4] The required computing time is about 100 CPU second on the VAX11/750 for each burnup step with 36 variables and 78 constraint equations.

A schematic calculational process of the burnable poison assignment is depicted in Fig. 1. At first, the assembly positions are identified where burnable poisons can be assigned in the search

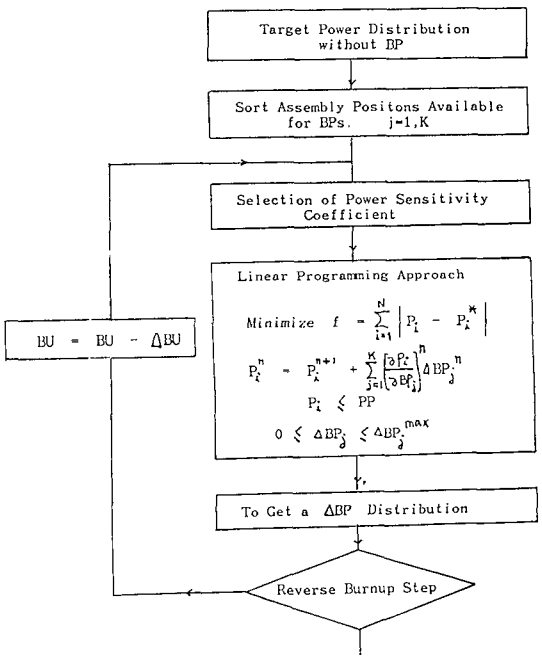


**Fig. 1. Schematic Poison Assignment Algorithm.**

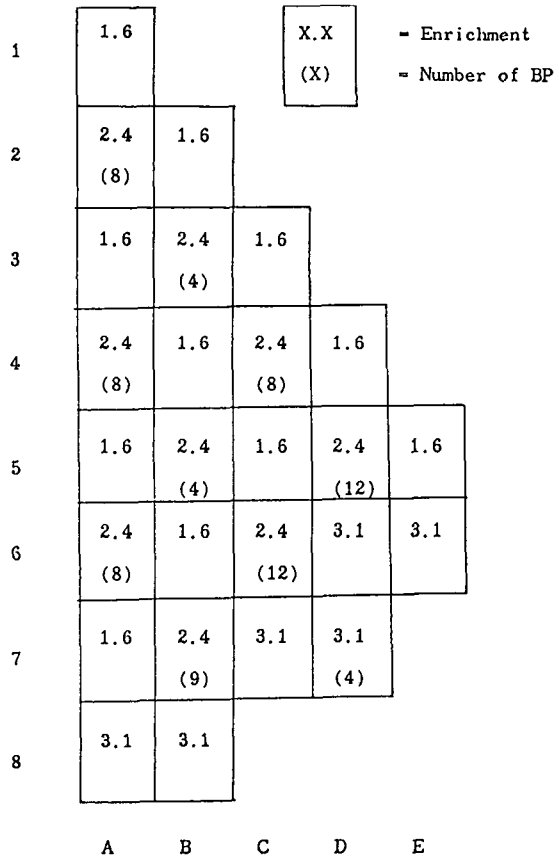


**Fig. 2. Cycle 1 Design Loading Pattern for KNU-7.**

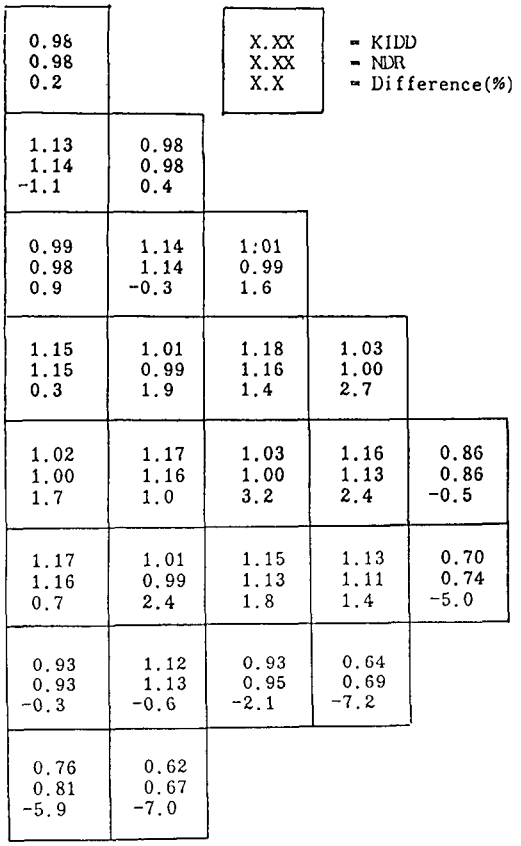


**Fig. 3. Comparison of Relative Power between KIDD and NDR at EOC of Cycle 1.**

process. In the next step, the optimal BP distribution at very EOC state is calulated. At (EOC-1) burnup step and up to the BOC, the necessary quantities in Eq. (4) are available. Therefore the optimal solution of $\Delta$BP for every burnup step is obtained. The BP distribution to be loaded at the BOC is the simple summation of $\Delta$BPs.

## III. Application to KNU-7

The design method proposed here is applied to cycle 1 of the KNU-7. The designed core loading pattern is shown in Fig. 2. In the octant core calculation, it is modeled that only ten assemblies can accommodate BPs and the number of burnup steps are ten, from 0 to 14,400 MWD/MTU.

As the diffusion depletion code KIDD is a well tuned code for a core calculation, it is used to calculate the reference power distribution. The power distribution between the KIDD calculation and the Nuclear Design Report of KNU-7 cycle 1[5] is compared in Fig. 3 for the EOC. The power distribution of the first cycle of the KNU-7 in this paper is not equal to the real power distribution of cycle 1. Although the actual type of BP in the cycle 1 core is the WABA(Wet Annual Burnable Absorber), an ordinary BP type is considered in this work. But this is not important to justify the usefulness of the reverse depletion method.

As the first stage of this study, the PSCs for each assembly and burnup step are computed from the results of the KIDD depletion calculation and the following assumptions are used:

1) The power sensitivity coefficient is a function

of BPs only and the burnup effect of fuel, enrichment, geometry factor, etc. are negligible.

2) The PSC at the BP inserted assembly is not affected by the BPs in other assemblies.

3) The PSC at the assembly without BP is affected by the BPs in the adjacent four assemblies or by the BPs in the nearest BP inserted assembly.

In order to generate PSCs, the spatial range of BP to affect on the power distribution is studied previously. For this purpose, 8-BPs are inserted at the A4 position in Fig. 4 and the variation of the power distribution during depletion is checked. Fig. 4 shows assembly positions where power decrease. The power decreases of the assemblies adjacent to the A4 position are due to the effect of BP insertion. However, power decreasing in some distant assemblies are considered to be attributable to the power normalization. According to this result, the effective range of BPs in A4 is considered to be the neighboring assemblies such as A3, A5 and B4. Therefore, the core region is divided along to the BP inserted assembly positions and PSCs generated in each divided region using Eq. (5). After generation of PSCs in each divided region at each burnup step, we could select these data sets adequately.

The variations of power sensitivity coefficients during depletion are shown in Figs. 5, 6, and 7. Fig. 5 shows the change of coefficients according to the difference of number of BP rods, 4, 8, and 12. Fig. 6 illustrates the influence of neighboring assemblies to assembly A5. These effects are similar except for the beginning-of-cycle. However, Fig. 7 shows the difference of coefficients despite equal BP rods. It means that the effects of BPs are different where BPs are inserted in the core. Therefore, the geometry factor at the BP inserted assembly position must be considered for a more fine calculation.

The next stage in the design procedure is to determine the BP distribution that controls the

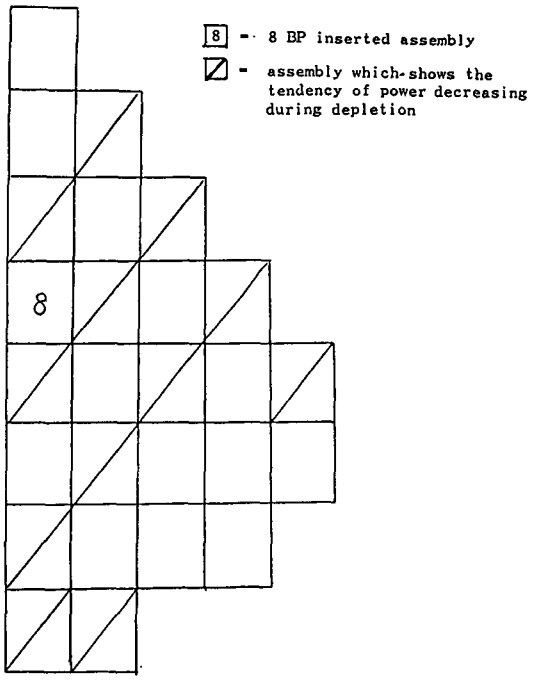


Fig. 4. The Variation of Power Distribution due to Insertion of BP at A4.

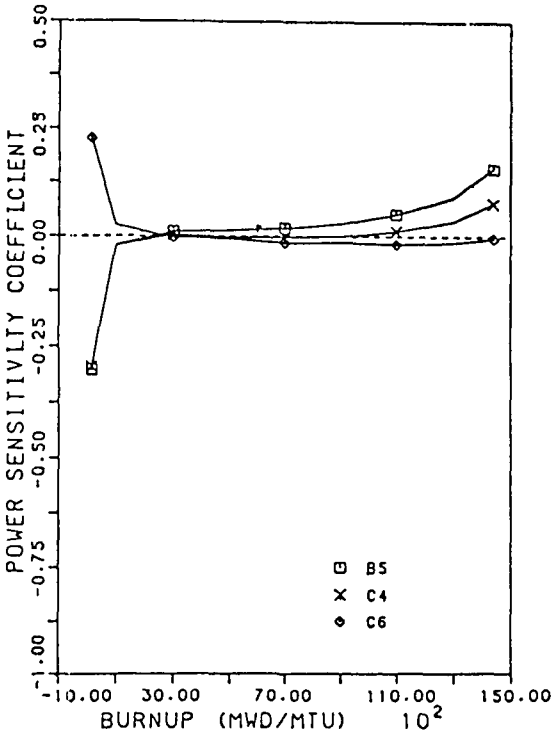


Fig. 5. The Power Sensitivity Coefficient vs. Burnup.

**Table 1. Burnable Poison Requirement for Cycle 1.**

| ASS<br>BU(MWD/MTU) | A2 | A4 | A6 | B3 | B5 | B7 | C4 | C6 | D5 | D7 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.24 | 0.24 | 0.23 | 0.09 | 0.09 | 0.16 | 0.16 | 0.25 | 0.23 | 0.04 |
| 150 | 0.95 | 0.97 | 0.89 | 0.51 | 0.48 | 0.82 | 0.93 | 1.29 | 1.22 | 0.21 |
| 1,000 | 2.20 | 2.12 | 1.91 | 1.12 | 1.03 | 1.74 | 2.03 | 2.71 | 2.62 | 0.46 |
| 3,000 | 1.79 | 1.78 | 1.70 | 0.91 | 0.89 | 1.64 | 1.76 | 2.45 | 2.42 | 0.46 |
| 5,000 | 1.21 | 1.23 | 1.28 | 0.60 | 0.63 | 1.40 | 1.28 | 1.94 | 1.97 | 0.43 |
| 7,000 | 0.73 | 0.76 | 0.85 | 0.36 | 0.40 | 1.08 | 0.82 | 1.36 | 1.41 | 0.39 |
| 9,000 | 0.41 | 0.44 | 0.53 | 0.20 | 0.23 | 0.78 | 0.49 | 0.89 | 0.94 | 0.34 |
| 11,000 | 0.22 | 0.23 | 0.30 | 0.11 | 0.13 | 0.54 | 0.26 | 0.53 | 0.57 | 0.29 |
| 13,000 | 0.09 | 0.09 | 0.08 | 0.04 | 0.05 | 0.38 | 0.10 | 0.06 | 0.24 | 0.18 |
| Estimation | 7.8 | 7.9 | 7.8 | 3.9 | 3.9 | 8.5 | 7.8 | 11.5 | 11.6 | 2.8 |
| Reference | 7.9 | 7.9 | 7.8 | 3.9 | 3.9 | 8.9 | 7.8 | 11.6 | 11.6 | 2.8 |

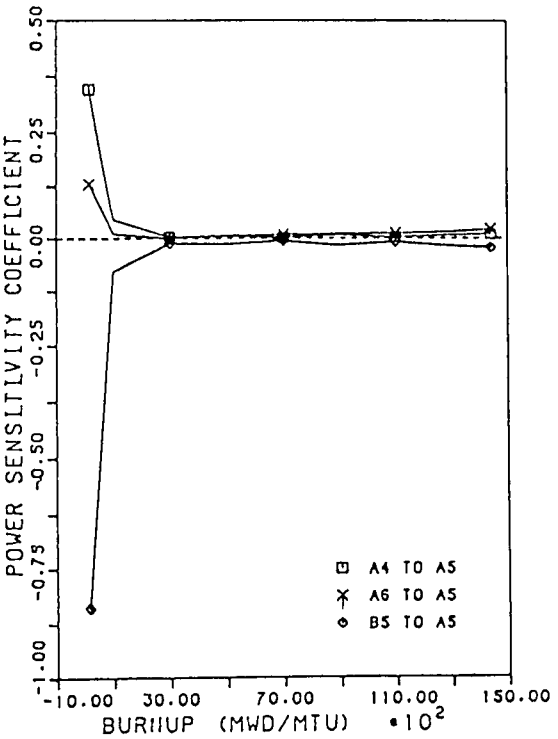


**Fig. 6. The Power Sensitivity Coefficient vs. Burnup.**

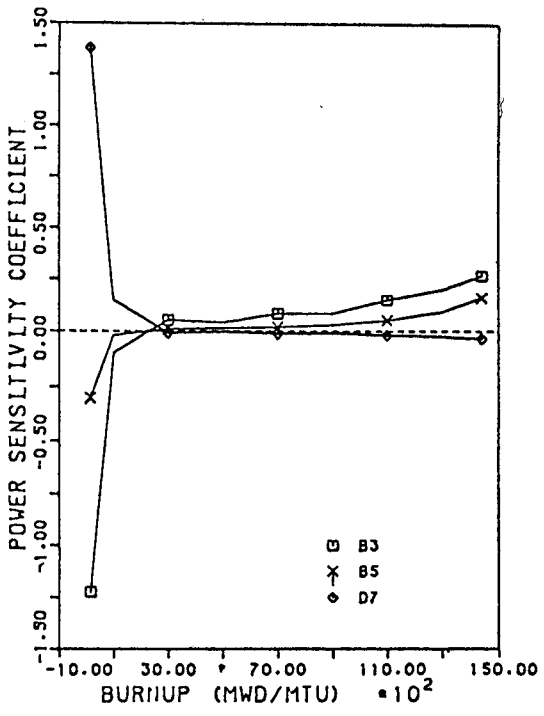


**Fig. 7. The Power Sensitivity Coefficent vs. Burnup.**

power distribution as similar as possible to the target power distribution using the method described in chapter II.

As seen from Table 1 the estimated are nearly the same as the reference BP distributions. The result shows maximum 4.5% error in the number of burnable poisons between the estimated and the refernece values.

## IV. Conclusions and Recommendations

The objective of this study is to verify the usefulness of the reverse depletion method to determine the optimal BP distributon. Before the reverse depletion method is applied to cycle 1 of KNU-7, PSCs are generated from the forward de-

pletion calculation using KIDD code. And those coefficients are used to justify if the estimated BP distribution is close to the reference BP distribution.

As seen previously, if PSC data sets and the target power distribution are given, the optimal burnable poison distribution can be obtained without time-consuming core calculation. The result shows maximum 4.5% error in the number of BP rods between calculated and the reference values. It is concluded that the reverse depletion method can be used to find the optimal burnable poison distribution.

The following recommendations are made to improve the present work further. In this work, the variation of power distribution is assumed to be a function of BP only. But in a rigourous manner, the change of power distribution is the function of burnup, enrichment, geometry, etc. Therefore, these factors should be considered in the PSC data sets generation. In this case, however, the PSCs should be calculated for each burnup step, each fuel cycle, and each reactor unit. Therefore, a large number of data sets would be required. A study on the construction of PSC data sets which are as small as possible but can be used for a wide spectrum of core condtion is recommended.

## References

1. Thomas J. Downar, Young Jin Kim, "A Reverse Depletion Method for Pressurized Water Reactor Core Reload Design," *Nucl. Tech.*, **73,** 42. (1986).
2. Y. J. Kim, "Optimization of Core Reload Design for Low-Leakage Fuel management in Pressurized Water Reactor," Ph.D. Thesis, Purdue University (1985).
3. K. S. Moon, *et al.*, "KIDD, A KAERI Improved Diffusion Depletion Program for Nuclear Reactor Analysis," KAERI.
4. 박순달, "선형계획법(및 그 관련분야)," 대영사, 1987.
5. S. S. Witter, *et al.*, "Nuclear Design Report of Korea Nuc ear Units 7 and 8 Cycle 1," Westinghouse Electric Corporation, 1985.
6. Y. A Chao, *et al.*, "A Theory of Fuel Management via Backward Diffusion Calculation," *Nucl. Sci. Eng.*, **93,** 78 (1986).
7. K. C. Murty, "Linear and Combinatorial Programming," John Wiley & Sons Inc., 1976.
8. John O. Mingle, "In-Core Fuel Management via Perturbation Theory," *Nucl. Tech.*, **27,** 248 (1975).
9. T. J. Downar and J. A. Stillman, "Explicit Burnable Absorber Modeling for Pressurized Water Reactor Core Reload Design Application," *Nucl. Sci. Eng.*, **96,** 85 (1987).
10. T. O. Sauar, "Application of Linear programming to In-Core Fuel Management Optimization in Light Water Reactor," *Nucl. Sci. Eng.*, **46,** 274 (1971).
11. Y. C. Chang and A. Sesonske, "Optimization and Analysis of Low-Leakage Core Management for Pressurized Water Reactor," *Nucl. Tech.*, **65** (1983).