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Development of a Method for Optimal Fuel Distribution in 1-D Cylindrical Geometry

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일차원 cylinder구조에서의 최적 연료분포를 구하는 방법의 개발

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Abstract

Previously determining the fuel loading pattern is based on the trial and error method. For a candidate pattern, the core analysis is performed and the pattern is examined whether it satisfies the imposed constraints such as the power peaking or not. The pattern, then, is revised by the shuffling of assemblies and the revision is repeated until all of the conditions are met. This method unavoidably requires many iterative diffusion calculations, computing times and accumulated experiences.

To overcome these disadvantages, a new method which is called backward diffusion calculation is introduced. If the most desirable power distribution is already known, the optimal loading pattern can be obtained by solving the backward diffusion equation with simple calculation.

In this study, the basic equation for the backward diffusion calculation is derived and the optimal power and fuel distributions are searched in one-dimensional cylindrical geometry by using the proposed method. In additon, the basis to determine the optimal power and fuel distributions is suggested for the real core geometry.

요 약

지금까지는 노심의 장전 패턴을 찾는데 있어서, 임의로 대략 노심 장전 패턴을 가정한 후 출력 분포를 구하여 보고 peaking limit와 같은 여러가지 제약 조건을 만족시키지 못하면 모든 조건을 만족할 수 있도록 다시 다른 형태의 노심 장전 패턴을 시도하여 구하는 방법을 사용하였으나, 이 방법은 축적된 경험과 반복적인 많은 노력과 시간을 요구하는 단점이 있다.

본 논문에서는 후방확산 계산 이론을 이용하여 1차원 원통형 노심에 대해 첨두출력 제약 조건을 만족시키면서 가장 적은 연료를 필요로하는 출력 분포를 찾고, 그에 대응하는 노심 장전 패턴을 구할 수 있는 방법을 검토하였다. 이는 후방 확산 이론을 실제 노심구조에 적용하기 위한 전 단계로서 수행되었다.

Nomenclature

∇^2 : Laplacian operator
D_1 : fast neutron diffusion coefficient
D_2 : thermal neutron diffusion coefficient
Σ_1 : fast neutron removal cross section
Σ_{12} : removal cross section
$\nu\Sigma_{f1}$: fast neutron fission yield
$\nu\Sigma_{f2}$: thermal neutron fission yield
Φ_1 : fast neutron flux
Φ_2 : thermal neutron flux
L_1 : fast neutron diffusion length
L_2 : thermal neutron diffusion length
ϵ : fast fission factor
K_∞ : infinite multiplication factor.

1. Introduction

Optimization of fuel management is closely connected with economy of nuclear power plant. Saving in fuel is directly related to low electricity cost. Therefore, minimum fuel loading is one of the most important requirements in designing the reactor core. So optimization of fuel loading is needed.

In the aspect of optimization, object functions can be varied and subsequent various constraints can be defined according to object function. There are many different kinds of optimization problems in the reactor design. Some of them are exampli-fied as follows.

1. flux flattening problem.
2. minimum critical mass problem.
3. maximum power problem.
4. control rod sequence problem.

Each problem has various constraints which must be satisfied for specific purpose. In this study, power distribution and according loading pattern that require minimum core fuel loading are searched over the whole core to meet the criticality condition. For this purpose, a new concept, which is called fuel potential, is introduced. To obtain such object function, "effective fast group model" is developed to represent the behavior of fast neutron and fuel enrichment by modifying the two group diffusion equation.

Search for minimum object function must be carried out in more realistic geometry. But there are many difficulties in guessing various power shapes. In this research, for simplicity one dimensional geometry is adopted to obtain optimal power shape and minimum fuel loading. The "optimal power" stands for the power distribution which minimizes object function under imposed constraints.

A new method which determines optimal loading pattern using specified power distributions is introduced in chapter 2. Then the concept of fuel potential is discussed in slab and cylindrical geometry in chapter 3. Chapter 4 presents the method of mapping from 1-D to 2-D geometry and chapter 5 and chapter 6 present results and conclusions respectively. This study mainly follows the backward diffusion method of Chao,[7] however the different method to choosing optimal power shape is adopted. The results show that K* distributions are somewhat different and the peripheral region should be loaded with low enrichment fuels.

2. Bakcward Diffusion Method

To determine loading pattern conveniently, backward diffusion method is used in this study. This method is based on the assumption that fast group corss sections are nearly constant over the core and that mesh size is determined so as that meshes are coupled only via fast neutrons.

In reality, however, thermal group cross sections can not be constant over the core. Spatial dependent variation of thermal group cross section, however, is assumed not to affect the fast neutron distribution signifcantly. This statement can be verified by Eq. (2-9). It, then, can be said the above approximation is legitimate. By using the above assumption, the backward diffusion method can be derived as follows.

Two-group diffusion equations for the critical system are derived as follows;

$$-D_1\nabla^2\Phi_1+\Sigma_1\Phi_1=\nu\Sigma_{f1}\Phi_1+\nu\Sigma_{f2}\Phi_2, \quad (2\text{-}1)$$

$$-D_2\nabla^2\Phi_2+\Sigma_2\Phi_2=\Sigma_{12}\Phi_1. \quad (2\text{-}2)$$

Eq. (2-2) can be modified as follows;

$$\Phi_2=\frac{\Sigma_{12}}{\Sigma_2}\cdot\frac{1}{1-L_2^2\nabla^2\Phi_2/\Phi_2}\Phi_1. \quad (2\text{-}3)$$

If Eq. (2-3) is inserted into Eq. (2-1), Eq. (2-1) can be rewirtten as follows;

$$-D_1\nabla^2\Phi_1+\Sigma_1\Phi_1 = \nu\Sigma_{f1}\Phi_1+\nu\Sigma_{f2}\frac{\Sigma_{12}}{\Sigma_2}\cdot\frac{\Phi_1}{1-L^2{}_2\nabla^2\Phi_2/\Phi_2} \quad (2\text{-}4)$$

Rearranging both sides of Eq. (2-4), it is possible to represent the above equation only with fast neutron flux if the infinite multiplication factor and fast fission factor are introdued.

$$-L^2{}_1\nabla^2\Phi_1+\Phi_1 =\frac{\nu\Sigma_{f1}}{\Sigma_1}\Phi_1+\frac{\nu\Sigma_{f2}}{\Sigma_1}\cdot\frac{\Sigma_{12}}{\Sigma_2}\cdot\frac{\Phi_1}{1-L^2{}_2\nabla^2\Phi_2/\Phi_2} \quad (2\text{-}5)$$

Meanwhile the infinite multiplication factor can be stated as follows;

$$K_\infty=\frac{\nu\Sigma_{f1}}{\Sigma_1}+\frac{\nu\Sigma_{f2}}{\Sigma_2}\cdot\frac{\Sigma_{12}}{\Sigma_1}, \quad (2\text{-}6)$$

$$K_\infty/\epsilon=\frac{\nu\Sigma_{f2}}{\Sigma_2}\cdot\frac{\Sigma_{12}}{\Sigma_1}. \quad (2\text{-}7)$$

Thermal neutron leakage probability, δ_2, is the ratio of thermal neutron leakage to absorption. This term is introduced to consider the effect of the thermal neutron flux to fast neutron flux distribution.

$$\delta_2\equiv-\frac{L^2{}_2\nabla^2\Phi_2}{\Phi_2}. \quad (2\text{-}8)$$

It is well known that this thermal neutron leakage probability is very small compared with fast neutron leakage probability. Then taking account of thermal leakage probability, two-group diffusion equations can be rewritten as follows

$$-L^2{}_1\nabla^2\Phi_1+\Phi_1=[1-\frac{\delta_2/\epsilon}{1+\delta_2}]K_\infty\Phi_1, \quad (2\text{-}9)$$

$$-L^2{}_2\nabla^2\Phi_2+\Phi_2=\frac{\Sigma_{12}}{\Sigma_2}\Phi_1. \quad (2\text{-}10)$$

In the similar way, fast neutron leakage probability, δ_1, can be defined as

$$\delta_1\equiv-\frac{L^2{}_1\nabla^2\Phi_1}{\Phi_1}. \quad (2\text{-}11)$$

Using Eqs. (2-8), (2-10), and (2-11), Eqs. (2-9) and (2-10) can be represented in terms of thermal and fast neutron leakage probabilities.

$$\delta_1+1=[1-\frac{\delta_2/\epsilon}{1+\delta_2}]K_\infty \quad (2\text{-}12)$$

$$\delta_2+1=\frac{\Sigma_{12}}{\Sigma_2}\cdot\frac{\Phi_1}{\Phi_2}. \quad (2\text{-}13)$$

Dividing Eq. (2-8) by Eq. (2-11), it is obtained as

$$\frac{\delta_2}{\delta_1}=\frac{\Phi_1L^2{}_2\nabla^2\Phi_2}{\Phi_2L^2{}_1\nabla^2\Phi_1}. \quad (2\text{-}14)$$

Using Eq. (2-13), Eq. (2-14) can be expressed with fast neutron flux only as follows;

$$\Phi_2=\frac{\Sigma_{12}}{\Sigma_2}\cdot\frac{1}{(1+\delta_2)}\Phi_1, \quad (2\text{-}15)$$

$$\frac{\delta_2}{\delta_1}=\frac{(1+\delta_2)L^2{}_2\nabla^2[\Phi_1/(1+\delta_2)]}{L^2{}_1\nabla^2\Phi_1}. \quad (2\text{-}16)$$

Because the thermal neutron leakage probability is small quantity, the thermal neutron leakage

Table 1. Fast Neutron Diffusion Length L, for Typical PWR

Enrichment (wt%)	Burnup (MWD/MTU)	D_1 (cm)	Σ_1 (cm^{-1})	L_1 (cm)
2.1	0.0	1.4676	0.025788	7.544
2.1	12,000	1.4739	0.026180	7.503
2.1	24,000	1.4571	0.026776	7.377
2.6	0.0	1.4622	0.025578	7.651
2.6	12,000	1.4695	0.026014	7.516
2.6	24,000	1.4677	0.026593	7.429
3.1	0.0	1.4556	0.025443	7.572
3.1	12,000	1.4661	0.025892	7.525
3.1	24,000	1.4773	0.026446	7.474
3.6	0.0	1.4562	0.025356	7.578
3.6	12,000	1.4635	0.025807	7.531
3.6	24,000	1.4757	0.026331	7.486

probability can be kept only first order terms. Therefore the ratio of both leakage probabilities is same as the ratio of both migration areas. Then Eq. (2-12) can be represented in terms of thermal neutron leakage probability as follows;

$$\delta_2=\frac{(L^2_2/L^2_1)\cdot(K_\infty-1)}{1+(L^2_2/L^2_1)\cdot(1-K_\infty+K_\infty/\epsilon)}. \qquad (2\text{-}17)$$

Inserting Eq. (2-17) into Eq. (2-9), two group diffusion equations can be restated with fast neutron flux only.

$$-L^2_1\nabla^2\Phi_1+\Phi_1=K^*\Phi_1 \qquad (2\text{-}18)$$

where

$$K^*=\frac{1+(L^2_2/L^2_1\,\epsilon)}{1+(L^2_2/L^2_1\,\epsilon)K_\infty}K_\infty. \qquad (2\text{-}19)$$

Fast neutron diffusion length, thermal neutron diffusion length, and fast fission factor are assumed constant so that K* in Eq. (2-19) is independent of thermal neutron flux. Eq. (2-18) which is called "effective fast group model" is expressed as if it were 1-group diffusion equation.

In deriving the backward diffusion equation, thermal group cross sections except for fission cross section are assumed constant, but in real core they are different in each assembly. However, the variation of the thermal neutron leakage probability due to the change in thermal neutron cross section does not affect fast neutron distribution. Even if there are burnable poisons which influence thermal neutron absorption only, thermal neutron leakage probability becomes smaller so that effective fast group diffusion is more legitimate.

The physical meaning of K* in the effective fast group model is the same as the multiplication factor, excpet for including of the thermal neutron diffusion correction term, and represents the usability of fuel.

From the above effective fast group model, K* distribution can be determined by the predetermination of fast flux distribution, and vice versa.

In this derivation fast diffusion length, L_1, is also used as constant. The values of L_1 are listed in the following Table 1. It can be seen that L_1's are nearly constant regardless of burn-up and enrichment.

3. Determination of Optimal Power Distrbution and Fuel Loading Pattern

A. Optimal Power Distribution

In the sense that K* is analogous to the infinite multiplication factor, the optimal power distribuion is the one that necessitates minimum global fuel loading to achieve the criticality of a system having fixed dimensions.

To obtain these optimal power distribution, the concept of fuel potential is introduced. The fuel potential is the references to select the optimal power distribution. From the effective fast group model, derivation of fuel potential in the slab and cylindrical geometry is performed.

B. Fuel Potential

b-1. Slab geometry

The effective fast group equation is expressed in slab geometry as

$$L^2{}_1\frac{d^2}{dx^2}\Phi_1(x)-\Phi_1(x)=-K^*(x)\Phi_1(x) \qquad (3\text{-}1)$$

For convenience, the spatial variable, x, is made dimensionless by dividing the variable by the fast neutron diffusion length. Boundary conditions are adopted as the finite fast flux at the core boundary and the zero neutron current at the core center.

$$\frac{d\Phi_1(x)}{dx}\mid_{x=0}=0, \qquad (3\text{-}2)$$

$$\Phi_1(x=x_0)=\Phi_{10}. \qquad (3\text{-}3)$$

Eq. (3-1) is a kind of Sturm-Liouville nonhomogeneous boundary value problem. This problem is solved analytically by introducing Green function as follows.

The effective fast group equation is deduced as Green function G(x);

$$\frac{d^2}{dx^2}G(x)-G(x)=0. \qquad (3\text{-}4)$$

The constraints that Green function G(x) should satisfy are as follows;

$$\frac{dG(x)}{dx}\mid_{x=0}=0,\ G(x)\mid_{x=x_0}=0, \qquad (3\text{-}5)$$

$$G(x)\mid_{x=x'+0}=G(x)\mid_{x=x'-0}, \qquad (3\text{-}6)$$

$$\frac{dG(x)}{dx}\mid_{x=x'+0}-\frac{dG(x)}{dx}\mid_{x=x'-0}=1, \qquad (3\text{-}7)$$

Above constraints are derived from the fact that Green function should vanish at boundary, that Green function must be continuous and that derivatives of Green function must be discontinuous at one interior source point.

Then the above Green function can be solved depending on the position of the source point, x′, and the detection point, x

$$G(x,x')=A(e^x+e^{-x})(e^{x'}-e^{2x_0}e^{-x'}),\ x<x' \qquad (3\text{-}8)$$

$$G(x,x')=A(e^x-e^{2x_0})(e^{x'}+e^{-x'}),\ x>x' \qquad (3\text{-}8)$$

where

$$A=[2(1+e^{2x_0})]^{-1}.$$

Therefore the fast neutron flux can be expressed in terms of Green function.

b-2. Cylinderical geometry

For a cylindrical model, the effective fast group equation is written as follows:

$$\frac{1}{r}\frac{d}{dr}r\frac{d}{dr}\Phi_1(r)-\Phi_1(r)$$

$$=-K^*(r)\Phi_1(r) \qquad (3\text{-}14)$$

The boundary conditions which fast flux must satisfy are the same as those for the slab geometry

$$\frac{d\Phi_1(r)}{dr}\mid_{r=0}=0, \qquad (3\text{-}15)$$

$$\Phi_1(r=r_0)=\Phi_{10}. \qquad (3\text{-}16)$$

In the same way, the Green function can be written as follows;

$$\frac{1}{r}\frac{d}{dr}r\frac{d}{dr}G(r)-G(r)=0, \qquad (3\text{-}17)$$

$$\frac{dG(r)}{dr}\mid_{r=0}=0,\ G(r)\mid_{r=r0}=0, \qquad (3\text{-}18)$$

$$G(r)\mid_{r=r'+0}=G(r)\mid_{r=r'-0}, \qquad (3\text{-}19)$$

$$\frac{dG(r)}{dr}\mid_{r=r'+0}-\frac{dG(r)}{dr}\mid_{r=r'-0}$$

$$=\frac{1}{2\pi r}. \qquad (3\text{-}20)$$

The Green function which fulfills the above constraints in the cylindrical geometry is written in terms of modified Bessel functions.

$$G(r,r')=\frac{1}{2\pi}I_0(r)[\frac{K_0(r_0)}{I_0(r_0)}I_0(r')-K_0(r')],$$

$$r<r' \qquad (3\text{-}21)$$

$$G(r,r)=\frac{1}{2\pi}I_0(r')[\frac{K_0(r_0)}{I_0(r_0)}I_0(r)-K_0(r)],$$

$$r>r' \qquad (3\text{-}22)$$

where,

I_0=zeroth order modified Bessel function of first kind,

K_0=zeroth order modified Bessel function of second kind.

With the Green function for cylindrical geometry, fast neutron flux can be expressed as follows;

$$\Phi_1(r)=-\int_0^{r_0}K^*(r')\Phi_1(r')G(r,r')2\pi r'dr' + 2\pi r_0\Phi_1(r_0)G'_{r'}(r,r_0) \qquad (3\text{-}23)$$

where

$$G'_r(r,r_0)=\frac{dG(r,r')}{dr'}\Big|_{r'=r_0}$$

Boundray condition can be defined as follows;

$$\frac{-(D_1/L_1)\Phi'_1(r_0)}{\Phi_1(r_0)}=\alpha_1 \qquad (3\text{-}24)$$

Then the fast flux in the cylindrical geometry is

$$\Phi_1(r)=-\int_0^{r_0}K^*(r')\Phi_1(r')G(r,r')2\pi r'dr' + DI_1(r)\int_0^{r_0}K^*(r')\Phi_1(r')r'dr' \qquad (3\text{-}25)$$

And the power distribution, P(r), is:

$$P(r)=K^*(r)\Phi_1(r) \qquad (3\text{-}26)$$

$$K^*(r)=P(r)/[DI_0(r)\int_0^{r_0}P(r')I_0(r')r'dr' - \int_0^{r_0}G(r,r')p(r')p(r')2\pi r'dr'] \qquad (3\text{-}27)$$

where

$$D=\{r_0I_0(r_0)[I_1(r_0)+(\alpha_1L_1/D_1)I_0(r_0)]\}^{-1}$$

Since the object of this study is to reduce the amount of total fuel loading as low as possible, the object function can be expressed in K*(r) integrated over the core volume.

$$J=\int_0^{r_0}K^*(r)2\pi r dr. \qquad (3\text{-}28)$$

Because K*(r) is functional of P(r), optimal power distribution can be determined via selecting the power distribution which minimizes the object function.

This object function is notated as "J" and is called as the fuel potential which means the amount of total fuel loading to meet criticality.

b-3. Determination of power shape

It is possible to express the power shape in analytical form, since K*(r) is dependent on the power distribution. In determining the global power distribution, considered constraints are; the power shape is symmetric and the peak power does not exceed the limit. Taking into account of these constraints, the trial power distribution is expanded in 3rd order polynomials.

To search for the optimal power distribution, the position of a peak of the power distribution is moved from core centre to boundary by adjusting the coefficients of the polynomials.

The power distribution, of course, can be expanded with other function such as trigonometric functions. But when other expansion function is used and peak position and global power shape is similar, the value of the object function must not be appreciably different from that of the expansion case. For demonstration of above statement, the cosine function has been checked as an expansion function.

4. Method of Mapping from 1-D to 2-D

For practical use of the 1-D calculation, following algorithm, mapping the result of the x-y coordinates, is proposed. First, consider a uniform distribution of fuel potential throughout the two-dimensional core in question and calculate

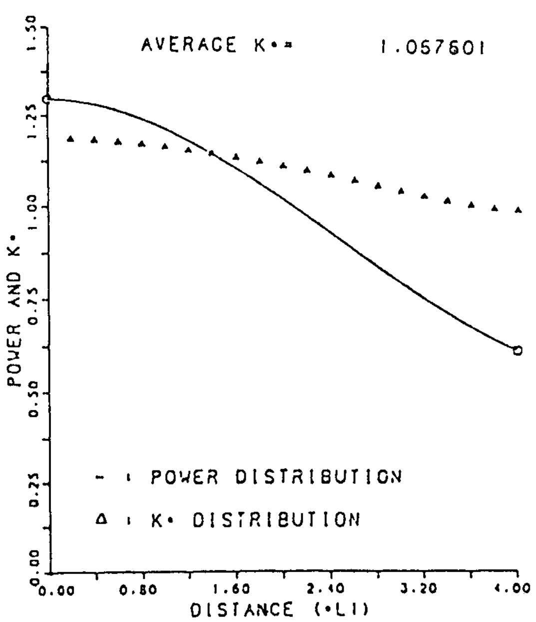

Fig. 1. K* and Power Distribution

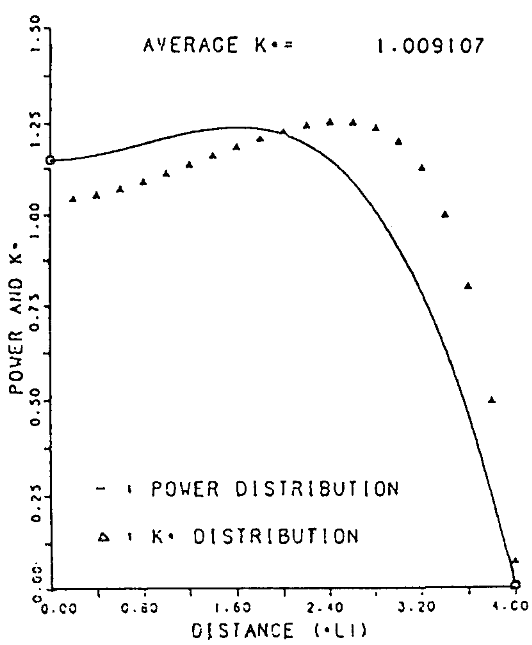

Fig. 2. K* and Power Distribution.

the critical value of the fuel potential and the corresponding two-dimensional power distribution. Then find an equivalent one-dimensional cylindrical core by adjusting its radius so that it has the same density of fuel potential at critical condition as for the two-dimensional core.

Now compare the normalized power distribution for the two cores to determine the mapped positions of the assemblies. This is done at first by matching the positions where the power levels are in its peak in 1-D and 2-D geometry. The same process is then repeated for the other assemblies by descending order of power level. Fig. 10 shows how the 1-D results are mapped on the 2-D plane when the above strategy is used.

5. Results

Using the effective fast group model and the procedure described in the previous chapter, the optimal power distribution is predetermined and accordingly optimal fuel enrichment distribution is obtained for a reactor whose core radius is four times of fast neutron diffusion length.

Calculated power distribution and K* distribution are plotted in Figs. 1 to 7. From Fig. 1 to 4 a polynomial power shape is used in cylindrical core, and cosine power shape is used in from Fig. 5 to 7.

As a summary, the variation of core average K* with power peak position and with power peaking limit is plotted in Figs. 8 and 9. It can be seen in these figures that the optimal loading is obtained when power peak is positioned at around a half of the radius, and that the average K* is decreasing as the power peaking limit is relaxed.

6. Conclusions

From the above figures, it can be concluded as follows.

1) If high enrichment fuels are loaded in core center region, the value of object function is usually high. Thus, it can be said that high enrichment fuels must not be loaded at central region.

2) In case of high enrichment fuels loaded in

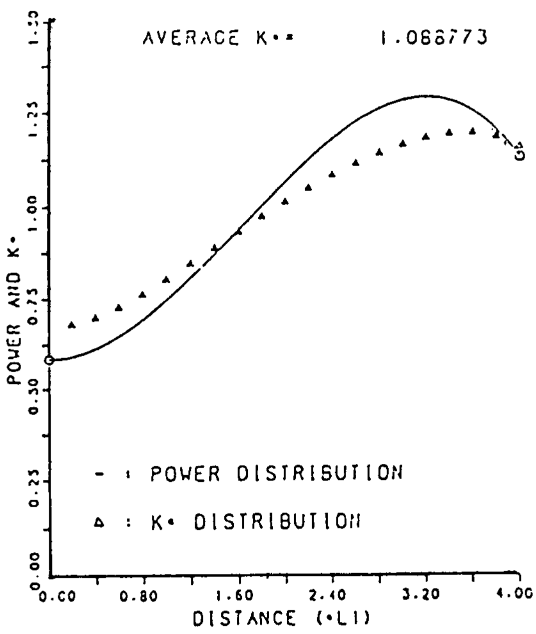

Fig. 3. K* and Power Distribution.

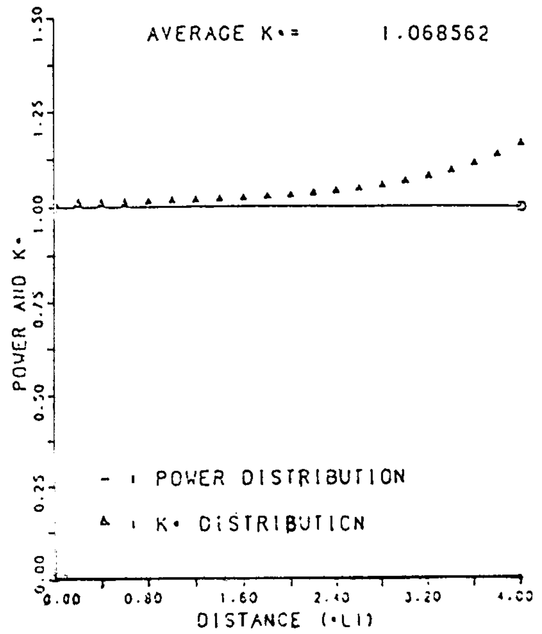

Fig. 4. K* and Power Distribution.

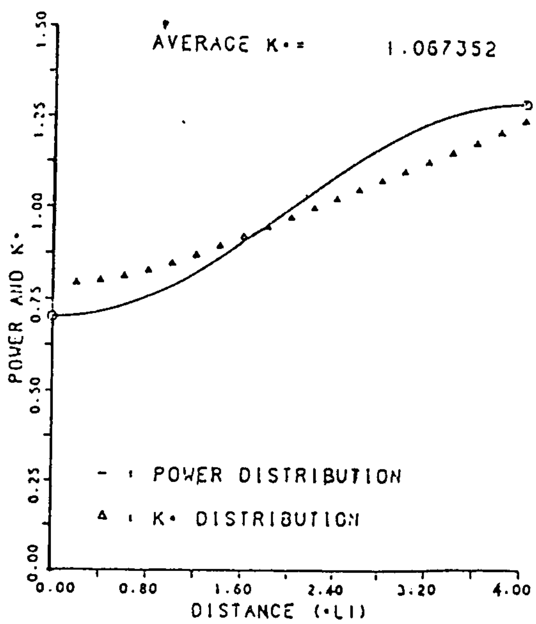

Fig. 5. K* and Power Distribution.

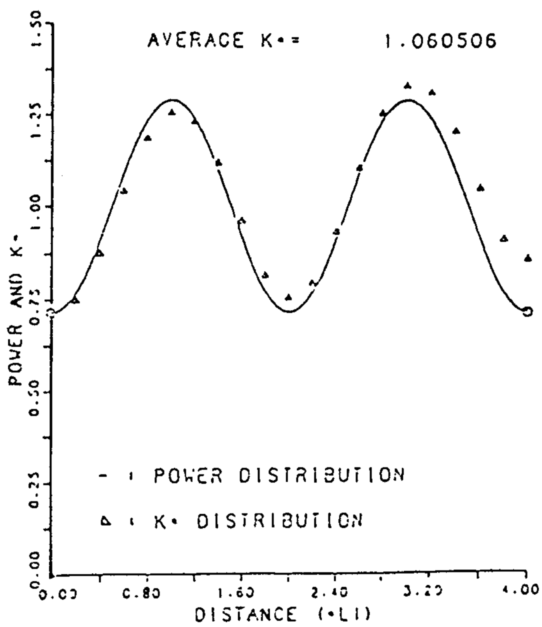

Fig. 6. K* and Power Distribution.

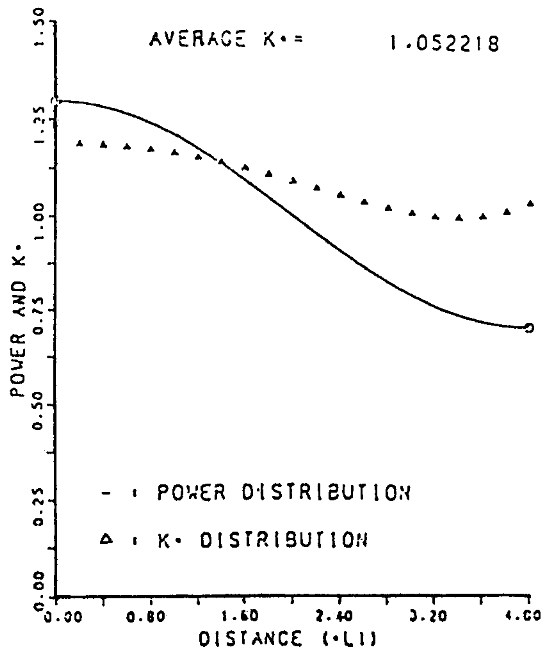

Fig. 7. K* and Power Distribution.

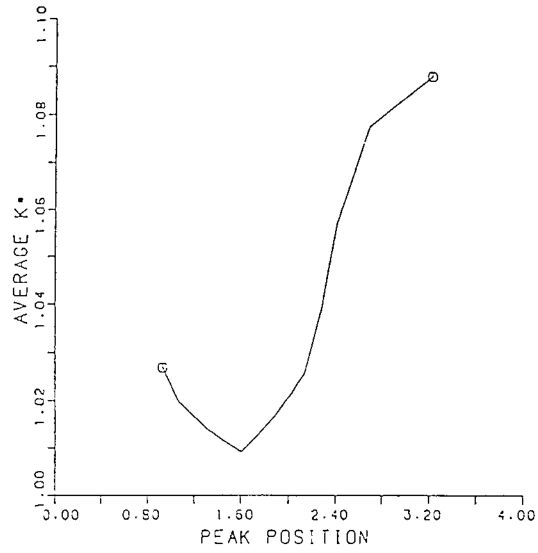

Fig. 8. K* Sensitivity to Peak Position.

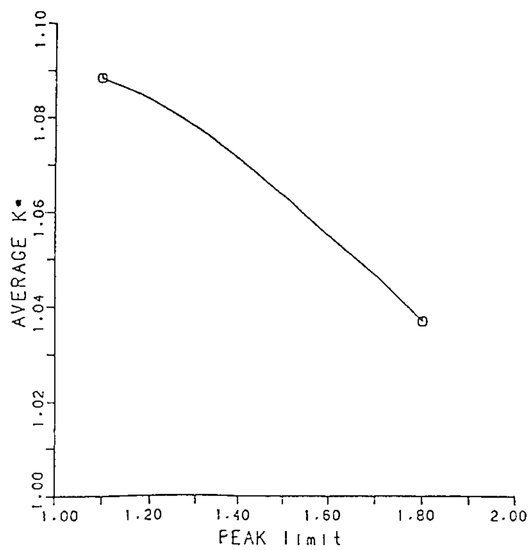

Fig. 9. K* Sensitivity to Peak Limit.

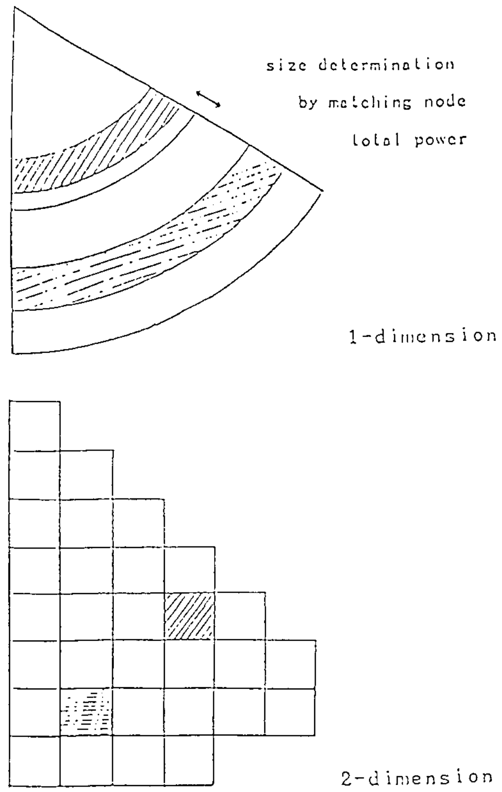

Fig. 10. Diagram of Mapping from 1-D to 2-D Geometry

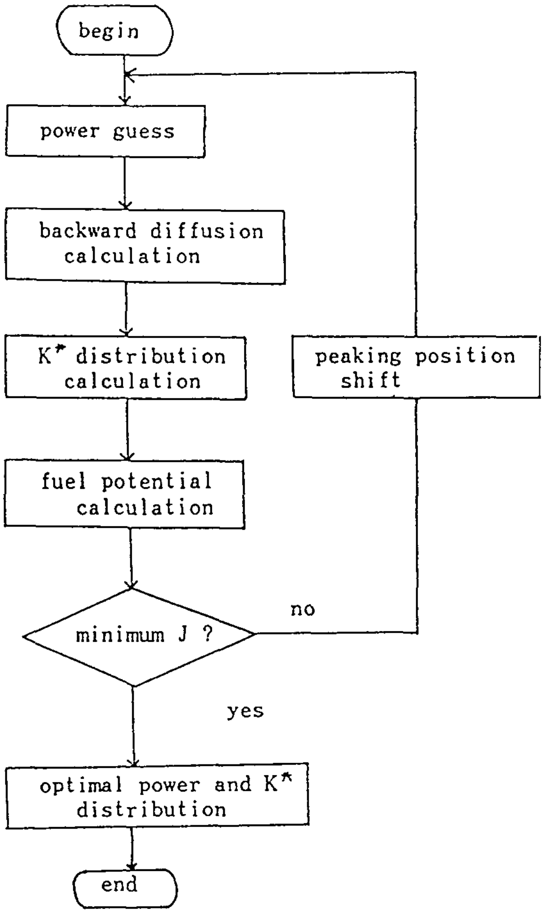

Fig. 11. Flow Diagram of Calculation.

core periphery, it is also undesirable, since at boundary the gradient of flux is so high that neutron leakage rate is high.

3) High enrichment fuel, therefore, should be loaded in the middle of the core and low enrichment fuel should be located in the peripheral and central region.

4) It can be seen that the fuel potential is inversely proportional to the power peak limit so that the more reduction in fuel potential leads to the higher power peak.

Up to now, the optimal power distribution and K* distribution are obtained for a 1-D cylindrical reactor. The entire calculation sequence is diagramed in Fig. 11. However, the difficulties imposed in real reactor application are that Green function should be seeked in real core geometry and power shape is to be expressed in multidimension.

To surmount these difficulties, a mapping which matches 1-dimensional ring with 2-dimensional node is suggested.

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