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An Effective Method in Analyzing a Class of Bilinear Systems via Taylor Polynomials

(Taylor 다항식에 의한 양선형 시스템의 효과적인 해석법)

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要 約

본 논문에서는 Taylor 다항식에 의해 양 선형 시스템을 해석하는 효과적인 방법이 제안된다. Yang 과 Chen에 의해 유도된 결과는 미지의 상태 벡타가 닫혀진 형태로 구해지지 않고 또한 사용하는 항이 증가할 때 큰 차원의 선형 대수 방정식을 풀어야하는 문제점을 가지고 있지만, 본 논문에서 제안된 방법은 미지의 상태 벡타가 닫혀진 형태로 구해지며 선형 대수 방정식을 풀 필요가 없다.

Abstract

In this paper, an effective method in analyzing a class of bilinear systems via Taylor polynomials is proposed. The result derived by Yang and Chen shows an implicit form for unknown state vector and requires to solve a linear algebraic equation with large dimension when the number of terms used increase. In comparison to the result of Yang and Chen, the method in this paper gives a closed form for unknown state vector and does not need to solve any linear algebraic equation.

I. Introduction

There are many practical systems whose mathematical model may be expressed by bilinear forms, such as nuclear reactor kinetic equations, biological, socio-economic or ecological systems [1]-[3]. The analytic solutions for time-varying bilinear systems cannot be found in general [4]-[5]. However, when those systems are to be analyzed, it is desirable to obtain an analytic solution since numerical solution cannot be compared with analytic one [6].

Recently, as a method of approximately finding an analytic solution of time-varying bilinear systems, series expansion approach has been proposed [7]-[11]. Yang and Chen [11] employed the Taylor polynomials in analyzing a class of time-varying bilinear systems as a better alternative than Block pulse functions [7], Walsh functions [8], or Chebyshev polynomials [9]-[10]. According to the results in Yang and Chen [11], the final linear algebraic equation for the analysis of a class of time-varying bilinear systems is given by

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$$H-[\underline{X}(0),\underline{0},\cdots,\underline{0}]=F^*\begin{bmatrix}H_1\\H_2\\\vdots\\H_n\end{bmatrix}\underline{P}_{(r\times r)}+B^*\begin{bmatrix}u_1\\u_2\\\vdots\\u_n\end{bmatrix}\underline{P}_{(r\times r)} \quad (1)$$

Here H and H_i, i = 1,2, ... ,n, are unknown matrices to be determined, $P_{(rxr)}$ is the operational matrix of integration for the Taylor polynomials. Also, the i-th row and j-th column of an nxn matrix, F^*, and nxqr matrix, B^*, is an 1xr coefficient vector of the Taylor series expansion for the function in the i-th row and j-th column of an nxn matrix F(t) and an nxq matrix B(t), respectively. In (1), it should be observed that (1) is not a closed form for unknown matrices H and H_i, i=1,2, ..., n, and the dimension of (1) is larger when the number r of terms used increase. Therefore it becomes very difficult to solve (1).

In this paper, we suggest an another approach of analyzing a class of time-varying bilinear systems via Taylor polynomials in order to overcome the problems indicated above. The final form derived in this paper is a closed form for unknown state vector and does not include the operation of solving any linear algebraic equation. Also, the inverse of a matrix is not needed in this approach.

II. Problem Formulation

When a function f(t) is expanded at the neighborhood of t = 0 as

$$f(t)=\sum_{i=0}^{\infty} f_i \phi_i(t), \qquad t\in[0,1) \quad (2)$$

where

$$\phi_i(t)=t^i, \quad f_i=\frac{1}{i!}\left(\frac{d^i f(t)}{dt^i}\right)\Big|_{t=0} \quad (3)$$

$\phi_i(t)$, for i=0,1, ..., are called Taylor series basis functions [12]. Also a linear combination of finite number of Taylor series basis functions is called a Taylor polynomial. For an integer $m \geqslant 1$, let $\phi_{(m)}(t)$ be the m-vector function defined by

$$\phi^T_{(m)}(t)=[\phi_0(t),\phi_1(t),\cdots,\phi_{m-1}(t)], \; t\in[0,1) \quad (4)$$

where the superscript T denotes the transpose.

Let us consider the following bilinear systems

$$\dot{\underline{x}}(t)=A(t)\underline{x}(t)+B(t)\underline{u}(t)+\sum_{k=1}^{r} N_k(t)\underline{x}(t)u_k(t)$$

$$\underline{x}(0)=\text{initial vector} \quad (5)$$

where $\underline{x}(t)$ is an n-dimensional state vector, $\underline{u}(t)$ is an r-dimensional input vector, and A(t), B(t), and $N_k(t)$ are nxn, nxr, and nxn time-varying matrices, respectively, and $U_k(t)$, k=1,2,..., r, are the entries of $\underline{u}(t)$.

Now let the matrix F(t) be defined as

$$F(t)=A(t)+\sum_{k=1}^{r} N_k(t)u_k(t) \quad (6)$$

Then eqn. (5) becomes

$$\dot{\underline{x}}(t)=F(t)\underline{x}(t)+B(t)\underline{u}(t) \quad (7)$$

For each i, j=1,2,..., n, let $f_{ij}(t)$ be the entry in i-th row and j-th column of F(t) and for each i=1,2,..., n and k=1,2,...,r, let $b_{ik}(t)$ be the element in i-th row and k-th column of B(t). Assume that all elements of F(t), B(t) and U(t) are analytic in the time interval [0,1); then the expansions of those elements via m Taylor series basis functions are as follows:

$$f_{ij}(t)\cong[f_{ij0}\; f_{ij1}\cdots f_{ij,m-1}]\,\phi_{(m)}(t) \quad (8)$$

$$b_{ik}(t)\cong[b_{ik0}\; b_{ik1}\cdots b_{ik,m-1}]\,\phi_{(m)}(t) \quad (9)$$

and

$$u_k(t)\cong[u_{k0}\; u_{k1}\cdots u_{k,m-1}]\,\phi_{(m)}(t) \quad (10)$$

Then, for each i=1,2,..., n, the expansion of $x_i(t)$ via m Taylor series basis functions is expressed as

$$x_i(t)\cong[x_{i0}\; x_{i1}\cdots x_{i,m-1}]\,\phi_{(m)}(t) \quad (11)$$

Our problem is to find a formula, which has an explicit form and does not require the inverse of large matrix, for the vectors $[x_{io}\; x_{i1} \ldots x_{i,m-1}]$ for i=1,2,...,n, in terms of $\underline{x}(o)$, $[f_{ijo}\; f_{ij1} \ldots f_{ij,m-1}]$ for i,j=1,2,...,n, $[b_{iko}\; b_{ik1} \ldots b_{ik,m-1}]$ for i=1,2, ...,n and k=1,2, ...,r, and $[u_{ko}\; u_{k1} \ldots u_{k,m-1}]$ for k=1,2, ..., r.

III. A Mathematical Preliminary

Let there be given the following two arbitrary

Taylor polynomials each of which is a linear combination of m Taylor series basis functions;

$$p(t) = p_0\phi_0(t) + p_1\phi_1(t) + \cdots + p_{m-1}\phi_{m-1}(t) \triangleq p^T\phi_{(m)}(t) \quad (12)$$

and

$$q(t) = q_0\phi_0(t) + q_1\phi_1(t) + \cdots + q_{m-1}\phi_{m-1}(t) \triangleq q^T\phi_{(m)}(t) \quad (13)$$

where

$$p^T = [p_0 \ \ p_1 \cdots p_{m-1}] \quad (14)$$

$$q^T = [q_0 \ \ q_1 \cdots q_{m-1}] \quad (15)$$

Then, the multiplication of two functions p(t) and q(t) can be approximated as a linear combination of m Taylor series basis functions as follows [12]:

$$p(t)\,q(t) \cong h^T\,\phi_{(m)}(t) \quad (16)$$

where

$$h = \begin{bmatrix} p_0\,q_0 \\ p_0\,q_1 + p_1 q_0 \\ \vdots \\ p_0 q_{i-1} + p_1 q_{i-2} + \cdots + p_{i-1} q_0 \\ \vdots \\ p_0 q_{m-1} + p_1 q_{m-2} + \cdots + p_{m-1} q_0 \end{bmatrix} \begin{matrix} \\ \\ \\ \text{i-th row} \\ \\ \\ \end{matrix} \quad (17)$$

In the above, the terms whose indices are higher than $\phi_{m-1}(t)$ are neglected. The validity of this approximation [12] is confirmed by the fact that the considered time domain is $t \in [0, 1)$.

Now, the integration of the multiplication of two functions p(t) and q(t) can be approximated as a linear combination of m Taylor series basis functions via the operational matrix for forward integration [12], $P_{(mxm)}$, as

$$\int_0^t p(s)\,q(s)\,ds \cong \int_0^t h^T\,\phi_{(m)}(s)\,ds \cong h^T\,P_{(m\times m)}\,\phi_{(m)}(t) \quad (18)$$

where

$$\underline{P}_{(m\times m)} = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \cdots & 0 & 0 & 0 \\ & & & & \vdots & & & \\ 0 & 0 & 0 & 0 & \cdots & 0 & \frac{1}{m-2} & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \frac{1}{m-1} \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \end{bmatrix} \quad (19)$$

We can show that when p(t) is a nxn matrix and q(t) is a nx1 vector, $\int_0^t P(s)\,q(s)\,ds$ can be similarly expressed.

Theorem 1.

Let there be given F(t) and $\underline{x}(t)$ as in (7). For each i,j=1,2,...,n, let $f_{ij}(t)$ be the entry in i-th row and j-th column of F(t). And let the expansions of $f_{ij}(t)$ and the each function of $\underline{x}(t)$ be expressed as follows:

$$f_{ij}(t) \cong [f_{ij0} \ \ f_{ij1} \cdots f_{ij,m-1}]\,\phi_{(m)}(t), \ \ i, j = 1, 2, \cdots, n \quad (20)$$

$$x_i(t) \cong [x_{i0} \ x_{i1} \cdots x_{i,m-1}]\,\phi_{(m)}(t), \ i = 1, 2, \cdots, n \quad (21)$$

Also, let us define the nxn matrix F_w and nx1 vector $\underline{X}_w$ as

$$F_w = \begin{bmatrix} f_{11w} & f_{12w} & \cdots & f_{1nw} \\ f_{21w} & f_{22w} & \cdots & f_{2nw} \\ \vdots & \vdots & & \vdots \\ f_{n1w} & f_{n2w} & \cdots & f_{nnw} \end{bmatrix} \quad (22)$$

$$\underline{X}_w = \begin{bmatrix} x_{1w} \\ x_{2w} \\ \vdots \\ x_{nw} \end{bmatrix} \quad (23)$$

where w=0,1,...,m-1.

Then

$$\int_0^t F(s)\,\underline{X}(s)\,ds \cong [F_0\underline{X}_0, \ \ F_0\underline{X}_1 + F_1\underline{X}_0, \cdots, F_0\underline{X}_{m-1} + F_1\underline{X}_{m-2} + \cdots + F_{m-1}\underline{X}_0]\,\underline{P}_{(m\times m)}\,\phi_{(m)}(t) \quad (24)$$

Proof:

Since

$$F(t)\,\underline{X}(t) = \begin{bmatrix} f_{11}(t) & f_{12}(t) & \cdots & f_{1n}(t) \\ f_{21}(t) & f_{22}(t) & \cdots & f_{2n}(t) \\ \vdots & \vdots & & \vdots \\ f_{n1}(t) & f_{n2}(t) & \cdots & f_{nn}(t) \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} = \begin{bmatrix} f_{11}(t)x_1(t) + f_{12}(t)x_2(t) + \cdots + f_{1n}(t)x_n(t) \\ f_{21}(t)x_1(t) + f_{22}(t)x_2(t) + \cdots + f_{2n}(t)x_n(t) \\ \vdots \\ f_{n1}(t)x_1(t) + f_{n2}(t)x_2(t) + \cdots + f_{nn}(t)x_n(t) \end{bmatrix} \quad (25)$$

the integration of $F(t)\,\underline{x}(t)$ is

$$\int_0^t F(s)\underline{x}(s)\,ds = \begin{bmatrix} \int_0^t \{f_{11}(s)x_1(s)+f_{12}(s)x_2(s)+\cdots+f_{1n}(s)x_n(s)\}\,ds \\ \int_0^t \{f_{21}(s)x_1(s)+f_{22}(s)x_2(s)+\cdots+f_{2n}(s)x_n(s)\}\,ds \\ \vdots \\ \int_0^t \{f_{n1}(s)x_1(s)+f_{n2}(s)x_2(s)+\cdots+f_{nn}(s)x_n(s)\}\,ds \end{bmatrix} \tag{26}$$

The first row in (26) is

$$\begin{aligned} &\int_0^t \{f_{11}(s)x_1(s)+f_{12}(s)x_2(s)+\cdots+f_{1n}(s)x_n(s)\}\,ds \\ &= \int_0^t f_{11}(s)x_1(s)\,ds+\int_0^t f_{12}(s)x_2(s)\,ds+\cdots \\ &\quad+\int_0^t f_{1n}(s)x_n(s)\,ds \end{aligned} \tag{27}$$

By using the property of (16) and (18), we obtain

$$\left.\begin{aligned} &\int_0^t f_{11}(s)\,x_1(s)\,ds \\ &\cong [f_{110}x_{10},\ f_{110}x_{11}+f_{111}x_{10},\ \cdots, \\ &\quad f_{110}x_{1,m-1}+f_{111}x_{1,m-2}+\cdots+f_{11,m-1}x_{10}] \\ &\quad \underline{P}_{(m\times m)}\ \phi_{(m)}(t) \\ &\int_0^t f_{12}(s)\,x_2(s)\,ds \\ &\cong [f_{120}x_{20},\ f_{120}x_{21}+f_{121}x_{20},\ \cdots, \\ &\quad f_{120}x_{2,m-1}+f_{121}x_{2,m-2}+\cdots+f_{12,m-1}x_{20}] \\ &\quad \underline{P}_{(m\times m)}\ \phi_{(m)}(t) \\ &\qquad\vdots \\ &\int_0^t f_{1n}(s)\,x_n(s)\,ds \\ &\cong [f_{1n0}x_{n0},\ f_{1n0}x_{n1}+f_{1n1}x_{n0},\ \cdots, \\ &f_{1n0}x_{n,m-1}+f_{1n1}x_{n,m-2}+\cdots+f_{1n,m-1}x_{n0}] \\ &\quad \underline{P}_{(m\times m)}\ \phi_{(m)}(t) \end{aligned}\right| \tag{28}$$

Therefore the first row in (26) becomes

$$\begin{aligned} &\int_0^t \{f_{11}(s)x_1(s)+f_{12}(s)x_2(s)+\cdots+f_{1n}(s)x_n(s)\}\,ds \\ &\cong [\underline{f}_{10}\underline{X}_0,\ \underline{f}_{10}\underline{X}_1+\underline{f}_{11}\underline{X}_0,\ \cdots, \\ &\quad \underline{f}_{10}\underline{X}_{m-1}+\underline{f}_{11}\underline{X}_{m-2}+\cdots+\underline{f}_{1,m-1}\underline{X}_0]\,\underline{P}_{(m\times m)}\ \phi_{(m)}(t) \end{aligned} \tag{29}$$

where

$$\underline{f}_{1w} = [f_{11w}\ \ f_{12w}\cdots f_{1nw}] \tag{30}$$

$$\underline{X}_w^T = [x_{1w}\ \ x_{2w}\cdots x_{nw}] \tag{31}$$

for w=0,1, ..., m-1.

Similarly, for the second row in (26), we can obtain easily

$$\begin{aligned} &\int_0^t \{f_{21}(s)x_1(s)+f_{22}(s)x_2(s)+\cdots+f_{2n}(s)x_n(s)\}\,ds \\ &\cong [\underline{f}_{20}\underline{X}_0,\ \underline{f}_{20}\underline{X}_1+\underline{f}_{21}\underline{X}_0,\ \cdots,\ \underline{f}_{20}\underline{X}_{m-1}+\underline{f}_{21}\underline{X}_{m-2}+\cdots \\ &\quad+\underline{f}_{2,m-1}\underline{X}_0]\ \underline{P}_{(m\times m)}\ \phi_{(m)}(t) \end{aligned} \tag{32}$$

Where

$$\underline{f}_{2w} = [f_{21w}\ \ f_{22w}\cdots f_{2nw}] \tag{33}$$

$$\underline{X}_w^T = [x_{1w}\ \ x_{2w}\cdots x_{nw}] \tag{34}$$

for w=0,1, ..., m-1, and so forth.
Finally, we find

$$\int_0^t F(s)\,x(s)\,ds = \begin{bmatrix} \underline{f}_{10}\underline{X}_0 & \underline{f}_{10}\underline{X}_1+\underline{f}_{11}\underline{X}_0 & \cdots & \underline{f}_{10}\underline{X}_{m-1}+\underline{f}_{11}\underline{X}_{m-2}+\cdots+\underline{f}_{1,m-1}\underline{X}_0 \\ \underline{f}_{20}\underline{X}_0 & \underline{f}_{20}\underline{X}_1+\underline{f}_{21}\underline{X}_0 & \cdots & \underline{f}_{20}\underline{X}_{m-1}+\underline{f}_{21}\underline{X}_{m-2}+\cdots+\underline{f}_{2,m-1}\underline{X}_0 \\ & & \vdots & \\ \underline{f}_{n0}\underline{X}_0 & \underline{f}_{n0}\underline{X}_1+\underline{f}_{n1}\underline{X}_0 & \cdots & \underline{f}_{n0}\underline{X}_{m-1}+\underline{f}_{n1}\underline{X}_{m-2}+\cdots+\underline{f}_{n,m-1}\underline{X}_0 \end{bmatrix} \underline{P}_{(m\times m)}\ \phi_{(m)}(t) \tag{35}$$

Since

$$\begin{bmatrix} \underline{f}_{1w} \\ \underline{f}_{2w} \\ \\ \underline{f}_{nw} \end{bmatrix} = \begin{bmatrix} f_{11w} & f_{12w} & \cdots & f_{1nw} \\ f_{21w} & f_{22w} & \cdots & f_{2nw} \\ & & \vdots & \\ f_{n1w} & f_{n2w} & \cdots & f_{nnw} \end{bmatrix} = F_w \tag{36}$$

we obtain

$$\begin{aligned} &\int_0^t F(s)\,x(s)\,ds \\ &\cong [F_0\underline{X}_0\ \ F_0\underline{X}_1+F_1\underline{X}_0\cdots F_0\underline{X}_{m-1}+F_1\underline{X}_{m-2}+\cdots+ \\ &\quad F_{m-1}\underline{X}_0]\ \underline{P}_{(m\times m)}\ \phi_{(m)}(t) \end{aligned} \tag{37}$$

This completes the proof.

Q.E.D

IV. Main Result

In the following, we present an efficient method in analyzing a class of bilinear systems via Taylor polynomials.

Theorem 2.

For the system in (7), let the entries of F(t), B(t), and $\underline{U}(t)$ be expanded as in (8), (9), and (10), respectively, and let the expansion of each function of $\underline{x}(t)$ via m Taylor series basis functions be expressed as

$$x_i(t) \cong [x_{io}\ x_{i1} \cdots x_{i,m-1}]\ \phi_{(m)}(t),\ i=1,2,\cdots,n \quad (38)$$

Also, let us define the nxr matrix B_w and rx1 vector $\underline{U}_w$ as

$$B_w = \begin{bmatrix} b_{11w} & b_{12w} \cdots b_{1rw} \\ b_{21w} & b_{22w} \cdots b_{2rw} \\ & \vdots \\ b_{n1w} & b_{n2w} \cdots b_{nrw} \end{bmatrix} \quad (39)$$

$$\underline{U}_w = \begin{bmatrix} u_{1w} \\ u_{2w} \\ \vdots \\ u_{rw} \end{bmatrix}, \quad \text{for } w=0,1,\cdots,m-1 \quad (40)$$

Then the 1xm vectors $[x_{io}\ x_{i1}\ \ldots\ x_{i,m-1}]$ for i= 1,2,...,n can be obtained by

$$\underline{X}_0 = \underline{x}(0)$$

$$\underline{X}_{v-1} - \underline{0} = \frac{F_0\underline{X}_{v-2} + F_1\underline{X}_{v-3} + \cdots + F_{v-2}\underline{X}_0}{v-1} + \frac{B_0\underline{U}_{v-2} + B_1\underline{U}_{v-3} + \cdots + B_{v-2}\underline{U}_0}{v-1} \quad (41)$$

where v=2,3,...,m and the nxn matrix F_w and nx1 vector $\underline{X}_w$ were given in (22) and (23).

Proof:

Integrating (7) from o to t, we obtain

$$\underline{x}(t) - \underline{x}(0) = \int_0^t F(s)\underline{x}(s)ds + \int_0^t B(s)\underline{u}(s)ds \quad (42)$$

Expanding the state variables with m Taylor series basis functions

$$\underline{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} \cong \begin{bmatrix} x_{10}\phi_0(t) + x_{11}\phi_1(t) + \cdots + x_{1,m-1}\phi_{m-1}(t) \\ x_{20}\phi_0(t) + x_{21}\phi_1(t) + \cdots + x_{2,m-1}\phi_{m-1}(t) \\ \vdots \\ x_{n0}\phi_0(t) + x_{n1}\phi_1(t) + \cdots + x_{n,m-1}\phi_{m-1}(t) \end{bmatrix}$$

$$= \underline{X}_0\phi_0(t) + \underline{X}_1\phi_1(t) + \cdots + \underline{X}_{m-1}\phi_{m-1}(t) \quad (43)$$

is obtained.

Substituting this into (42) and using the Theorem 1, we have

$$[\underline{X}_0\underline{X}_1\cdots\underline{X}_{m-1}]\phi_{(m)}(t) - [\underline{x}(0)\ \underline{0}\cdots\underline{0}]\ \phi_{(m)}(t)$$
$$= [F_0\underline{X}_0\ F_0\underline{X}_1 + F_1\underline{X}_0 \cdots F_0\underline{X}_{m-1} + F_1\underline{X}_{m-2} + \cdots + F_{m-1}\underline{X}_0]\ \underline{P}_{(m\times m)}\ \phi_{(m)}(t)$$
$$+ [B_0\underline{U}_0\ B_0\underline{U}_1 + B_1\underline{U}_0 \cdots B_0\underline{U}_{m-1} + B_1\underline{U}_{m-2} + \cdots + B_{m-1}\underline{U}_0]\ \underline{P}_{(m\times m)}\ \phi_{(m)}(t) \quad (44)$$

Since (44) should be satisfied for all t in the time interval t ∈ [0, 1),

$$[\underline{X}_0\underline{X}_1\cdots\underline{X}_{m-1}] - [\underline{x}(0)\ \underline{0}\cdots\underline{0}]$$
$$= [F_0\underline{X}_0\ F_0\underline{X}_1 + F_1\underline{X}_0 \cdots F_0\underline{X}_{m-1} + F_1\underline{X}_{m-2} + \cdots + F_{m-1}\underline{X}_0]\ \underline{P}_{(m\times m)}$$
$$+ [B_0\underline{U}_0\ B_0\underline{U}_1 + B_1\underline{U}_0 \cdots B_0\underline{U}_{m-1} + B_1\underline{U}_{m-2} + \cdots + B_{m-1}\underline{U}_0]\ \underline{P}_{(m\times m)} \quad (45)$$

in obtained.

Substituting (19) into (45), we find

$$[\underline{X}_0\ \underline{X}_1\cdots\underline{X}_{m-1}] - [\underline{x}(0)\ \underline{0}\cdots\underline{0}]$$
$$= \left[\underline{0}\ F_0\underline{X}_0\ \frac{F_0\underline{X}_1 + F_1\underline{X}_0}{2} \cdots \frac{F_0\underline{X}_{m-2} + F_1\underline{X}_{m-3} + \cdots + F_{m-2}\underline{X}_0}{m-1}\right.$$
$$+ \left[\underline{0}\ B_0\underline{U}_0\ \frac{B_0\underline{U}_1 + B_1\underline{U}_0}{2} \cdots \frac{B_0\underline{U}_{m-2} + B_1\underline{U}_{m-3} + \cdots + B_{m-2}\underline{U}_0}{m-1}\right] \quad (46)$$

By equating each colum in (46), we can obtain finally

$$\underline{X}_0 - \underline{x}(0) = \underline{0}$$

$$\underline{X}_1 - \underline{0} = F_0\underline{X} + B_0\underline{U}_0$$

$$\underline{X}_2 - \underline{0} = \frac{F_0\underline{X}_1 + F_1\underline{X}_0}{2} + \frac{B_0\underline{U}_1 + B_1\underline{U}_0}{2}$$

$$\vdots$$

vth cloumn

$$\underline{X}_{v-1} - \underline{0} = \frac{F_0\underline{X}_{v-2} + F_1\underline{X}_{v-3} + \cdots + F_{v-2}\underline{X}_0}{v-1} + \frac{B_0\underline{U}_{v-2} + B_1\underline{U}_{v-3} + \cdots + B_{v-2}\underline{U}_0}{v-1}$$

$$\vdots$$

$$\underline{X}_{m-1} - \underline{0} = \frac{F_0\underline{X}_{m-2} + F_1\underline{X}_{m-2} + \cdots + F_{m-2}\underline{X}_0}{m-1} + \frac{B_0\underline{U}_{m-2} + B_1\underline{U}_{m-3} + \cdots + B_{m-2}\underline{U}_0}{m-1} \quad (47)$$

This completes the proof.

Q.E.D

V. An Example

Let us consider the following bilinear system [11]

$$\dot{\underline{x}}(t)=A(t)\underline{x}(t)+B(t)\underline{u}(t)+\sum_{K=1}^{2}N_K(t)\underline{x}(t)\underline{u}_K(t),\ x(0)=\begin{bmatrix}0\\1\end{bmatrix}$$

where

$$A(t)=\begin{bmatrix}-4 & 1\\-20 & 3t\end{bmatrix},\quad B(t)=\begin{bmatrix}t & 1\\1 & t\end{bmatrix},$$

$$N_1(t)=\begin{bmatrix}2 & 3t\\-5 & t\end{bmatrix}$$

$$N_2(t)=\begin{bmatrix}3 & -1\\20 & -5t\end{bmatrix},\quad \underline{u}(t)=\begin{bmatrix}0\\1\end{bmatrix}$$

From (6), we obtain the matrix F(t) as

$$F(t)=\begin{bmatrix}-1 & 0\\0 & -2t\end{bmatrix}$$

When the number of terms used is 4, we obtain

$$f_{11}(t)=[-1\ 0\ 0\ 0]\,\phi_{(4)}(t)$$

$$f_{12}(t)=f_{21}(t)=[0\ 0\ 0\ 0]\,\phi_{(4)}(t)$$

$$f_{22}(t)=[0\ -2\ 0\ 0]\,\phi_{(4)}(t)$$

Therefore

$$F_0=\begin{bmatrix}-1 & 0\\0 & 0\end{bmatrix},\quad F_1=\begin{bmatrix}0 & 0\\0 & -2\end{bmatrix},$$

$$F_2=F_3=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

Similary, we have

$$b_{11}(t)=b_{22}(t)=[0\ 1\ 0\ 0]\,\phi_{(4)}(t)$$

$$b_{12}(t)=b_{21}(t)=[1\ 0\ 0\ 0]\,\phi_{(4)}(t)$$

$$u_1(t)=[0\ 0\ 0\ 0]\,\phi_{(4)}(t)$$

$$u_2(t)=[1\ 0\ 0\ 0]\,\phi_{(4)}(t)$$

Thus

$$B_0=\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix},\ B_1=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix},\quad B_2=B_3=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

$$\underline{U}_0=\begin{bmatrix}0\\1\end{bmatrix},\quad \underline{U}_1=\underline{U}_2=\underline{U}_3=\begin{bmatrix}0\\0\end{bmatrix}$$

Equation (47) yields

$$\underline{X}_0=\begin{bmatrix}0\\1\end{bmatrix}$$

$$\underline{X}_1=\begin{bmatrix}0\\1\end{bmatrix}$$

$$\underline{X}_2=\begin{bmatrix}-\frac{1}{2}\\-\frac{1}{2}\end{bmatrix}$$

$$\underline{X}_3=\begin{bmatrix}\frac{1}{6}\\0\end{bmatrix}$$

Therefore an approximate solution of this bilinear system is

$$\underline{x}(t)=\begin{bmatrix}x_1(t)\\x_2(t)\end{bmatrix}\cong\begin{bmatrix}0\\1\end{bmatrix}\phi_0(t)+\begin{bmatrix}1\\0\end{bmatrix}\phi_1(t)+\begin{bmatrix}-\frac{1}{2}\\-\frac{1}{2}\end{bmatrix}\phi_2(t)+\begin{bmatrix}\frac{1}{6}\\0\end{bmatrix}\phi_3(t)$$

$$=\begin{bmatrix}t-\frac{1}{2}t^2+\frac{1}{6}t^3\\1-\frac{1}{2}t^2\end{bmatrix}$$

It is remarked that while an approach presented in this paper is extremely simpler than that of [11], the solution obtained by (47) is equal to that of [11].

V. Concluding Remarks

In this paper, we have proposed an effective way of analyzing a class of bilinear systems via Taylor polynomials. The approach suggested in this paper gives a closed form for unknown state vector and does not need to solve any linear algebraic equation, while the method based upon the product and coefficient matrix derived by Yang and Chen is an implicit form for unknown state vector and requires to solve a linear algebraic equation with large dimension when the number of terms employed increase.

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