# Phase Error Variation of Timing Recovery Circuit in Optical Communication

# (광통신에서 타이밍 복원 회로의 위상 오차 변화)

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## 要 約

가우시안 열 잡음, 패턴 잡음 그리고 산탄 잡음이 존재하며 또한 타이밍 복원 루우프가 완전히 조율되지 않았을 경우 광수신기의 광검파 전류에 의해 구동되는 PLL(phase-locked loop)의 성능을 분석하였다. 위상 오차의 분산은 회로의 형태와 생성된 잡음 모델에 따라 변화한다. 그리고 개선된 형태의 능동 비반전 1차의 루우프 여파기를 갖는 90.194Mbps의 광 시스템에 분석된 결과를 응용해 보았다.

## Abstract

It is analyzed how performance of phase-locked loop driven by photodetector current in optical receiver will be changed under the condition that Gaussian thermal noise, pattern noise and shot noise are present and the loop has the nonzero detuning frequency. The phase error variance changes with the circuit configuration and the produced noise models. The analyzed results are applied to the previously implemented 90.194Mbps optic system whose loop filter is the improved active noninverting 1-st order lag-lead type.

## I. Introduction

For conventional channels disturbed by additive Gaussian noise, there are satisfactory results in analyzing and testing the performance of a variety of synchronization circuits[1]. In optical communications the situation is different because the other noises -shot noise and pattern noise are accompanied [2-3]. These properties degrade the performances for the timing recovery circuit to estimate the transition times of the incoming data and to compensate for frequency drifts [4]. Without the loss of generality, the detuning frequency is nonzero so that the stochastic characteristic of each noise is more complex than the results of the U. Mengali. As the detuning frequency becomes larger, the variances of phase error due to each noises also increase. Especially, the increased variances owing to Gaussian thermal noise, pattern noise and shot noise are found to be 18.67[dB], 5.12[dB] and 27.91[dB] when the detuning frequency is 0.06% of the symbol rate.

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## II. First-Order Approximation

From the photodetector model used by S.D.

Personick the output current is expressed as follows [5].

$$i(t) = s(t) + n_g(t) + n_p(t) + n_s(t) \quad (1)$$

where s(t) is data signal,

$n_g(t)$ is zero-mean wide-sense stationary Gaussian noise,
$n_p(t)$ is pattern noise which is deviations from the timing wave produced by the data pattern,
$n_s(t)$ is shot noise which is time-varying owing to its impulsive character.

We shall look at s(t) as the synchronizing signal to be tracked by the PLL in the presence of such noises which have zero-mean and orthogonal. Since s(t) is the input of PLL timing recovery circuit, the timing wave s(t) can be expanded into Fourier series.

$$s(t) = \sum_k I_k \sin(kw_s t + \theta_k) \quad (2)$$

where $w_s = 2\pi/T$, T is symbol period.

The fundamental mode of s(t) has the following amplitude and phase.

$$I_1 = \frac{2G1C1}{T} \left| P_1(1/T) \right| \quad (3)$$

$$\theta_1 = \arg\{P_1(1/T)\} - 2\pi\tau/T + \pi/2 \quad (4)$$

where G1 = E { $g_k$ } is the expected value of avalanche gains,
C1 = E { $c_k$ } is the expected value of transmitted data,
$\tau$ is an unknown, constant timing parameter tracted by the PLL.

$$P_1(f) = P_o(f)\ W(f) \quad (5)$$

$P_o(f)$ is the Fourier Transform of the optical pulse shape and W(f) is the response of the photodetector circuitry to the generation of a single charge-carrier.

The equivalent model of PLL recovery circuit is shown in Fig.1.

$\Omega$ is loop-detuning, $\phi(t)$ is phase error of loop which is the difference between the instantaneous

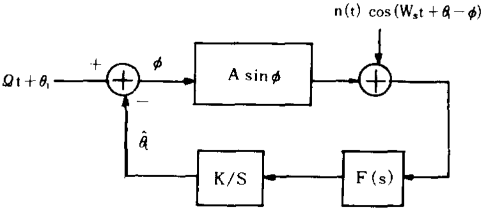


**Fig.1.** Equivalent model of PLL.

phase $w_s t+\theta_1$ and $w_o t+\hat{\theta}_1$ of the instantaneous phase of VCO, K is the DC gain of loop,

$A=I_1/2$, F(s) is the loop filter

$$n(t) = n_g(t) + n_p(t) + n_s(t) \quad (6)$$

$$\frac{d\phi}{dt} = \Omega - KF(p)\,[\,A\sin\phi + n(t)\cos(w_s t+\theta_1-\phi)\,] \quad (7)$$

where p=d/dt

The eq.(7) is the stochastic differential equation governing the model of Fig.1. Based upon the pertubation techniques and the Taylor series expansion [6], the eq.(7) can be represented as eqs.(8), (9) assuming that the phase error is not so much fluctuated.

$$\frac{d\phi_o}{dt} = \Omega - AKF(p)\sin\phi_o \quad (8)$$

$$\frac{d\phi_1}{dt} = -KF(p)\,[\,A\phi_1\cos\phi_o + n(t)\cos(w_s t+\theta_1-\phi_o)\,] \quad (9)$$

For simplicity, the original eq.(7) is the same as eq.(9) on the condition that $\phi(t)$ is much small, which is the case in general practice.
From eq.(8)

$$\frac{d}{dt}\phi_o(t) \cong \Omega - AKF(p)\,\phi_o(t) \quad (10)$$

In nonzero detuning frequency,

$$\Phi_o(s) = \frac{\Omega}{s(s+AKF(s))} \quad (11)$$

From eq.(9)

$$\phi_1(t) = h(t) \circledast [n(t)\cos(w_s t+\theta_1) + \phi_o(t)n(t)\sin(w_s t+\theta_1)] \quad (12)$$

where $h(t) = \mathcal{L}^{-1}[-KF(s)/s+AKF(s)]$ (12a)

$$\phi_o(t) = \mathcal{L}^{-1}[\Phi_o(s)] \quad (12b)$$

And overall noise n(t) is

$$n(t) = \sum_{x=g,p,s} n_x(t) = n_g(t) + n_p(t) + n_s(t) \quad (13)$$

## III. Noise Power Spectra

$$\phi(t) = h(t) \circledast q(t) \quad (14)$$

where $q(t) = n(t)\cos(w_s t+\theta_1) + \phi_o(t)n(t)\sin(w_s t+\theta_1)$

$$R_\phi(t_1, t_2) = E\{\phi(t_1)\,\phi(t_2)\} \quad (15)$$

Through the simple procedure, $R_\phi(t_1, t_2)$ is confirmed to be cyclostationary.

If the cutoff frequency of H(f) is much less than 1/2T, $\phi(t)$ is wide-sense stationary so that variance of phase error due to each noises and loop detuning can be depicted as eq. (16).

$$\sigma^2 = \sum_{x=g,p.s} \int |H(f)|^2\, \tilde{S}x(f)\, df \quad (16)$$

where $\tilde{S}x(f)$ is the spectral density of $\tilde{q}_x(t)$.

The cyclostationary process (17) $\tilde{q}(t)$ can be replaced by q(t+v), where v is uniformly distributed random variable [O,T]. By using the properties of the cyclostationary process,

$$q(t) = n(t)\cos(w_s t+\theta_1+\theta) + \phi_o(t)n(t)\sin(w_s t+\theta_1+\theta) \quad (17)$$

where $\theta$ is uniformly distributed $[0,2\pi]$.

$$R_{\tilde{q}}(t_1, t_2) = 1/2\, E\{n(t_1)\, n(t_2)\}\cos w_s(t_1-t_2) + 1/2\, R_{\phi_o}(t_1, t_2)\, E\{n(t_1)\, n(t_2)\}\cos w_s(t_1-t_2) \quad (18)$$

$$R_{n_p}(t_1, t_2) = G1^2\,(C2-C1^2) \sum_k p(t_1-kT-\tau)\, p(t_2-kT-\tau) \quad (19)$$

$$R_{n_s}(t_1, t_2) = G2 \int w(t_1-t)\, w(t_2-t)\, E\{z(t)\}\, dt \quad (20)$$

where $C2= E\{c_k^2\}$, $G2= E\{g_k^2\}$ z(t) is a filtered, doubly stochastic Poisson process.

Therefore, we can obtain the following power spectral densities of each noises.

$$\tilde{S}n_g(f) = 1/4\,[N(f-1/T)+N(f+1/T)] \circledast [\delta(f) + |\Phi_o(f)|^2] \quad (21)$$

$$\tilde{S}n_p(f) = G1^2\,(C2-C1^2)/4T\, [\,|P_1(f-1/T)|^2 + |P_1(f+1/T)|^2 - 2\mathrm{Re}\{P_1(f-1/T)P_1^*(f+1/T)\, e^{j\alpha}\} \circledast [\delta(f) + |\Phi_o(f)|^2] \quad (22)$$

$$\tilde{S}n_s(f) = G2/4\,[z_o+C1P_o(0)/T]\,[\,|W(f-1/T)|^2 + |W(f+1/T)|^2] - G2C1/2T \cdot \mathrm{Re}\,[P_o^*(2/T)W(f-1/T)W^*(f+1/T)e^{j\alpha}] \circledast [\delta(f) + |\Phi_o(f)|^2] \quad (23)$$

where $z_o$ is the factor considering dark current effect,

$\alpha$ equals $2\arg\{P_1(1/T)\}$,

N(f) is the power spectral density of Gaussin noise.

## IV. 90Mbps Fiber-Optic Recovery System

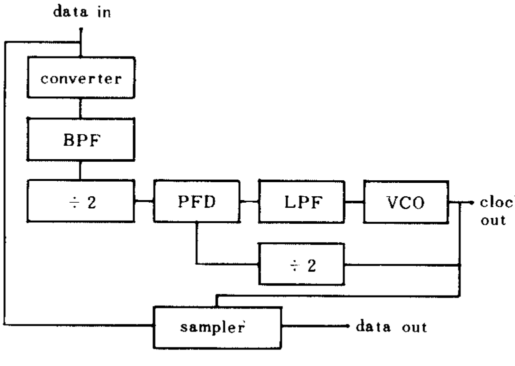


**Fig.2.** Clock recovery system.

In Fig.2 the clock recovery system is shown which adopts the PLL (phase-locked loop) method.

The clock signal is produced at the output of VCO and obtained more precisely by "divide and multiply (by 2) technique" PLL is made up of PFD (phase frequency detector), VCO (voltage controlled oscillator) and the LPF (low pass filter) which governs the dynamic charateristics of the loop. The other part of the system in Fig.2 is the data retiming circuit.

MECL12040 has been used for PFD (phase frequency detector) with transfer gain of 0.1194 [v/rad], ECL line receiver MC10216 has been practiced in VCO (voltage controlled oscillator) with the average sensitivity of 62800 [rad/v] and the 1-st order active noninverting lead-lag loop filter is shown in Fig.3.

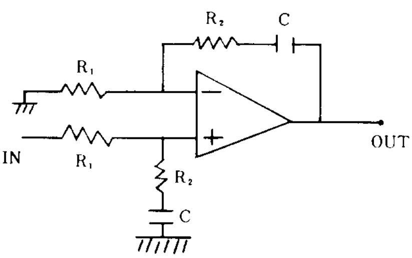


**Fig.3.** Loop filter circuit.

At this time the transfer function of loop filter is

$$F(s) = \frac{1 + sCR2}{sCR1} \tag{24}$$

In order to improve the transient response characteristics, we place the RC LPF in front of loop filter but the global closed loop transfer function is negligibly affected. The improved loop filter is in Fig.4.

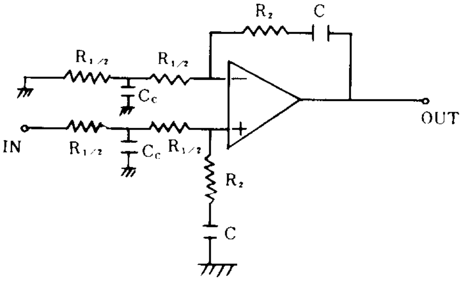


**Fig.4.** Improved loop filter circuit.

But if the cutoff frequency $w_c$ is close to the natural frequency $w_n$, pole is expected to be added and produce even more oversoot. Therefore, to avoid this damage, $w_c$ must be at least 10 times more than $w_n$.

$$w_c = 4/(R1Cc) \tag{25}$$

$$w_c = 10w_n \tag{26}$$

We have chosen the damping factor of 0.8 with 1.5 % overshoot and natural frequency of 3653.2 [rad/sec]. To recover the better timing signal, we have used frequency divider circuit before the recovery circuit and the frequency multiplication has made in PLL circuit. As the factor of 2.0 is taken, the practical transfer gain of PFD, VCO will be considered to be half times of original values.

## V. Simulation and Results

The variances of phase error due to the each noises are shown in Figs, 5,6 and 7. The magnitudes of variances increase with the detuning frequency.

## VI. Conclusion

We have derived the phase error variance from the Gaussian, pattern, shot noises through the first-order approximation on the pertubation techniques and the Taylor series in case of nonzero detuning loop.

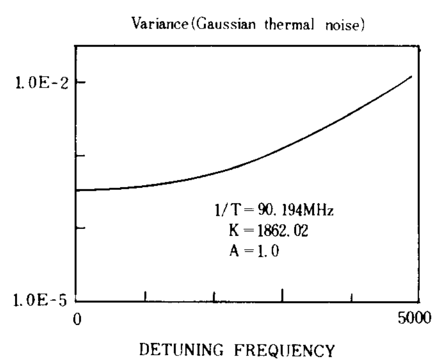


**Fig.5.** Variance due to Gaussian noise.

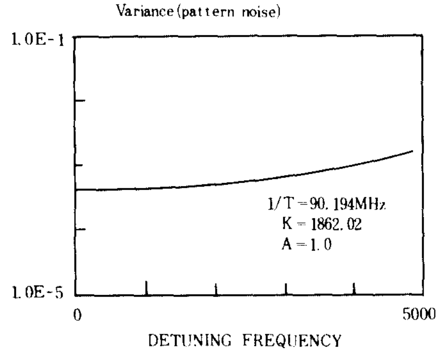


**Fig.6.** Variance due to pattern noise.

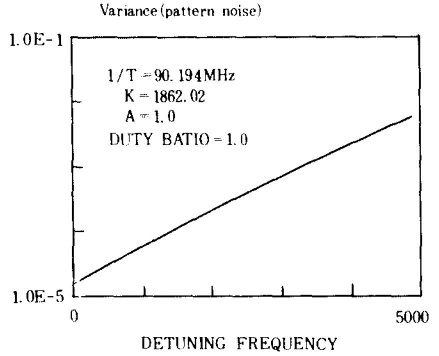


**Fig.7.** Variance due to shot noise.

The poreviously designed and implemented 90.194Mbps (1344 voice channels) fiber-optic system has been adopted in this analysis and through the computer simulation its performance variation can be obtained. The augmented variances due to Gaussian, pattern, shot noise are 18.67 [dB], 5.12[dB] and 27.91[dB] in case 0.06% detuning frequency of symbol rate is compared to zero detuning. The further research will be remained to minimize the phase error variance and to optimize the retiming clock systems.

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