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An Application of Linear Singular System Theory To Electric Circuits

(선형 Singular 시스템 이론의 전기 회로에의 적용)

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要 約

본 논문은 선형 singular 시스템, 기하학적 구조, 그리고 feedback의 개념을 소개할 뿐 아니라, 어떤 전자 기기에 쓰일지 모르는 전기 회로에 다변수 선형 singular 시스템 이론이 적용될 수 있음을 보인다. 바람직하지 못한 충격적인 혹은 불연속적인 동작을 허용초기 조건 set에 의해 없앨 수가 있다. 출력제거 supremal (A, E, B) invariant subspace와 singular 시스템 구조 알고리듬이 이 2입력 2출력 전기회로에 적용되었다.

Pencil(SE-A)의 Weierstrass형식이 출력제거 supremal(A, E, B) invariant subspace와 관련되어 서술되었고, 이로부터 유한 subsystem과 무한 subsystem의 시간영역에서의 해(解)가 구해졌다. 이 feedback을 가진 적용문제를 위해 일반화된 Lyapunov의 식이 연구되었고, singular 시스템에서의 직교함수들의 사용이 논의되었다.

Abstract

This paper aims not only to introduce the concept of linear singular systems, geometric structure, and feedback but also to provide applications of the multivariable linear singular system theories to electric circuits which may appear in some electronic equipments. The impulsive or discontinuous behavior which is not desirable can be removed by the set of admissible initial conditions. The output-nulling supremal (A,E,B) invariant subspace and the singular system structure algorithm are applied to this double-input double-output electric circuit.

The Weierstrass form of the pencil (sE-A) is related to the output-nulling supremal (A,E,B) invariant subspace from which the time domain solutions of the finite and the infinite subsystems are found. The generalized Lyapunov equation for this application with feedback is studied and finally, the use of orthogonal functions in singular systems is discussed.

I. Introduction

Consider a differential equation of the form in eq (1).

$$f(x, \dot{x}, u, t) = 0 \tag{1}$$

$$df = \frac{\partial f}{\partial \dot{x}} d\dot{x} + \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial u} du \tag{2}$$

Let $E = \frac{\partial f}{\partial \dot{x}}$, $A = -\frac{\partial f}{\partial x}$, $B = -\frac{\partial f}{\partial u}$, then we can write

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$$E d\dot{x} = A dx + B du + \left(df - \frac{\partial f}{\partial t}\ dt\right) \qquad (3)$$

As $df \cong \dfrac{\partial f}{\partial t}\ dt$, a linear time invariant approximation of eq (1) with a control input u(t) can be described as eq (4).

$$E\dot{x} = Ax + Bu \qquad (4a)$$

$$y = Cx + Du \qquad (4b)$$

where $x(t) \in R^n$, $u(t) \in R^m$, $y(t) \in R^p$ and $x(o)=X_o$.

If $|E|=0$, we call eq (4) a linear singular system. In the frequency domain, eq (4a) is

$$(sE-A)X(s) = Ex_0 + BU(s) \qquad (5)$$

For existence of a solution x (t) for all u(t) when $x_0=0$, conditions of (6), (7) are necessary and sufficient.

$$\mathbf{R}(sE-A) \supset \mathbf{R}(B) \quad \text{a. e., or} \qquad (6)$$

$$\text{rank}[sE-A \quad B] = \text{rank}[sE-A] \quad \text{a. e} \qquad (7)$$

where R(A) denotes the range of A and N(A) represents the null space of A.

If the pencil (sE-A) is regular, i.e., if $\Delta(s) = |sE-A| \neq 0$, then the system eq (4a) is solvable. For the uniqueness of a solution to Eq (4), necessary and sufficient conditions are as follows;

$$\mathbf{N}(sE-A) \supset \mathbf{N}(C) \quad \text{a. e., or} \qquad (8)$$

$$\text{rank}\begin{bmatrix} sE-A \\ C \end{bmatrix} = \text{rank}[sE-A] \quad \text{s. e} \qquad (9)$$

The roots of $\Delta(s)$ are called finite relative eigenvalues of (E,A) while infinite zeros of (sE-A) are infinite relative eigenvalues of (E,A). The finite spectrum of (E,A) is denoted by σ_f (E,A) and the infinite spectrum of (E,A) is σ_∞ (E,A) and the relative spectrum of (E,A) is $\sigma(E,A) = \sigma_f(E,A) \cup \sigma_\infty(E,A)$.

The output-nulling (ON) (A,E,**B**) invariant subspace for the linear singular system in eq (4) satisfies

$$\begin{bmatrix} A \\ C \end{bmatrix} S \subset \begin{bmatrix} E \\ O \end{bmatrix} S + \begin{bmatrix} B \\ D \end{bmatrix} \qquad (10)$$

The supremal ON (A,E,**B**) invariant subspace L* can be computed recursively as follows;

$$X_{k+1} = \begin{bmatrix} A \\ C \end{bmatrix}^{-1} \left(\begin{bmatrix} E \\ O \end{bmatrix} X_k + \begin{bmatrix} B \\ D \end{bmatrix} \right) \qquad (11)$$

with $x_o = R^n$, then $L^* = x_\infty$ where ∞ is the first k such that $x_k = x_{k+1}$.

The supremal (A,E,**B**) invariant subspace contained in **k** which satisfies (10) with N(C)=**K** and D=0 is defined as

$$V^* = \sup\{S \subset \mathbf{K} \mid AS \subset ES + \mathbf{B}\} \qquad (12)$$

and V* can be found in the recursion

$$X_{k+1} = \tilde{\mathbf{K}} \cap A^{-1}(EX_k + \tilde{\mathbf{B}}), \quad X_0 = \mathbf{R}^n \qquad (13)$$

V* is used for finding the reachable and controllable subspaces for the system (4), and also for solving the disturbance decoupling problem for linear singular systems.

II. Mathematical Modeling for an Electric Circuit

Considering a double-input double-output circuit in Fig.1, we can describe this transistor circuit by the equivalent circuit of Fig.2. Here, u_1 and u_2 are system inputs, y_1 and y_2 are system outputs, and state-space variables are chosen as follows:

$$\begin{aligned} x_1 &= v_{c_1} \\ x_2 &= i_1 \\ x_3 &= v_{c_2} \\ x_4 &= i_2 \end{aligned} \qquad (14)$$

Furthermore, there are four state equations and two output equations which are

$$u_1 + v_{c_1} + r_1 i_1 = 0 \quad i_1 = c_1 \dot{v}_{c_1} \qquad (15)$$

$$u_2 + v_{c_2} = y_1 = r_1(a_1 i_1 - i_2) \quad i_2 = c_2 \dot{v}_{c_2}$$

By using eq's (14), eq's (15) can be described as a state-space representation.

$$\begin{bmatrix} c_1 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & r_1 & 0 & 0 \\ 0 & -r_2 a_2 & 1 & r_2 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u \qquad (16a)$$

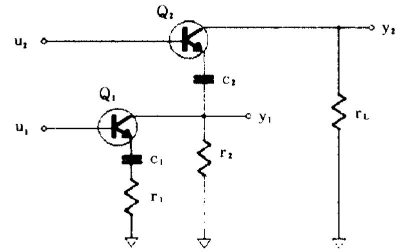

Fig.1. A double-input double-output transistor circuits.

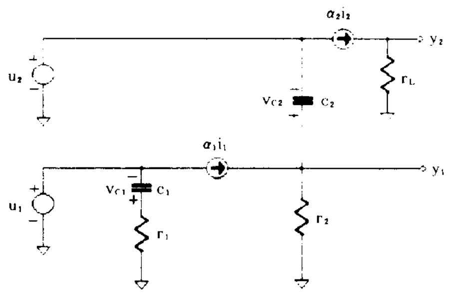

Fig.2. Equivalent circuit of Fig.1.

$$y=\begin{bmatrix} 0 & r_2a_2 & 0 & -r_2 \\ 0 & 0 & 0 & r_L\alpha_2 \end{bmatrix} x \tag{16b}$$

where $x=[x_1x_2x_3x_4]^T$, $u=[u_1u_2]^T$, and $y=[y_1y_2]^T$

Therefore, the system in eq (16) is singular since $|E|=0$ and if we assume that

$c_1=c_2=1[F]$, $r_1=r_2=r_L=1[\Omega]$, $\alpha_1=\alpha_2==1[A/A]$

then this results in a linear singular system in eq (17)

$$\Sigma: E\dot{x}=Ax+Bu \quad x(0)=x_0 \tag{17a}$$

$$y=Cx \tag{17b}$$

where $E=\begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \end{bmatrix}$ $A=\begin{bmatrix} 0&1&0&0 \\ 0&0&0&1 \\ 1&1&0&0 \\ 0&-1&1&1 \end{bmatrix}$ $B=\begin{bmatrix} 0&0 \\ 0&0 \\ 1&0 \\ 0&1 \end{bmatrix}$

and $C=\begin{bmatrix} 0&1&0&-1 \\ 0&0&0&1 \end{bmatrix}$

The pencil (sE-A) is

$$sE-A=\begin{bmatrix} s&-1&0&0 \\ 0&s&0&-1 \\ -1&-1&0&0 \\ 0&1&-1&-1 \end{bmatrix}$$

and $\Delta(s)=|sE-A|=-s-1$

A Test for regularity is called shuffle algorithm: For k=1..n do

$$T_k\begin{bmatrix} E_k & A_k \\ \underline{A}_k & 0 \end{bmatrix}=\begin{bmatrix} E_{k+1} & A_{k+1} \\ 0 & \underline{A}_{k+1} \end{bmatrix} \tag{18}$$

with E_{k+1} full row rank, T_k nonsingular row compression, and $E_0=E$, $A_0=A$, $\underline{A}_0=0$.
Then the pancil is regular iff $|E_n|\neq 0$.
In the above example, we get

$$T_2T_1T_0\left[\begin{array}{cccc:cccc} 1&0&0&0&0&1&0&0 \\ 0&1&0&0&0&0&0&1 \\ 0&0&0&0&1&1&0&0 \\ 0&0&0&0&0&-1&1&1 \\ \hdashline 0&0&0&0&0&0&0&0 \end{array}\right]=\left[\begin{array}{cccc:cccc} 1&0&0&0&0&1&0&0 \\ 0&1&0&0&0&0&0&1 \\ 0&0&1&2&0&0&0&0 \\ 0&0&0&-1&0&0&0&1 \end{array}\right]$$

Thus, (sE-A) is regular since $|E_2|\neq 0$, and E_2 has full row rank.

Ⅲ. Weierstrass Form of the pencil (sE-A)

Now, let us look at the relative eigenstructure of (E,A). As we know $\Delta(\lambda)=-\lambda-1$, $\lambda_1=-1$, $\eta_1=\dim N(\lambda_1 E-A)=1$, and a rank 1 finite relative eigenvector is obtained by eq (19),

$$(\lambda_1 E-A)v^1_{1j}=0 \quad \text{for} \quad 1\leq j\leq \eta_1 \tag{19}$$

and rank (k + 1) finite relative eigenvectors are given by

$$(\lambda_1 E-A)v^{k+1}_{1j}=-Ev^k_{1j} \quad \text{for} \ k\geq 1 \tag{20}$$

Since $|E|=0$, there exist infinite relative eigenvalues and if we define $\eta=\dim N(E)$ then the rank 1 infinite relative eigenvectors are given by

$$Ev^1_{\infty j}=0 \quad \text{for} \ 1\leq j\leq\eta \quad \eta=2 \ \text{in this example} \tag{21}$$

also, the rank (k + 1) infinite relative eigenvestors are derived by

$$Ev_{\infty j}^{k+1} = Av_{\infty j}^{k} \quad \text{for } k \geq 1 \tag{22}$$

In this multivariable system case, we have one rank 1 finite eigenvector and two rank 1 infinite and one rank 2 infinite eigenvectors.

So, in order to get the Weierstrass form in eq (23), e.g.,

$$W^{-1}(sE-A)V = \begin{bmatrix} sI-J & 0 \\ 0 & sN-I \end{bmatrix} \tag{23}$$

first of all, V and W matrices have to be obtained as

$$V = \begin{bmatrix} 1 & : & 0 & 0 & 0 \\ -1 & : & 0 & -1 & 0 \\ -2 & : & 1 & 0 & 0 \\ 1 & : & -1 & 0 & 1 \end{bmatrix} \quad W = \begin{bmatrix} 1 & : & 0 & -1 & 0 \\ -1 & : & -1 & 0 & 1 \\ 0 & : & 0 & -1 & 0 \\ 0 & : & 0 & 1 & 1 \end{bmatrix}$$

since $v_{11}^{1} = [1 \ -1 \ -2 \ 1]^T$

$v_{\infty 1}^{1} = [0 \ 0 \ 1 \ -1]^T$ and $v_{\infty 1}^{2} = [0 \ -1 \ 0 \ 0]^T$

$v_{\infty 2}^{1} = [0 \ 0 \ 0 \ 1]^T$

Thus, $W^{-1} \begin{bmatrix} 1 & 0 & -1 & 0 \\ -1 & -1 & 2 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ and from eq (23), the Weierstrass form is obtained.

$$\therefore W^{-1}(sE-A)V = \begin{bmatrix} s+1 & : & 0 & 0 & 0 \\ \hdashline 0 & : & -1 & s & 0 \\ 0 & : & 0 & -1 & 0 \\ 0 & : & 0 & 0 & -1 \end{bmatrix}$$

where $J = [-1]$, and $N = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$,

and $N^{\alpha} = 0$ with $\alpha = 2$.

By performing transformation, $X'(s) = V^{-1} X(s)$ and premultiplying W^{-1} in the Laplace domain of eq (24), we can separate the system into two subsystems.

$$(sE-A)X(s) = Ex_0 + BU(s) \tag{24}$$

$$W^{-1}(sE-A)VX'(s) = W^{-1}BU(s) + W^{-1}x_0 \tag{25}$$

Therefore, from eq (25)

$$\begin{bmatrix} sI-J & : & \\ \hline & : & sN-1 \\ & : & \end{bmatrix} X'(s) = \begin{bmatrix} B_1 \\ \hline B_2 \end{bmatrix} U(s) + W^{-1}Vx_0' \tag{26}$$

In the time domain, eq (26) has two subsystems, the finite (slow) subsystem $\underline{\Sigma}^{f}$ and $\underline{\Sigma}^{\infty}$ the infinite (fast) subsystem.

$$\underline{\Sigma}^{f} : \dot{x}_1 = Jx_1 + B_1 u \tag{27a}$$

$$\underline{\Sigma}^{\infty} : N\dot{x}_2 = x_2 + B_2 u \tag{27b}$$

where $x_1 \in R^{n}$, $x_2 \in R^{n2}$ and $n_1 = \deg |SE\text{-}A|$.

Then the solution of eq (27) becomes simple as in eq (28).

$$x_1(t) = e^{Jt} x_1(O) + \int_0^t e^{J(t-\tau)} B_1 u(\tau) d\tau \tag{28a}$$

$$x_2(t) = -\sum_{i=1}^{\alpha-1} \delta^{(i-1)} N^i x_2(O) - \sum_{i=0}^{\alpha-1} N^i B_2 u^{(i)}(t) \tag{28b}$$

In the above example, J, N, B_1 and B_2 are

$$J = [-1] \qquad E_1 = [-1 \ 0]$$

$$N = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad B_2 = \begin{bmatrix} 2 & 1 \\ -1 & 0 \\ 1 & 1 \end{bmatrix}$$

and $x^1(t) = V^{-1}x(t) = [x_1^T \ x_T^2]^T$.

The solution is as follows:

$$x_1(t) = e^{-t}\{x_1(0) - \int_0^t e^{\tau} u_1(\tau) d\tau\}$$

$$x_2(t) = -\delta(t) \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_2(0) - \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ -1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

When $u(t)=0$, impulsive behavior cannot appear from initial conditions $x_2(0)$ if $x_2(0) \in \mathbf{N}(E)$, and in the case $u(t) \neq 0$, we can choose $u(t)$ to eliminate the discontinuity.

$$H_{1u} = R^{n_1} \oplus \mathbf{R}[B_2 \ NB_2]$$

The set of $x(0)$ which does not show any impulsive behavior is $H_I + N(E)$.

Here $N(E) = \mathbf{R}[e_3\, e_4]$ and $H_I = \mathbf{R}[1\,\text{-}1\,\text{-}\text{-}1\,\text{-}1]^T$.

IV. Singular System Structure Algorithm

The singular system structure algorithm can be used in the optimal control problem and it generalizes Silverman's structure algorithm and Luenberger's shuffle algorithm. This generalized algorithm relates L^* in eq (11).

step (i) Set $k=0;\ E_0=E;\ A_0=A;\ B_0=B;\ C_0=C;$

$$D_0=D;\ \underline{C}_0=0$$

step (ii) Find constant unitary transformations T_k and S_k such that

$$T_k\left[\begin{array}{ccc} n & n & m \\ E_k & A_k & B_k \\ \underline{C}_k & 0 & 0 \end{array}\right]\begin{array}{c} \\ r_k \\ t_k \end{array} = \left[\begin{array}{ccc} n & n & m \\ E_{k+1} & A_{k+1} & B_{k+1} \\ 0 & \underline{A}_k & \underline{B}_k \end{array}\right]\begin{array}{c} \\ r_{k+1} \\ \\ \end{array} \tag{29a}$$

$$S_k\left[\begin{array}{cc} n & m \\ C_k & D_k \\ \underline{A}_k & \underline{B}_k \end{array}\right]\begin{array}{c} \\ s_k \\ \\ \end{array} = \left[\begin{array}{cc} n & m \\ C_{k+1} & D_{k+1} \\ \underline{C}_{k+1} & 0 \end{array}\right]\begin{array}{c} \\ s_{k+1} \\ t_{k+1} \end{array} \tag{29b}$$

with E_{k+1}, D_{k+1} having full row rank r_{k+1}, s_{k+1} resp.

step (iii) If $t_{k+1} = 0$ or $\underline{c}_{k+1} = 0$ then go to step (iv), else set k=k+1 and go to step (ii).

step (iv) Define L=k+1. End.

In the circuit example in Fig.1, the singular system structure is applied as follows:

step (i) k=0; same as above.

step (ii)

$$k=0:\quad T_0\begin{array}{l} \quad\ \ 4 \qquad\qquad\ \ 4 \qquad\quad 2 \\ \left[\begin{array}{cccc:cccc:cc} 1&0&0&0&0&1&0&0&0&0\\ 0&1&0&0&0&0&0&1&0&0\\ 0&0&0&0&1&1&0&0&1&0\\ 0&0&0&0&0&-1&1&1&0&1\\ \hdashline 0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0 \end{array}\right] \end{array} =$$

$$\begin{array}{l} \quad\ \ 4 \qquad\qquad\ \ 4 \qquad\quad 2 \\ \left[\begin{array}{cccc:cccc:cc} 1&0&0&0&0&1&0&0&0&0\\ 0&1&0&0&0&0&0&1&0&0\\ \hdashline 0&0&0&0&1&1&0&0&1&0\\ 0&0&0&0&0&-1&1&1&0&1 \end{array}\right]\begin{matrix}2\\ \\ \\ \\ \end{matrix} \end{array}$$

$$S_0\begin{array}{l} \qquad\quad 4 \qquad\quad 2 \\ \left[\begin{array}{cccc:cc} 0&1&0&-1&0&0\\ 0&0&0&1&0&0\\ \hdashline 1&1&0&0&1&0\\ 0&-1&1&1&0&1 \end{array}\right]\begin{matrix}2\\ \\ \\ \\ \end{matrix} \end{array} =$$

$$\begin{array}{l} \qquad\quad 4 \qquad\quad 2 \\ \left[\begin{array}{cccc:cc} 1&1&0&0&1&0\\ 0&-1&1&1&0&1\\ \hdashline 0&1&0&-1&0&0\\ 0&0&0&1&0&0 \end{array}\right]\begin{matrix}2\\ \\ \\ 2\end{matrix} \end{array}$$

$$k=1:\quad T_1\left[\begin{array}{cccc:cccc:cc} 1&0&0&0&0&1&0&0&0&0\\ 0&1&0&0&0&0&0&1&0&0\\ \hdashline 0&1&0&1&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0 \end{array}\right]\begin{matrix}2\\ \\ 2\\ \\ \end{matrix} =$$

$$\left[\begin{array}{cccc:cccc:cc} 1&0&0&0&0&1&0&0&0&0\\ 0&1&0&0&0&0&0&1&0&0\\ 0&0&0&1&0&0&0&0&0&0\\ \hdashline 0&0&0&0&0&0&0&-1&0&0 \end{array}\right]\begin{matrix}3\\ \\ \\ \\ \end{matrix}$$

$$S_1\left[\begin{array}{cccc:cc} 1&1&0&0&1&0\\ 0&-1&1&1&0&1\\ \hdashline 0&0&0&-1&0&0 \end{array}\right]\begin{matrix}2\\ \\ \\ \end{matrix} =$$

$$\left[\begin{array}{cccc:cc} 1&1&0&0&1&0\\ 0&-1&1&1&0&1\\ \hdashline 0&0&0&-1&0&0 \end{array}\right]\begin{matrix}2\\ \\ 1\end{matrix}$$

$$k=2:\ T_2\left[\begin{array}{cccc:cccc:cc} 1&0&0&0&0&1&0&0&0&0\\ 0&1&0&0&0&0&0&1&0&0\\ \hdashline 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0 \end{array}\right]\begin{matrix}2\\ \\ 2\\ \\ \end{matrix} =$$

$$\left[\begin{array}{cccc:cccc:cc} 1&0&0&0&0&1&0&0&0&0\\ 0&1&0&0&0&0&0&1&0&0\\ 0&0&0&1&0&0&0&0&0&0\\ \hdashline 0&0&0&0&0&0&0&0&0&0 \end{array}\right]\begin{matrix}3\\ \\ \\ \\ \end{matrix}$$

$$S_2\left[\begin{array}{cccc:cc} 0&1&0&0&1&0\\ 1&-1&1&1&0&1\\ \hdashline 0&0&0&0&0&0 \end{array}\right]\begin{matrix}2\\ \\ \\ \end{matrix} =$$

$$\left[\begin{array}{cccc:cc} 1&1&0&0&1&0\\ 0&-1&1&1&0&1\\ \hdashline 0&0&0&0&0&0 \end{array}\right]\begin{matrix}2\\ \\ 1\end{matrix}$$

since $t_{k+1} = t_3 = 0$ or $\underline{c}_3 = 0$, therefore $L = k+1 = 3$. The relationship between singular system structure algorithm and recursion in eq (11) is

$$X_k = \mathbf{N}\begin{bmatrix} \underline{C}_0 \\ \underline{C}^1 \\ \vdots \\ \underline{C}_k \end{bmatrix} = \bigcap_{i=0}^{k} \mathbf{N}(\underline{C}_i) \tag{30}$$

if $i \geq L$ then $\underline{C}_L = 0$.

In our example, $\underline{C}_0 = 0$, $\underline{C}_1 = \begin{bmatrix} 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

$\underline{C}_2 = [0\ 0\ 0\ -1]$, and $\underline{C}_3 = 0$, therefore,

$$X_3 = \bigcap_{i=1}^{3} \mathbf{N}(\underline{C}_i) = \mathbf{N}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix} = \mathbf{R}[e_1\ e_3]$$

Now, let's compare with recursion in eq (11).

$X_0 = R^4$ and $EX_0 = \mathbf{R}[e_1\ e_2]$, $EX_0 + \mathbf{B} = R^4$

$(EX_0 + \mathbf{B})^{\perp} A = 0$

$$X_1 = \mathbf{N}\begin{bmatrix} C \\ (EX_0 + B)^{\perp} A \end{bmatrix} = \mathbf{N}\begin{bmatrix} 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$= \mathbf{R}[e_1\ e_3]$

$EX_1 = \mathbf{R}[e_1$, $EX_1 + \mathbf{B} = \mathbf{R}[e_1 : e_3\ e_4]$

$(EX_1 + \mathbf{B})^{\perp} A = [0\ 1\ 0\ 0]\ \ A = [0\ 0\ 0\ 1]$

$$X_2 = \mathbf{N}\begin{bmatrix} C \\ (EX_0 + \mathbf{B})^{\perp} A \end{bmatrix} = \mathbf{N}\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ \hdashline 0 & 0 & 0 & 1 \end{bmatrix}$$

$\mathbf{R}[e_1\ e_3] = X_1$, stop

$\therefore L^* = X_1 = \mathbf{R}[e_1\ \ e_3] = \text{ON}\ \sup(A, E, \mathbf{B})$ invariant subspace

$AS \subset ES + \mathbf{B}$ and $CS \subset 0$ are satisfied since

$AS = \mathbf{R}[e_3\ \ e_4] \subset \mathbf{R}[e_1\ e_3\ e_4] = ES + \mathbf{B}$ and $CS = 0$.

V. Generalized Lyapunov Equation and Feedback

Consider the generalized Lyapunov equation in eq (31).

$$\begin{bmatrix} A \\ C \end{bmatrix} S = \begin{bmatrix} E \\ O \end{bmatrix} SF - \begin{bmatrix} B \\ D \end{bmatrix} G \tag{31}$$

After suitable manipulations, we can obtain eq (32).

$$(sE - A) S (sI - F)^{-1} = ES + BG (sI - F)^{-1} \tag{32a}$$

$$0 = CS (sI - F)^{-1} + DG (sI - F)^{-1} \tag{32b}$$

The Laplace transform of eq (4) becomes

$$(sE - A) X(s) = Ex(\underset{\sim}{0}) + BU(s) \tag{33a}$$

$$Y(s) = CX(s) + DU(s) \tag{33b}$$

Let's define a feedback $u(t) = Kx(t)$ (34)

which is applied to the system (4) then

$$E\dot{x}(t) = (A + BK)\, x(t) \tag{35a}$$

$$y(t) = (C + DK)\, x(t) \tag{35b}$$

There exists a K such that $K = GS^{+}$ with $S^{+}S = I$ (36)

and $(A+\underset{\sim}{E}K)\, S \subset ES$, $(C+DK)S = 0$ if and only if (37)

S satisfies (10), i.e., ON (A,E,**B**) inveriant subspace.

A feedback K satisfying (37) is called on ON friend of S, and $N(E) \cap S = 0$ should hold in order to guarantee regularity of $[sE - (A+BK)]$.

From eq (31), there is a unique solution to F given S and G

$$ESF = AS + BG \tag{38}$$

if and only if ES has full column rank, or $\mathbf{N}(E) \cap \mathbf{R}(S) = 0$.

Thus, an ON friend of S can be found as following:

$S = \mathbf{R}[e_1\ \ e_3]$ and $B = [e_3\ \ e_4]$

$$[ES\ -\ B_1]\begin{bmatrix} F \\ G_1 \end{bmatrix} = AS + B_2 G_2$$

In the electric circuit example, since $ES \cap \mathbf{B} = \phi$,

$$[ES - B]\begin{bmatrix} F \\ G \end{bmatrix} = AS \text{ with ES full column rank}$$

$$\begin{bmatrix} 1 & : & 0 & 0 \\ 0 & : & 0 & 0 \\ 0 & : & -1 & 0 \\ 0 & : & 0 & -1 \end{bmatrix}\begin{bmatrix} F \\ G \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

But, ES has a rank of 1. $\therefore$No spectrum assignability

$$\begin{bmatrix} F \\ G \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\therefore K = GS^{+} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

In general the gain $K = \begin{bmatrix} -1 & k_2 & 0 & k_4 \\ 0 & k_6 & -1 & k_8 \end{bmatrix}$

Check $(A+BK)\ S \subset ES$ and $CS=0$ are satisfied.

VI. Reachability and Controllability Subspaces

The reachability and controllability subspaces can be obtained by the following subspace recursion:

$$v_{k+1}(\mathbf{K}) = \mathbf{K} \cap A^{-1}(EV_k + \mathbf{B}) \tag{39}$$

$$W_{k+1}(\mathbf{K}) = \mathbf{K} \cap E^{-1}(AW_k + \mathbf{B}) \tag{40}$$

with $v_0 = \mathbf{K}$ and $w_0 = \mathbf{K}$

The reachability subspace is computed by

$$\bar{\mathbf{R}} = V_n(W_n(R^n)) \tag{41}$$

and the controllability subspace can be found by

$$\bar{C} = \bar{\mathbf{R}} + \mathbf{N}(E) \tag{42}$$

In the example, $\bar{\mathbf{R}} = R^4$ and $\bar{\mathbf{C}} = R^4$.
But, for the ON supremal reachability and controllability subspaces we can obtain $\bar{\mathbf{R}} = \mathbf{R}\ (e_3)$ and $\bar{\mathbf{C}} = \mathbf{R}\ (e_3\ e_4)$.
By subspace recursion algorithm, it is not necessary to convert the singular system to Weierstrass form.

VII. Use of Orthogonal Functions in Singular Systems

Considering the singular system of the form (4), given u(t) and x(0), the solution x(t) can be found alternatively and this method is based on approximation of x(t) by a truncated orthogonal series which forms the basis functions.
Let ϕ_0 (t), ϕ_1(t),... ϕ_{r-1} (t) be the basis functions that are orthogonal on the sampling interval and F is a constant matrix in R^{nxr}, then

$$x(t) = F\Phi(t) \quad \Phi(t) = [\phi_0(t)\ \phi_1(t) \cdots \phi_{r-1}(t)]^T \tag{43}$$

The examples of the orthogonal series are Walsh, block-pulse, Laguerre, Chebychev and Hermite. Some basis functions have the integral property of approximation such as

$$\int_\alpha^t \Phi(\sigma)\, d\sigma \simeq P\,\Phi(t) \quad t \le \beta \quad \text{on}\,[\alpha, \beta] \tag{44}$$

Integrating eq (4a) on $[\alpha,t]$ eq (45) is obtained

$$Ex(t) - Ex(0) = A\int_\alpha^t x(\sigma)\, d\sigma + B\int_\alpha^t u(\sigma)\, d\sigma \tag{45}$$

Let u(t) be approximated by another matrix H

$$u(t) = H\,\Phi(t) \tag{46}$$

Then, combining (43), (44), (46) into (45), we get

$$EF\,\Phi(t) - Ex(0) = AFP\,\Phi(t) + BHP\,\Phi(t) \tag{47}$$

By assuming ϕ_0 (t)=1, the term Ex(0) can be formulated as

$$Ex(0) = EQ\,\Phi(t) \quad \text{where} \quad Q = [x(0)\ \ 0 \cdots 0] \in R^{n \times r} \tag{48}$$

Substituting (48) into (47), generalized Lyapunov equation is obtained such as

$$EF - EQ = AFP + BHP \tag{49}$$

Here, F is solved to get an approximated version of x(t) and moreover, there is a more convenient form by the use of the Kronecker product in eq (50)

$$Mf = d \quad \text{where} \quad f, d \in R^r \tag{50}$$

f and d are the i-th column of F and D, respectively. M is given by

$$M = A \otimes P^T - E \otimes I^T \in R^{(nr) \times (nr)} \tag{51}$$

where $\otimes$ denotes the Kronecker product.
The Kronecker product is defined as follows:

$$A \otimes P^T = \begin{vmatrix} P_{11}^{A} & P_{21}^{A} & \cdots & P_{r1}^{A} \\ P_{12}^{A} & P_{22}^{A} & \cdots & P_{r2}^{A} \\ \vdots & \vdots & & \vdots \\ P_{1r}^{A} & P_{2r}^{A} & \cdots & P_{rr}^{A} \end{vmatrix} \tag{52}$$

Clearly, x(t) can be solved by finding f from the relation

$$f = M^{-1} d \qquad (53)$$

In this approximation, the physical meaning of $\Phi(t)$ may be interesting, but, at the risk of computational inaccuracy. Moreover, matrix M may be ill-conditioned or singular.

VIII. Discussion

As shown in the above procedures, we are able to analyze linear singular systems and the solution depends on initial conditions in order to eliminate the impulsive behavior. A simple and two-transistor electric network with double-input and double-output system is illustrated as a linear singular system. And output-nulling subspaces are derived.

If there exist some singularly perturbed dynamic systems, they can also be controlled optimally or adaptively by suitably chosen performance criteria with stability.

A brief introduction of use of orthogonal function in singular systems shows the flexibility in choosing arbitrary basis functions which is related to some physical meaning.

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