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A Robust Discrete-Time Model Reference Adaptive Control in the Presence of Bounded Disturbances

(制限된 外亂下에서의 強靭한 離散 時間 모델 追從 適應 制御)

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要 約

本 論文에서는 一般化 모델 追從 適應 알고리듬을 使用하여 單一 入出力 離散時間 工程에 대해 適用 可能한 強靭한(robust) 모델 追從 適應 制御器를 提案하였다. 本 制御器는 工程의 信號 成分 크기에 따라 死區間(dead-zone)의 크기가 決定되는 時變 死區間 方式을 모델 追從 適應 制御 構造에 導入한 것이다. 이러한 死區間 方式을 使用하는 適應 制御下에서는 制限된 外亂이 있는 境遇에서도 工程의 信號 成分 크기가 制限될 수 있음을 보였다.

Abstract

In this paper, a robust discrete model reference adaptive controller is proposed using a generalized model reference adaptive algorithm for single-input single-output discrete systems. A signal dependent time-varying dead-zone is employed in a generalized adaptive control structure. This adaptive controller is shown to assure the boundedness of the signals of the system even in the presence of bounded external disturbance.

I. Introduction

Much effort has been devoted in recent years to the design of model-reference adaptive control schemes, with the principal aim of establishing that the resulting systems are globally asymptoically stable. In order to achieve this objective, it is necessary that a great deal should be assumed about the plant transfer function which is never precisely available in practice. Moreover in reality, a real system is always subject to the effect of bounded external disturbance or unmodeled dynamics. Therefore in 1980s several different approaches concerning robustness in adaptive control have been proposed in the literature but it is still unsolved research field in adaptive control theory. Among the many modifications of controller Parameter adaptation law which have been suggested for adaptive control of an unknown plant in the presence of disturbances, three have gained wide acceptance. In [1] and [2] Peterson and Narendra, and G. Kreisselmeir and B.D.O. Anderson used a dead-zone in the controller parameter adaptive law to assure boundedness of all the signals in the adaptive system. More recently, Annaswarmy and Narendra [3] suggested a somewhat different adaptive law using a dead-zone which requires less prior information regarding the disturbance

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and provided greater flexibility in design. A second modification suggested by Kreisselmeier and Narendra [4] as well as by Egardt in [5], restricts the search region in parameter space by using prior information regarding bounds on the desired controller parameter. A third modification due to Ioannou and Kokotovic [6], generally referred to as σ-modification, introduces an additional term of the form $-\sigma\theta$ in the adaptive law for adjusting the controller parameter vector θ.

In this paper, to give a robustness to the model reference adaptive control in the presence of bounded external disturbance, a new dead-zone method is proposed. In this method, the size of dead-zone N_d in [2] is replaced by a term dependent upon both ζ (k) and n_o where ζ (k) is a filtered regressor signal vector and n_o is the maximum value of bounded external distrubance. This proposed method is shown to be able to assure the boundedness of all the signals in the adaptive system. One of the major features of this proposed dead-zone method arises from the fact that when the level of signals is larger than that of bounded external disturbance the dead-zone decreases but the dead-zone increases when the situation is reversed. From the results of computer simulation, it is also seen that the proposed dead-zone method has more improved output response than the fixed dead-zone method which uses the same control structure.

II. Adaptive Model Reference Control for Deterministic Case

In this section, we introduce adaptive control structure considered by Narendra et. al [8] for deterministic case. A single-output discrete linear time-invariant plant is described by the state equation

$$x_p(k+1)=A_p x_p(k)+b_p u_p(k)$$
$$y_p(k)=c_p^T x_p(k) \qquad (1)$$

where A_p is nxn unknown constant matrix, b_p and c_p are unknown constant vectors. The transfer function of plant is represented by

$$W_p(z)=k_p\frac{N_p(z)}{D_p(z)} \qquad (2)$$

where W_p (z) is proper, with D_p (z) a monic polynomial of degree n, N_p (z) a monic stable polynomial of degree $m \leqslant n$, and k_p a constant gain parameter. A reference model which is desired from the plant when this is augmented by a suitable controller is given by the transfer function

$$W_m(z)=k_m\frac{N_m(z)}{D_m(z)} \qquad (3)$$

where D_m (z) and N_m (z) are monic stable polynomials of degree n and $r \leqslant m$ respectively and k_m is contant. Hence the relative degree of the model is assumed to be greater than or equal to that of the plant. The reference input r(k) to the model is specified and is assumed to be uniformly bounded.

The objective of control is to determine u_p (k) such that

$$y_p(k)-y_m(k)\rightarrow 0 \quad \text{as } k\rightarrow\infty$$

and to make all the state variables bounded. Since for $m \leqslant (n-1)$ the transfer function of model W_m (z) may not generally be strictly positive real, some auxiliary inputs have to be fed into the reference model.

As shown in Fig. 1 two identical auxiliary signal generators of dimension n-1 having state variables v_1 (k), v_2 (k) and inputs u_p (k) and y_p (k) are used in the controller structure. If a vector ω (k) is definded as

$$\omega(k)^T=[v_1(k)^T, v_2(k)^T, y_p(k)] \qquad (4)$$

the control input u_p (k) into the plant may be represented by

$$u_p(k)=k_o(k)\ r(k)+\theta(k)^T\omega(k) \qquad (5)$$

where θ (k) is defined by

$$\theta(k)^T=[c_1(k),\cdots,c_{n-1}(k),d_1(k),\cdots,d_{n-1}(k),\ d_o(k)] \qquad (6)$$

and c_j (k), d_o (k), d_j (k), j=1,2,..., n-1 are the parameters used for implementing the auxiliary signal generators as in [8].

For simplicity, it is assumed that $k_p=k_m=1$, so $k_o(k)=1$. Then for constant values of θ, the overall transfer function of the controlled system is W(z) where

$$W(z)=\frac{W_p(z)}{1+W_1(z)+W_2(z)W_p(z)}$$
$$=\frac{N_p(z)N(z)}{[N(z)+C(z)]D_p(z)+N_p(z)[D(z)+d_oN(z)]} \qquad (7)$$

where the transfer functions of the auxiliary signal generators 1 and 2 are

$$W_1(z)=\frac{C(z)}{N(z)} \qquad (8)$$

$$W_2(z)=d_0+\frac{C(z)}{N(z)} \qquad (9)$$

respectively for constant values of θ. In order that the overall transfer function have the same zeros as the moel, N(z) is chosen to contain N_m (z) as a factor.

Thus if D_p (z) and N_p (z) are coprime and zeros of N_m (z) are the poles of auxiliary signal generator, there exists a unique controller parameter vector θ^* such that when θ (k) = θ^* the transfer function of the plant together with controller matches that of the model by the lemma 2 of [8].

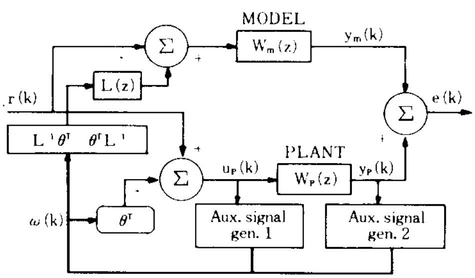

Fig.1. A general adaptive model reference control structure.

III. A Generalized Adaptive Model Reference Control Algorithm

Let the controller parameter θ (k) be expressed as

$$\theta(k)=\theta^*+\phi(k) \qquad (10)$$

where ϕ (k) represents the parameter error vector at time k. As shown in Fig. 1 the model output y_m (k) can be expressed as

$$\begin{aligned} y_m(k) &= W_m(z)\ r(k)+W_m(z)\,\omega(k)^T\theta(k) \\ &\quad -W_m(z)\ L(z)\ \theta(k)^T L(z)^{-1}\,\omega(k) \\ &= W_m(z)\ r(k)+W_m(z)\,\omega(k)^T(\theta^*+\phi(k)) \\ &\quad -W_m(z)L(z)\ (\theta^*+\phi(k))^T L(z)^{-1}\,\omega(k) \end{aligned} \qquad (11)$$

With the equality $W_m(z)\,L(z)\,\theta^{*T}L(z)^{-1}\,W(k) = W_m(z)\ \theta^{*T}\ W(k)$,

$$\begin{aligned} y_m(k) &= W_m(z)\,r(k)+W_m(z)\ \omega(k)^T\phi(k) \\ &\quad -W_m(z)\ L(z)\ \phi(k)^T L(z)^{-1}\,\omega(k) \end{aligned} \qquad (12)$$

Since when θ (k) = θ^* the transfer function of the overall system matches the model, the plant output y_p (k) can be expressed as [8]

$$y_P(k)=W_m(z)\ r(k)+W_m(z)\ \omega(k)^T\phi(k) \qquad (13)$$

Therefore, output error e(k) between plant and model can be expressed as

$$e(k)=y_P(k)-y_m(k)=W_m(z)L(z)\ \phi(k)^T L(z)^{-1}\,\omega(k) \qquad (14)$$

System structure equivalent to Fig. 1 in error analysis is shown in Fig. 2 where L(z) is a prefilter such that

$$L(z)^{-1}\,\omega(k)=\zeta(k) \qquad (15)$$

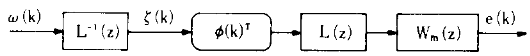

Fig.2. The error model equivalent to Fig.1 for error analysis.

Then a constant gain generalized parameter adaptation law is employed as [9]

$$\begin{aligned} \text{Algorithm}: \ &\theta_P(k)=-\mu\Gamma\ \zeta(k)\ e(k),\quad \mu>\frac{1}{2} \\ &\theta_I(k)=\theta_I(k-1)-\Gamma\ \zeta(k-1)\ \ e(k-1) \\ &\theta(k)=\theta_P(k)+\theta_I(k) \\ &\Gamma=\Gamma^T>0 \end{aligned} \qquad (16)$$

As shown in (16), the controller parameter θ (k) consists of both a proportional term θ_p (k) and a summation term θ_i (k) up to k-1. The parameter error vector ϕ_i (k) is defined as

$$\phi_I(k)=\theta_I(k)-\theta^* \qquad (17)$$

Therefore

$$\phi(k)=-\mu\Gamma\ \zeta(k)\ e(k)+\phi_I(k) \qquad (18)$$

The overall system structure for error analysis by using this parameter adaptation law is shown in Fig. 3.

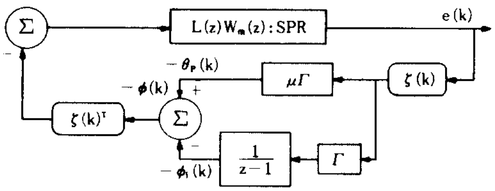

Fig.3. An equivalent error model with parameter adaptation law (16).

IV. Adaptive Control in the Presence of Bounded External Distrubance

In this section, we are going to deal with the robustness problem of the adaptive system by modifying the parameter adaptation law to assure the boundedness of all signals in the presence of bounded external disturbance.

Now we express the plant as

$$x_P(k+1) = A_P x_P(k) + b_P u_P(k)$$
$$y_P(k) = c_P{}^T x_P(k) + n(k) \tag{19}$$

where n(k) is the bounded external disturbance. In the similar approach as the deterministic case we can obtain following equations.

$$y_P(k) = W_m(z)\ r(k) + W_m(z)\ \phi(k)^T \omega(k) + n_1(k) \tag{20}$$

$$y_m(k) = W_m(z)\ r(k) + W_m(z)\ \phi(k)^T \omega(k) - W_m(z)\ L(z)\ \phi(k)^T \zeta(k) \tag{21}$$

where n_1 (k) is the effect of disturbance n(k) at the output as in [1]. Therefore output error e(k) is described as

$$e(k) = y_P(k) - y_m(k) = W_m(z)\ L(z)\ \phi(k)^T\ \zeta(k) + n_1(k) \tag{22}$$

It is assumed that we can express e(k) as

$$e(k) = W_m(z)\ L(z)\ (\phi(k)^T\ \zeta(k) + n_2(k)) \tag{23}$$

where

$$n_1(k) = W_m(z)\ L(z)\ n_2(k) \tag{24}$$

To avoid the integration effect of the external disturbance, we employ a dead-zone in the feedback path of integral action in the parameter adaptation law (16). So a modified adaptation law for robustness is proposed as

$$\text{Algorithm}: \theta_P(k) = -\mu\Gamma\zeta(k)\ e(k),\ \mu > \frac{1}{2}$$
$$\theta_I(k) = \theta_I(k-1) - \Gamma\ \zeta(k-1)\ \eta(k-1)$$
$$\theta(k) = \theta_P(k) + \theta_I(k)$$
$$\Gamma = \Gamma^T > 0 \tag{25}$$

where

$$\eta(k) = e(k) \quad |e(k)| > N_a \text{(defined later)}$$
$$= 0 \quad \text{otherwise.} \tag{26}$$

By (25), we have

$$\phi_I(k) = \phi_I(k-1) - \Gamma\ \zeta(k-1)\ \eta(k-1) \tag{27}$$

where $\phi_i(k)$ is defined as in (17).

The equivalent error model for this algorithm is shown in Fig. 4. The linear time-invariant feedforward part of Fig. 4 can be described by state equations

$$x(k+1) = Ax(k) + b(\phi(k)^T\ \zeta(k) + n_2(k))$$
$$e(k) = c^T x(k) + d(\phi(k)^T\ \zeta(k) + n_2(k)) \tag{28}$$

where the transfer function is L(z) W_m (z) which is strictly positive real.

or

$$L(z)\ W_m(z) = c^T(zI - A)^{-1}b + d \tag{29}$$

By the Kalman-Yacubovich lemma it is known that for (29) there exist a real matrix $P = P^T > 0$, a vector q, and positive constants ϵ, w such that for any matrix $Q = Q^T > 0$

$$A^T P + PA = -qq^T - \epsilon Q$$
$$Pb = c - wq$$
$$w^2 = 2d \tag{30}$$

are simultaneously satisfied.

A Lyapunov function candidate for the set of eq. (27) and (28) is chosen as

$$V(k) = x(k)^T\ Px(k) + \phi_I(k)^T \Gamma^{-1} \phi_I(k),$$
$$\Gamma = \Gamma^T > 0 \tag{31}$$

Then $\triangle V(k)=V(k+1)-V(k)$ may be expressed as after some manipulations

$$\begin{aligned}\triangle V(k) = &-[x(k)^T q - w(\phi(k)^T \zeta(k) + n_2(k))]^2 \\ &-\varepsilon x(k)^T Qx(k) + 2e(k)n_2(k) + 2\phi_1(k)^T \\ &\zeta(k)(e(k)-\eta(k)) + \zeta(k)^T \Gamma \zeta(k) \\ &(\eta(k)^2 - 2\mu e(k)^2) \\ \leqq &-[x(k)^T q - w(\phi(k)^T \zeta(k) + n_2(k))]^2 \\ &-\varepsilon x(k)^T Qx(k) + 2|e(k)|n_o \\ &+2\phi_1(k)^T \zeta(k)(e(k)-\eta(k)) \\ &+\zeta(k)^T \Gamma \zeta(k)(\eta(k)^2 - 2\mu e(k)^2)\end{aligned} \tag{32}$$

where $n_o > |n_2(k)|$. If $|e(k)| > N_d$. We have

$$\begin{aligned}\triangle V(k) \leqq &-[x(k)^T q - w(\phi(k^T)\zeta(k) + n_2(k))]^2 \\ &-\varepsilon x(k)^T Qx(k) + 2|e(k)|n_o - (2\mu - 1) \\ &\zeta(k)^T \Gamma \zeta(k) e(k)^2\end{aligned} \tag{33}$$

Therefore if we slect the size of dead-zone N_d as

$$N_d(k) = \frac{2n_o}{(2\mu-1)\zeta(k)^T \Gamma \zeta(k)} \tag{34}$$

and $|e(k)| > Nd(k0)$. Thus we have

$$\begin{aligned}\triangle V(k) \leqq &-[x(k)^T q - w(\phi(k)^T \zeta(k) + n_2(k))]^2 \\ &-\varepsilon x(k)^T Qx(k)\end{aligned} \tag{35}$$

From the above analysis, we can see that if $|e(k)| > Nd(k)$ the V(k) decreases and therefore e(k) is bounded within approximately the dead-zone size $N_d(k)$. If the level of signal is less than that of external disturbance, the size of dead-zone increases relatively and e(k) is bounded within a larger value. On the other hand, if the level of signal is larger than that of external disturbance, the size of dead-zone decreases relatively and e(k) is bounded within a smaller value. This bounding nature ensures the signals of the system to be bounded in the presence of bounded external disturbance.

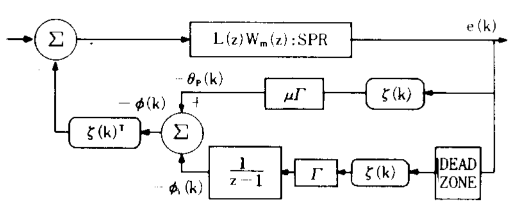

Fig.4. An equivalent error model with the modified algorithm (25).

V. Computer Simulations

The following simulations illustrate some features of the robust adaptive algorithm considered in section IV where $L(z)W_m(z) = 1$ is taken. Considered is a 2nd-order stable minimum phase plant described as

$$\begin{aligned}y_P(k) = &1.2y_P(k-1) - 0.35y_P(k-2) \\ &+u_P(k-1) + 0.4u_P(k-2) + n(k)\end{aligned}$$

here n(k) is an unbiased uniformly distributed random sequence [0,5] with the mean 2.5. The reference model is given by

$$\begin{aligned}y_m(k) = &y_m(k-1) - 0.24y_m(k-2) \\ &+r(k-1) + 0.5r(k-2)\end{aligned}$$

The initial controller parameter $\theta(0)$ is taken as 0 vector. To compare the effects of the size-variable dead-zone, a comparison is performed for the fixed dead-zone controller to the variable dead-zone one. In Fig. 5 is shown the output response of plant controlled by the same algorithm but has a fixed dead-zone whose size is $N_d=2$ with $\mu=1$, $\Gamma=0.01$ and $r(k)=10.0$ square wave. Fig. 6 shows the output response of plant controlled by the proposed algorithm where a time-varying dead-zone (refer to (33)) is used with $\mu=1$, $\Gamma=0.01$, $n_o=2$ and $r(k)=10.0$ square wave.

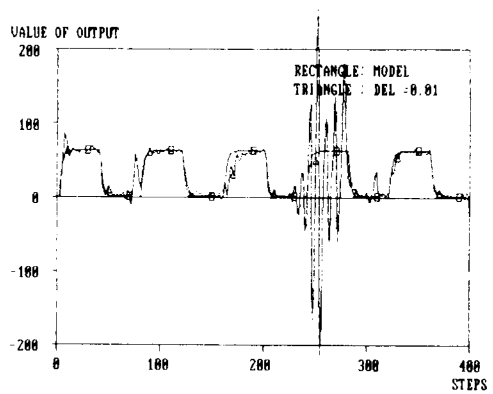

Fig.5. Output response for plant (Fixed dead-zone)

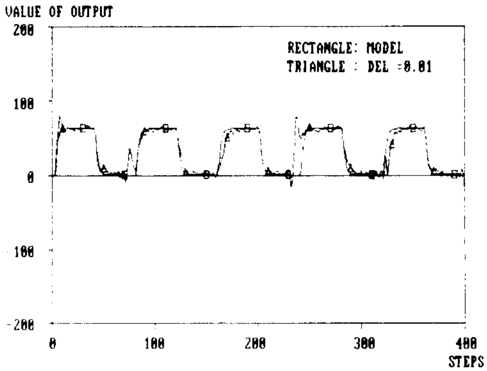

Fig.6. Output response for plant (time-varying dead-zone).

VI. Conclusions

This paper presents a robust model reference adaptive control having a modified parameter adaptation law with a generalized model reference adaptive structure. This new parameter adjustment law which employs a time-varying dead-zone method is shown to be able to maintain the stability of the adaptive control by assuring the boundedness of the signals in the presence of bounded external disturbance. It is also shown by an illustrative computer simulation that with this time-varying dead-zone method the output response of the system would be better than with the fixed dead-zone method

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