Equivalent Circuit Description for a Parallel-Plate Waveguide with a Transverse Slit in its Upper Plate

(한 면에 슬릿이 있는 평행-평판 도파관에 대한 등가회로)

金承珏,** 趙榮基,* 金昌熹,** 尹明漢,** 洪在杓,* 孫 鉉*

(Seung Gak Kim, Young Ki Cho, Chang Hee Kim, Myoung Han Yoon, Jae Pyo Hong and Hyon Son)

要 約

평행 평판 도파관을 해석하고 등가 자기전류에 대한 적분 방정식의 해는 모멘트법을 이용해서 구했다.

수치적 결과는 자기 전류, 반사 및 투과계수, 슬릿에서의 복사전력, 그리고 등가회로 파라미터 등으로 나타냈다. 등가회로 파라미터는 E-plane 결합 마이크로스트립 안테나 연구에 유용하다.

Abstract

A parallel-plate waveguide with a slit in its upper plate is analysed. An integral equation is formulated for the equivalent magnetic current and solved by the conventional moment method.

Numerical results are presented for the magnetic current, reflection and transmission coefficients, a normalized radiated power in the slit, and equivalent circuit parameters. The equivalent circuit parameters are an useful quantities in the study of the E-plane coupled microstrip antennas.

I. Introduction

The analysis of the slit in the parallel-plate waveguide was undertaken many years ago by R.F. Millar[1] but the results of this early work are valid only for wide slit.

Recently, some authors [2]-[4] analysed the narrowly slitted parallel-plate waveguide filled with a homogeneous media.

In this paper, an efficient computation technique in representing the equivalent circuit parameters of the slit in the wall of the parallel-plate waveguide is considered for the arbitrary slit width.

An integral equation is formulated for the equivalent magnetic current and is solved numerically. From knowledge of the equivalent magnetic current, quantities of interest such as reflection and transmission coefficients, a normalized radiated power in the slit, and equivalent circuit parameters are calculated. The equivalent circuit parameters are an useful quantities in the study of the E-plane coupled microstrip antennas.

II. Formulation of the Problem

The waveguide is illustrated in Fig. 1 where

*正會員, **準會員, 慶北大學校 電子工學科

(Dept. of Elec. Eng., Kyungpook Nat'l Univ.)

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one sees a pair of perfectly conducting plates of zero thickness and of infinite extent with the upper plate slitted by an infinite slit width units wide and parallel to the y axis. The problem is entirely independent of y and is therefore two-dimensional.

When a TEM wave whose electric field amplitude is assumed to be unity is incident upon the slit as shown in Fig.1, electromagnetic fields scattered from the slit may be calculated from the equivalent magnetic currents on the shorted slit. The time harmonic dependence factor e^{jwt} is suppressed throughout and $kb < \pi$ such that only the TEM mode can propagated along the waveguide.

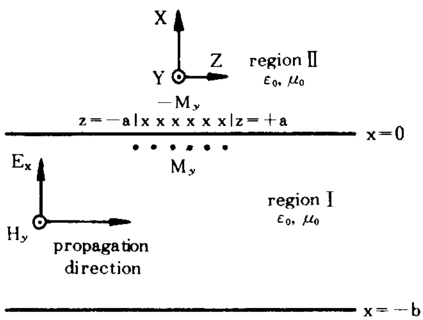

Fig.1. Equivalent magnetic currents on the slit.

The desired integral equation which is formulated in terms of the equivalent magnetic current M_y is obtained by resorting to the equivalent principle of electromagnetics. This means that if the slit is shorted by a perfect electric conductor, the fields in regions I and II will remain unchanged if appropriate magnetic currents are placed over the shorted slit on either side of the upper plate.

Choosing the appropriate Green's functions in each region I, II in Fig.1 and imposing the continuity of the tangential electromagnetic field in the slit, one can obtain

$$\frac{k}{2\eta}\int_{-a}^{+a} M_y(z')H_0^{(2)}(k\,|z-z'|)dz' = \int_{-a}^{+a} M_y(z')G(z,z')dz' + \frac{1}{\eta}e^{-jkz},$$
$$-a \le z, z' \le a, \quad x, x' = 0 \qquad (1)$$

in which the Green's function $G(z, z')$ [5] in region I is given by

$$G(z,z') = -\frac{1}{2\eta b}e^{-jk|z-z'|} - \frac{jk}{\eta b}\sum_{n=1}^{\infty}\frac{e^{-\sqrt{\left(\frac{n\pi}{b}\right)^2-k^2}\,|z-z'|}}{\sqrt{\left(\frac{n\pi}{b}\right)^2-k^2}} \qquad (2)$$

Here k and η are the propagation constant and intrinsic impedance, respectively, of the free space. $H_0^{(2)}$ is the Hankel function of the second kind and zero order.

The unknown $M_y(z')$ is approximated by pulses and the resulting approximate equation is subjected to collocation [6].

By partitioning the interval ($-a$, $+a$) into N equal segments of length $\Delta = 2a/N$ and by selecting the match-point locations z_m at pulse centers according to $z_m = -a+(m-\frac{1}{2})\Delta$, one may establish the following approximation of the integral equation,

$$\sum_{n=1}^{N} M_{y,n}\left[Y^{I}_{mn}+Y^{II}_{mn}\right] = H_{yI,inc}(z_m) = \frac{1}{\eta}e^{-jkz_m}, \quad m=1, 2, 3\cdots N \qquad (3)$$

in which $M_{y,n}$ is the unknown coefficient of the n-th pulse located at z_n.
The matrix elements in equation (3) are defined as

$$Y^{I}_{mn} = -\int_{z_n-\Delta/2}^{z_n+\Delta/2} G(z_m, z')dz' \qquad (4)$$

$$Y^{II}_{mn} = \frac{k}{2\eta}\int_{z_n-\Delta/2}^{z_n+\Delta/2} H_0^{(2)}(k\,|z_m-z'|)dz' \qquad (5)$$

Green's function $G(z_m, z')$ in the slit in equation (4) can be replaced by the small argument approximation [3] near the singular point $z = z'$ as follows

$$G(z,z') = -\frac{e^{-jk|z-z'|}}{2\eta b} + \frac{jk}{\eta\pi}\left\{\ln\left(\frac{\pi|z-z'|}{b}\right) - \sum_0\left(\frac{kb}{\pi}\right)\right\} \qquad (6)$$

where $\sum_0(x) = \sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n^2-x^2}} - \frac{1}{n}\right) : \ |x| < 1$

Therefore, in case of $m = n$, equation (4) becomes after some algebraic manipulations

$$Y^{I}_{mn} = \frac{1 - e^{-jk\Delta/2}}{j\eta kb} - \frac{jk\Delta}{\eta\pi}\left\{\ln\left(\frac{\Delta\pi}{2eb}\right) - \sum_{0}\left(\frac{kb}{\pi}\right)\right\} \tag{7}$$

where $e = 2.718$ . . . . .

For $m \neq n$, equation (4) is represented by [7]

$$\begin{aligned} Y^{I}_{mn} = {} & \frac{1}{\eta kb}\sin(k\Delta/2)\, e^{-jk|m-n|\Delta} \\ & - j\frac{kb}{\pi^2\eta}\Big[f(\pi[|m-n|+1/2]\Delta/b) \\ & - f(\pi[|m-n|-1/2]\Delta/b) \\ & + \sum_{g=1}^{\infty}\Big\{ \Big(\frac{1}{g^2-\left(\frac{kb}{\pi}\right)^2} e^{-\pi\sqrt{g^2-\left(\frac{kb}{\pi}\right)^2}[|m-n|+1/2]\Delta/b} \\ & - \frac{1}{g^2-1/4} e^{-g\pi[|m-n|+1/2]\Delta/b} \Big) \\ & - \Big(\frac{1}{g^2-\left(\frac{kb}{\pi}\right)^2} e^{-\pi\sqrt{g^2-\left(\frac{kb}{\pi}\right)^2}[|m-n|-1/2]\Delta/b} \\ & - \frac{1}{g^2-1/4} e^{-g\pi[|m-n|-1/2]\Delta/b} \Big) \Big\} \Big] \end{aligned} \tag{8}$$

where $f(x) = 2[1 - \sinh(x/2)\ln(\coth(x/4))],\ x > 0$

Evaluation of integral of equation (5) for both $m = n$ case and $m \neq n$ case is performed by standard technique [6] and omitted here.

Solving the linear simultaneous equation whose coefficient elements Y^{I}_{mn}, Y^{II}_{mn} are given above, one obtains the unknown coefficients $M_{y.n}$.

From knowledge of $M_{y.n}$, one can compute the TEM magnetic field reflection coefficient Γ_H, transmission coefficient T_H, equivalent circuit parameters and radiated power of the slit discontinuity as shown in Fig.2

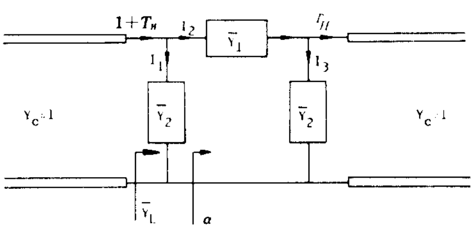

Fig.2. Equivalent π - network.

To obtain the equivalent π – network parameters in Fig.2, one uses the relation among the reflection coefficient, the transmission coefficient, and the normalized admittances in the transmission line, with a characteristic admittance Y_c unity.

The reflection coefficient Γ_H is defined as the ratio of reflected to incident magnetic field, and the transmission coefficient T_H as the ratio of transmitted to incident field at a reference plane located at $z = -a$; thus

$$\Gamma_H = -\frac{1}{2b} e^{-j2ka} \int_{-a}^{+a} M_y(z') e^{-jkz'} dz' \tag{9}$$

$$T_H = \left\{ 1 - \frac{1}{2b}\int_{-a}^{+a} M_y(z') e^{jkz'} dz' \right\} e^{-j2ka} \tag{10}$$

Using the conventional moment method, one can write

$$\Gamma_H = -\frac{\sin(k\Delta/2)}{kb} e^{j2ka} \sum_{n=1}^{N} M_{y,n} e^{-jkz_n} \tag{11}$$

$$T_H = \left\{ 1 - \frac{\sin(k\Delta/2)}{kb} \sum_{n=1}^{N} M_{y,n} e^{jkz_n} \right\} e^{-j2ka} \tag{12}$$

So the normalized admittances are given by

$$\bar{Y}_L = \frac{1+\Gamma_H}{1-\Gamma_H} = \bar{G}_L + j\bar{B}_L \tag{13}$$

$$\bar{Y}_2 = \frac{1+\Gamma_H - T_H}{1-\Gamma_H + T_H} = \bar{G}_2 + j\bar{B}_2 \tag{14}$$

$$\bar{Y}_1 = \frac{(\bar{Y}_2+1)(\bar{Y}_L - \bar{Y}_2)}{2\bar{Y}_2 + 1 - \bar{Y}_L} = \bar{G}_1 + j\bar{B}_1 \tag{15}$$

The branch currents I_1, I_2, and I_3 in Fig.2 are given as follows

$$I_1 = (1+\Gamma_H)\frac{\bar{Y}_2}{\bar{Y}_L} \tag{16}$$

$$I_2 = (1+\Gamma_H)\left(1 - \frac{\bar{Y}_2}{\bar{Y}_L}\right) \tag{17}$$

$$I_3 = I_2 \frac{\bar{Y}_2}{\bar{Y}_2 + 1} \tag{18}$$

The dissipated powers in each branch are given below in terms of the network parameters and the branch currents

$$P_1 = \mathrm{Re}\left\{\frac{|I_1|^2}{\overline{Y}_2}\right\} \quad (19a)$$

$$P_2 = \mathrm{Re}\left\{\frac{|I_2|^2}{\overline{Y}_1}\right\} \quad (19b)$$

$$P_3 = \mathrm{Re}\left\{\frac{|I_3|^2}{\overline{Y}_2}\right\} \quad (19c)$$

Therefore the dissipated power in the slit is given as follows

$$P = P_1 + P_2 + P_3 \quad (20)$$

The dissipated power in the slit means the normalized radiated power $(1 - |\Gamma_H|^2 - |T_H|^2)$ in the slit.

III. Numerical Results and Discussion

The numerical calculations were carried out by the digital computer CYBER 170-815. The summation of the infinite series of equation (14) and (15) was truncated when the last term added was less than one-10^8 th of the first term.

Numerical results are shown in Fig.3 through 11. In Fig.3 and 4, the equivalent magnetic currents induced in the slit are illustrated respectively for the narrow slit width and the wide slit width. The narrow slit magnetic currents are, as expected, relatively uniform except near the edges. The wide slit magnetic currents decay rapidly for $z > -a$ and show the edge condition at $z = -a$ and $z = +a$.

Values of $|\Gamma_H|$ and $|T_H|$ obtained here are illustrated in Fig.5 and 6 respectively and compared with the theoretical results of Millar and the experimental determinations of Simmons [1] for the slit range of $2ka \leqslant 10$. Our results show good agreement with their results.

Load admittance Y_L (per unit width along y) for wide slit is presented and compared with the previous results[5] in Fig.7, where the conductance is observed to vary slowly as a function of b while the susceptance is seen to vary rapidly.

In case of $kb = 0.09738$, the reflection and transmission coefficients and the normalized radiated power are given in Fig.8 and the normalized admittances are illustrated in Fig.9 for the slit range of $0 < ka \leqslant 1.3$. An interesting observation made during the course of this investigation is that, as the slit width goes to zero, the valve of $\overline{G}_1$ is constant $(kb/2)$ and nearly

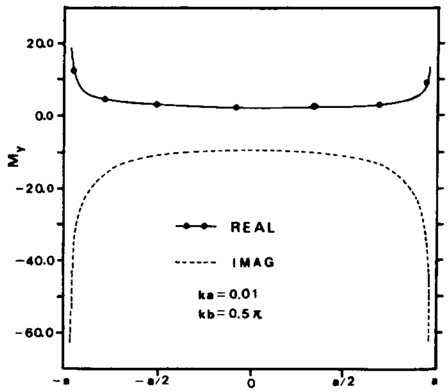

Fig.3. Magnetic current distribution in a narrow slit.

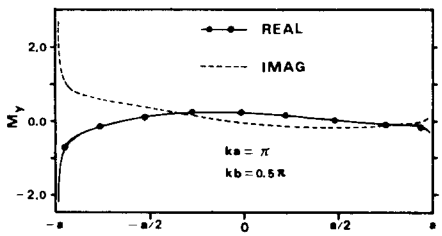

Fig.4. Magnetic current distribution in a wide slit.

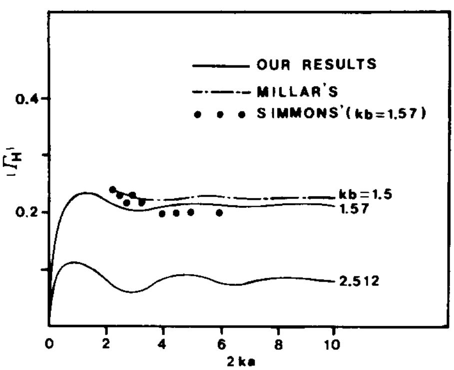

Fig.5. Reflection coefficient with slit width.

same to that of $\overline{G}_L$ for the range of $ka \geqslant 0.1$ while $\overline{Y}_2$ becomes negligible as shown in Fig.9 This is valuable in the characterization of the E-plane coupling phenomena between two microstrip rectangular patch antennas.

In Fig.9, one also observes that $\overline{Y}_L$ is nearly

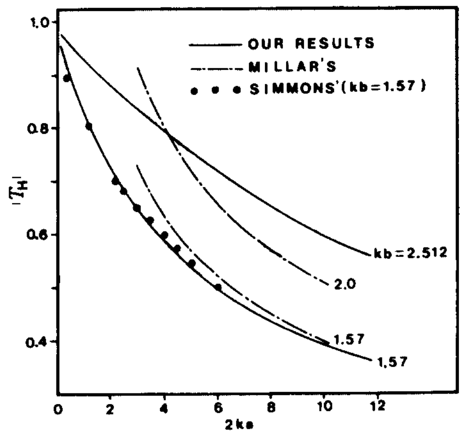

Fig.6. Transmission coefficient with slit width.

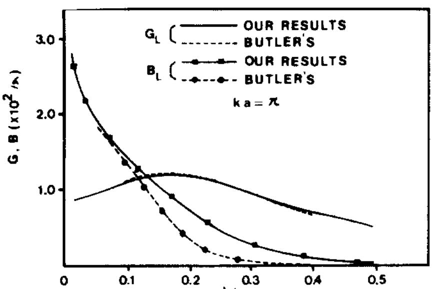

Fig.7. Load admittance $\overline{Y}_L$(per unit width along y).

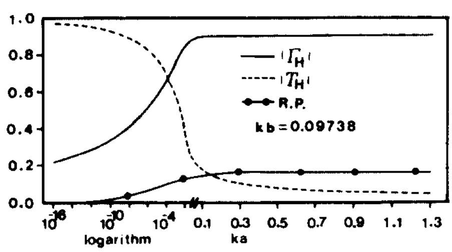

Fig.8. Reflection and transmission coefficients, and normalized radiated power with slit width.

constant in the range of $ka \geqslant 0.1$ The fact that the amplitude and the phase are seen to be nearly constant in the same range ($ka \geqslant 0.1$) in Fig.8 shows the reason for such behavior of $\overline{Y}_L$.

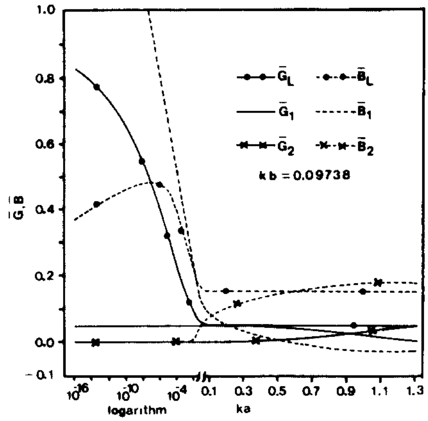

Fig.9. Normalized admittances with slit width.

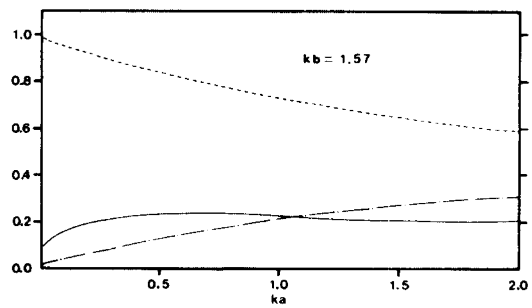

Fig.10. Reflection and transmission coefficients, and normalized radiated power with slit width. (— Γ_H , – – – T_H , —·—R.P.)

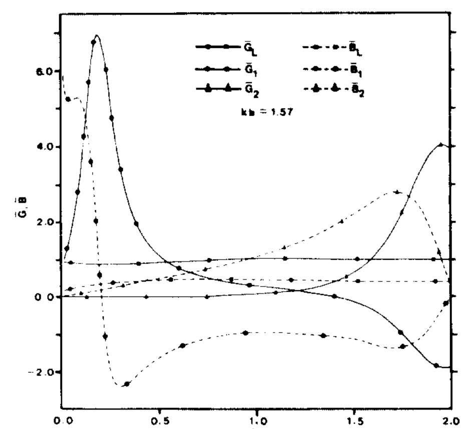

Fig.11. Normalized admittances with slit width.

In case of $kb = 1.57$, the reflection and transmission coefficients and the normalized radiated power are given in Fig.10, and the normalized admittances are illustrated in Fig.11 for the slit range of $0 < ka \leqslant 2$. In this case, the normalized admittances exhibit a similar behavior to the case of $kb = 0.09738$.

IV. Conclusion

Here an efficient computation technique in calculating the equivalent circuit parameters of the transverse slit in the wall of the parallel-plate waveguide is proposed. By use of the technique, equivalent π - network parameters are obtained both for the microstrip case (kb=0.09738) and for the waveguide case (kb=1.57).

Through this study some properties of the admittance parameter are pointed out.

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