Channel Distortion Effects on a BPSK DS/SS and a QPSK DS/SS Signal Demodulation

(BPSK DS/SS와 QPSK DS/SS신호 복호에서 채널 왜곡의 영향)

朴 鎭 秀*
(Jin Soo Park)

要 約

직접확산 스펙트럼 신호가 대역폭을 넘어선 이상정수값들로 부터 직각변조 시스템에서 채널 왜곡에 따르는 감손을 생각하였고, 채널이득과 위상응답에 대한 직렬 전개식을 사용하여 수신에서 상관 출력의 감손에 대하여 위상오율, 지연오율, 선형 이득편차, 직각 이득편차, 그리고 직각 위상편차를 포함하는 상관함수를 제안하였다.

Abstract

The degradation due to channel distortion in a quadrature modulation system from the ideal constant values over the bandwidth of a direct sequence spread spectrum signal are considered. Through using series expansion for the channel gain and phase response, the degradation in the correlator output at the receiver is found as a function of the parameters involved, including phase error, delay error, linear gain variation, quadratic gain variation, and quadratic phase variation.

I. Introduction

Spread-Spectrum techiques have found extensive applications in present day communication and navigation systems. In direct sequence spread - specturm digital modulation systems, it is important to consider the effects of channels that introduce amplitude and phase distortion. In this paper, series representations are used for the amplitude and phase characteristics of the channel to analyze the degradation due to channel distortion on DS/SS signal demodulation. Both direct sequence, biphase - shift keyed (DS/BPSK) and dual channel, direct sequence, quadriphase - shift keyed (DS/QPSK) signaling formats are considered.

*正會員, 淸州大學校 電子工學科
(Dept. of Elec. Eng., Chongju Univ.)
接受日字 : 1988年 3月 30日

II. Analysis

The complex exponential form of the channel transfer function from Figure 1 is represented as

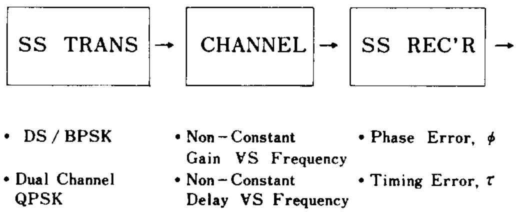


Fig.1. Illustration of non-ideal channel effects in DS/SS.

$$H(f) = A(f)\exp[j\theta(f)] \quad (1)$$

where A(f) and θ (f) are channel amplitude response and phase response respectively. They may be expressed as

$$A(f) = A_0 + \triangle A(f) \quad (2)$$

$$\theta(f) = -2\pi t_0 f + \triangle\theta(f) \quad (3)$$

where $\triangle$A(f) and $\triangle\theta$ (f) are deviations from the ideal characteristics. The output of the filter due to an input signal x(t) with Fourier transform X(f) is

$$y(t) = \mathscr{F}^{-1}\{X(f)\,A(f)\exp[j\theta(f)]\} \quad (4)$$

For $\triangle$ A(f) and $\triangle\theta$ (f) small over the passband of the signal, the filter transfer function is well approximated by

$$A(f)\,\exp[j\theta(f)] \simeq [A_0 + \triangle A(f) + jA_0\triangle\theta(f)]\,\exp(-j2\pi f t_0) \quad (5)$$

where (2) and (3) have been substituted, exp (j$\triangle\theta$) was expanded in a power series, and all terms above first order dropped.

The deviation functions of the amplitude and phase responses from the ideal constant and linear channel responses, respectively, are represented in series form as

$$\triangle A(f) = a_1 f + a_2 f^2 + \quad (6)$$

and

$$\triangle\theta(f) = b_0 + b_2 f^2 + \quad (7)$$

Since, the channel transfer function (1) can, for small gain and phase variations from the ideal, be expanded in series form as [1]

$$H(f) = [A_0 + \triangle A(f)]\,\exp(-j2\pi t_0 f) \times [1 + \triangle\theta(f) + \frac{1}{2!}\triangle\theta^2(f) + \cdots\cdots] \quad (8)$$

Thus, the low - pass equivalent envelope of the signal at the channel output can be written in term of on inverse Fourier transform as

$$\begin{aligned} y(t) &= \mathscr{F}^{-1}[H(f)\,X(f)] \\ &= \mathscr{F}^{-1}[A(f)e^{j\theta(f)}\,X(f)] \\ &= \mathscr{F}^{-1}\Big[A_0(1 + jb_0 - \frac{1}{2}b_0^2) \\ &\quad + \frac{a_1(1 + jb_0 - \frac{1}{2}b_0^2)}{j2\pi}(j2\pi f) \\ &\quad + \frac{a_2(1 - \frac{1}{2}b_0^2) - A_0 b_0 b_2 + j(A_0 b_2 + b_0 a_2)}{(j2\pi)^2}(j2\pi f)^2 \\ &\quad + \cdots\Big]X(f)e^{-j2\pi t_0 f} \end{aligned} \quad (9)$$

where, from the series expansion of (8), the coefficients are

$$\bar{A}_0 = A_0(1 - \frac{1}{2}b_0^2 + jb_0) \quad (10)$$

$$\bar{A}_1 = a_1(1 - \frac{1}{2}b_0^2 + jb_0)/j2\pi \quad (11)$$

and

$$\bar{A}_2 = \frac{a_2(1 - \frac{1}{2}b_0^2) - A_0 b_0 b_2 + j(A_0 b_2 + b_0 a_2)}{(j2\pi)^2} \quad (12)$$

Using the differentiation and time delay theorems of fourier transforms, the complex envelop of the output signal from the channel is

$$\tilde{y}(t) = \bar{A}_0\tilde{x}(t - t_0) + \bar{A}_1\,\tilde{x}'(t - t_0) + \bar{A}_2\,\tilde{x}''(t - t_0) \quad (13)$$

where the expansion has been stopped at second order terms and the primes represent differentiation with respect to time. The real channel output is obtained by multiplying (13) by exp (j2πft) and taking the real part.

To obtain the output of a coherent demodulator with phase error ϕ in response to the channel output, (13) is multiplied by exp ($-$ jϕ) and the real and imaginary parts taken. The real part is the response of the demodulator for in-phase carrier demodulation (with phase error ϕ) and the imaginary part is the response of the demodulator to quadrature carrier demodulation. The results for these operations are given by

$$\begin{aligned} Z_1(t) = [&A_{0r}\,\tilde{x}_r(t-t_0) - A_{0i}\,\tilde{x}_i(t-t_0) + A_{1r}\tilde{x}'_r(t-t_0) \\ &- A_{1i}\,\tilde{x}_i(t-t_0) + A_{2r}\tilde{x}''_r(t-t_0) \\ &- A_{2i}\tilde{x}''_i(t-t_0)]\cos\phi + [A_{0i}\,\tilde{x}_r(t-t_0) \\ &+ A_{0r}\,\tilde{x}_i(t-t_0) + A_{1i}\,\tilde{x}'_r(t-t_0) \\ &+ A_{1r}\,\tilde{x}''_i(t-t_0) + A_{2i}\,\tilde{x}''_r(t-t_0) \\ &+ A_{2r}\,\tilde{x}''_i(t-t_0)]\sin\phi \end{aligned} \tag{14}$$

and

$$\begin{aligned} Z_2(t) = -[&A_{0r}\,\tilde{x}_r(t-t_0) - A_{0i}\,\tilde{x}_i(t-t_0) + \\ &A_{1r}\,\tilde{x}'_r(t-t_0) - A_{1i}\,\tilde{x}'_i(t-t_0) + A_{2r}\,\tilde{x}''_r(t-t_0) \\ &- A_{2i}\,\tilde{x}''_i(t-t_0)]\sin\phi + [A_{0i}\,\tilde{x}_r(t-t_0) + \\ &A_{0r}\,\tilde{x}_i(t-t_0) + A_{1i}\,\tilde{x}_r(t-t_0) + A_{1r}\,\tilde{x}'_i(t-t_0) \\ &+ A_{2i}\tilde{x}''_r(t-t_0) + A_{2r}\,\tilde{x}''_i(t-t_0)]\cos\phi \end{aligned} \tag{15}$$

respectively, where the constants are the real and imaginary parts of (10) – (12) and the subscript "r" and "i" on x(t) denote the real and imaginary parts of the complex envelope of the input to the channel.

The above result can be specialized to Direct Sequence Spread Spectrum modulation by letting

$$x_1(t) = C_1(t)\, d_1(t) \tag{16}$$

and

$$x_2(t) = C_2(t)\, d_2(t) \tag{17}$$

where Ci (t) and di (t), i = 1,2, represent spreading codes and data signals, respectively. The code rates are assumed to be much greater than the data rates, and the data signals and codes are assumed to be independent ± 1 – Valued random binary processes. For code correlation, expressions like

$$\overline{C(t-\tau)C(t)d(t)} \tag{18}$$

$$\overline{C'(t-\tau)C(t)d(t)} \tag{19}$$

and

$$\overline{C''(t-\tau)C(t)d(t)} \tag{20}$$

result, where the overbar denotes averaging over times long compared with the code chip period, but short compared with the data bit period. Let

$$R_{ci}(\tau) = \overline{C_i(t-\tau)C_i(t)} \tag{21}$$

$(i = 1, 2, \cdots\cdots)$

represent the code correlation function, and assume that the cross-correlation function between the two code is zero. Also, recall the theorems

$$\overline{C'(t-\tau)C(t)} = -\frac{dRc(\tau)}{d\tau} \tag{22}$$

and

$$\overline{C''(t-\tau)\,C(t)} = -\frac{d^2Rc(\tau)}{d\tau^2} \tag{23}$$

By using the chain rule, the various derivatives in (14) and (15) can be expanded in the fashion

$$\tilde{x}'_r(t-t_0) = C'_1(t-t_0)\, d_1(t-t_0) + C_1(t-t_0)\, d'_1(t-t_0) \tag{24}$$

etc. When correlated with local code references, the result is a time average over a time interval long with respect to the code chip period but short with respect to the data bit period. This correlation process when carried out on (24) result in

$$\begin{aligned} \overline{\tilde{x}_r(t-t_0)}\,\overline{C_1(t-\tau)} &= \overline{C'_1(t-t_0)C_1(t-\tau)}d_1(t-t_0) \\ &\quad \overline{+C_1(t-t_0)}\,\overline{C_1(t-\tau)}\; d'_1(t-t_0) \\ &\cong -\frac{dRc(\tau-t_0)}{d\tau}d_1(t-t_0) \end{aligned} \tag{25}$$

where it is assumed that Rc (τ - to) d'_1 (t - t_0) is negligible (it consists of spikes at each data transition and is otherwise zero). Therefore, when (14) and (15) are multiplied by local codes and averaged, the approximate results are

$$\begin{aligned} Rz_1(\tau_1 t) \cong \{&[A_{0r}Rc_1(\tau-t_0) + A_{1r}\frac{dRc_1(\tau-t_0)}{d\tau} \\ &+ A_{2r}\frac{d^2Rc_1(\tau-t_0)}{d\tau^2}]\, d_1(t-t_0) \\ &[A_{0i}\,Rc_2(\tau-t_0) + A_{1i}\frac{dRc_2(\tau-t_0)}{d\tau} \\ &+ A_{2i}\frac{d^2Rc_2(\tau-t_0)}{d\tau^2}]\, d_2(t-t_0)\}\cos\phi \\ &+ \{[A_{0i}\,Rc_1(\tau-t_0) \\ &+ A_{1i}\frac{dRc_1(\tau-t_0)}{d\tau} \\ &+ A_{2i}\frac{d^2Rc_1(\tau-t_0)}{d\tau^2}]d_1(t-t_0) \\ &+ [A_{0r}Rc_2(\tau-t_0) + A_{1r}\frac{dRc_2(\tau-t_0)}{d\tau} \\ &+ A_{2r}\frac{d^2Rc_2(\tau-t_0)}{d\tau^2}]\, d_2(t-t_0)\}\sin\phi \end{aligned} \tag{26}$$

$$Rz_2(\tau,t) \simeq \{-[A_{0r}Rc_1(\tau-t_0)+A_{1r}\frac{aRc_1(\tau-t_0)}{d\tau}$$

$$+A_{2r}\frac{d^2Rc_1(\tau-t_0)}{d\tau^2}]d_1(t-t_0)+[A_{0i}Rc_2$$

$$\times(\tau-t_0)+A_{1i}\frac{dRc_2(\tau-t_0)}{d\tau}$$

$$+A_{2i}\frac{d^2Rc_2(\tau-t_0)}{d\tau^2}]d^2(t-t_0)\}\sin\phi$$

$$+\{[A_{0i}Rc_1(\tau-t_0)+A_{1i}\frac{dRc_1(\tau-t_0)}{d\tau}$$

$$+A_{2i}\frac{d^2Rc_2(\tau-t_0)}{d\tau^2}]d_1(t-t_0)$$

$$+[A_{0r}Rc_2(\tau-t_0)+A_{1r}\frac{dRc_2(\tau-t_0)}{d\tau}$$

$$+A_{2r}\frac{d^2Rc_2(\tau-t_0)}{d\tau^2}]\times d_2(t-t_0)\}\cos\phi \quad (27)$$

respectively.

The degradation due to channel distortion can be evaluated by substituting expressions for the code correlation functions and their derivatives in (26) and (27) and comparing the values for $\tau - t_0 = o$ with the results for no channel distortion.

III. The two Special Cases of DS/BPSK and Dual - Channel DS/QPSK

To obtain representative degradation result, the spreading code spectra are approximated as

$$Sc(f)=K\,\mathrm{sinc}^2(Tcf) \quad (28)$$

$$(\sin c(u)=\frac{\sin(\pi u)}{\pi u},\ K=\text{cons tant})$$

That is, the spectra of pseudo-noise spreading codes are approximated. These are assumed to be passed through an ideal lowpass filter of bandwidth w/2 (i.e, the IF filter is bandpass of width w). Thus, the correlation function of the filtered codes can be written as the inverse Fourier transform

$$Rc(\tau)=F^{-1}[K\,\mathrm{sinc}^2(Tcf)\,\pi(f/w)] \quad (29)$$

$$\pi(u)=1,\quad |u|\le\frac{1}{2}$$
$$=0,\ \text{Otherwise}$$

The first and second derivatives to the code autocorrelation functions are obtained by differentiating under the integral sign. Evaluation of these integral sign. Evaluation of these integrals is done numerically.

The two special case of DS/BPSK and dual channel DS/QPSK are considered next under the assumption that the channel phase response is zero at the midband of the channel. Under this assumption, $b_o = \phi$ and the constants in (26) and (27) simplify to

$$A_{0r}=A_0;A_{0i}=0$$

$$A_{1r}=0;A_{1i}=-\frac{a_1}{2\pi}$$

$$A_{2r}=-\frac{a_2}{(2\pi)^2};A_{2i}=-\frac{A_0b_0}{(2\pi)^2}$$

In DS/BPSK case $d_2(t) = o$ and $Rc_2(\tau) = o$ so that the code correlator output is

$$Rz_1(\tau,t)=\{[A_0Rc(\tau-t_0)-\frac{a_2}{(2\pi)^2}$$

$$\times\frac{d^2Rc(\tau-t_0)}{d\tau^2}]\cos\phi-[\frac{a_1}{2\pi}\frac{dRc(\tau-t_0)}{d\tau}$$

$$+\frac{A_0b_2}{(2\pi)^2}\frac{d^2Rc(\tau-t_0)}{d\tau^2}]\sin\phi\}\,d(t-t_0) \quad (30)$$

Dual Channel DS/QPSK. From (12) and (13), respectively, the outputs from the two channels in this case become

$$Rz_1(\tau,t)=\{[A_0Rc_1(\tau-t_0)-\frac{a_2}{(2\pi)^2}$$

$$\times\frac{d^2Rc_1(\tau-t_0)}{d\tau^2}]\cos\phi-[\frac{a_1}{2\pi}\frac{dRc_1(\tau-t_0)}{d\tau}$$

$$+\frac{A_0b_0}{(2\pi)^2}\frac{d^2Rc_1(\tau-t_0)}{d\tau^2}]\sin\phi\}\,d_1(t-t_0)$$

$$+\{[\frac{a_1}{2\pi}\frac{dRc_2(\tau-t_0)}{d\tau}$$

$$+\frac{A_0b_2}{(2\pi)^2}\frac{d^2Rc_2(\tau-t_0)}{d\tau^2}]\cos\phi$$

$$+[A_0Rc_2(\tau-t_0)-\frac{a^2}{(2\pi)^2}$$

$$\times\frac{d^2Rc_2(\tau-t_0)}{d\tau^2}]\sin\phi\}\,d_2(t-t_0) \quad (31)$$

and

$$Rz_2(\tau, t) = \{[A_0 Rc_1(\tau - t_0) - \frac{a_2}{(2\pi)^2} \times \frac{d^2 Rc_1(\tau - t_0)}{d\tau^2}]\sin\phi + [\frac{a_1}{2\pi}\frac{dRc_1(\tau - t_0)}{d\tau} + \frac{A_0 b_2}{(2\pi)^2}\frac{d^2 Rc_1(\tau - t_0)}{d\tau^2}]\cos\phi\} d_1(t - t_0)$$

$$- \{[\frac{a_1}{2\pi}\frac{dRc_2(\tau - t_0)}{d\tau} + \frac{A_0 b_2}{(2\pi)^2} \times \frac{d^2 Rc_1(\tau - t_0)}{d\tau^2}]\sin\phi + [A_0 Rc_2(\tau - t_0) + \frac{a_2}{(2\pi)^2}\frac{d^2 Rc_2(\tau - t_0)}{d\tau^2}]\cos\phi\} d_2(t - t_2) \quad (32)$$

Note that the term of (31) inside the first set of braces is the same as (30) with a similar observation holding for the second term of (32) inside braces.

IV. Computer Simulation

The computer evaluation of the linear amplitude distortion was done in two method by computer programs. In the one method, a simulation model developed in the study of degradation versus parabolic amplitude distortion and effect of demodulator phase error on degradation in a graphic format. In the second method, a simulation model developed in the study of the probability error versus signal-to-noise ratio under various distortion conditions and compare it with the ideal result.

In the simulation, the linear amplitude distortion ($a_1 \neq \phi$) has no effect since the derivative of the code autocorrelation is zero at zero delay. Thus, parabolic amplitude distortion ($a_2 \neq \phi$) has the major effect on of the degradation. Since correlation function is

$$Rz = F_1(\tau, \phi, a_1, a_2, b_0, b_2)d_1(t) + F_2(\tau, \phi, a_1, a_2, b_0, b_2)d_2(t) \quad (33)$$

Where a_1 is the linear amplitude distortion, a_2 is the parabolic amplitude distortion. Thus, the error probability function can be written by Q function [1].

$$P(E) = \frac{1}{2}Q[\frac{(F_1+F_2)}{\sigma_1}] + \frac{1}{2}Q[\frac{(F_1+F_2)}{\sigma_1}] \quad (34)$$

For no distortion can be written as

$$P(E) = \frac{1}{2}Q(\frac{A_0}{\sigma_1}) + \frac{1}{2}Q(\frac{A}{\sigma_1}) = Q(\frac{A_0}{\sigma_1}) \quad (35)$$

In the program sequence periods, the probability of symbol error is computed for each symbol using the fact that the degradation computed by use autocorrelation function (31)

V. Results

Representative result obtained through bound computation and computer simulation are shown in Figs. 2-7. From equation (28), it is seen that for zero phase error only parabolic amplitude distortion results in degradation. If the phase error is nonzero, but the code delay error is zero, additional degradation is introduced by parabolic phase distortion (linear delay distortion).

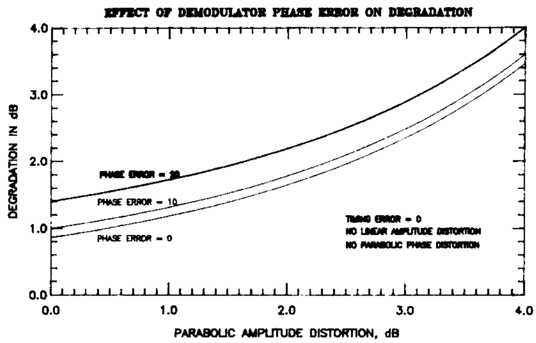


Fig.2. Effect of demodulation phase error on degradation.

Fig 2 shows degradation as a function of parabolic amplitude distortion for DS/BPSK with the product of code chip period and the IF filter bandwidth as a parameter. The larger the time-bandwidth product, the smaller the degradation as long as the parabolic amplitude distortion is small. As parabolic amplitude distortion gets larger, the curves cross over so that the larger the time-bandwidth product, the more the degradation, that is, for large channel distortions, more degradation is introduced into the code correlation for large time-bandwidth products than for small time-bandwidth. Products. Fig 3 shows the same type of curves, except demodulator phase error is the parameter rather than time-bandwidth product.

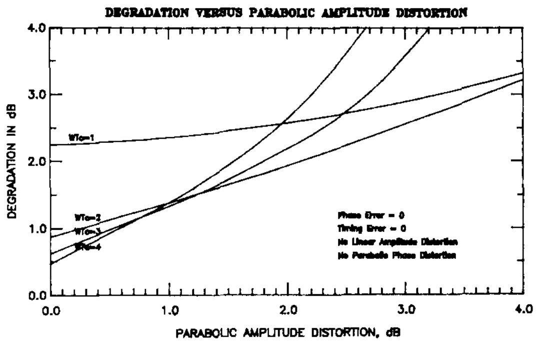


Fig.3. Degradation versus parabolic amplitude distortion.

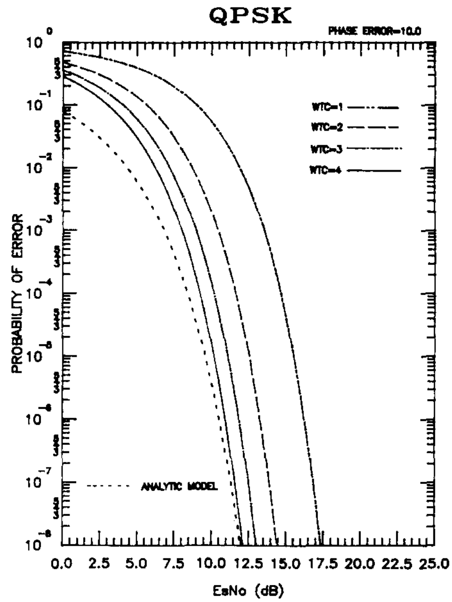


Fig.4. Probability error versus signal-to-noise due various code chip period (phase error=10).

Consider next equations (28) or (29). Now crosstalk between the quadrature channels is present (represented by the second term in brances in (28) and by the first term in (29)). Crosstalk takes place under the following conditions; 1. The usual case in quadrature modulation system of nonzero phase error; 2. Nonzero delay error and linear amplitude distortion; 3. Zero delay error and linear delay distortion; 4. Combinations of the above; Figs 4-7 shows probability error versus signal-to-noise ratio under various code chip period and phase error and compare it with the ideal result.

In order to obtain the degradation due to channel distortion in a quadrature modulation system it is necessary to plot the error probability versus signal-to-noise ratio amplitude and phase distortion conditions and compare it with the ideal result.

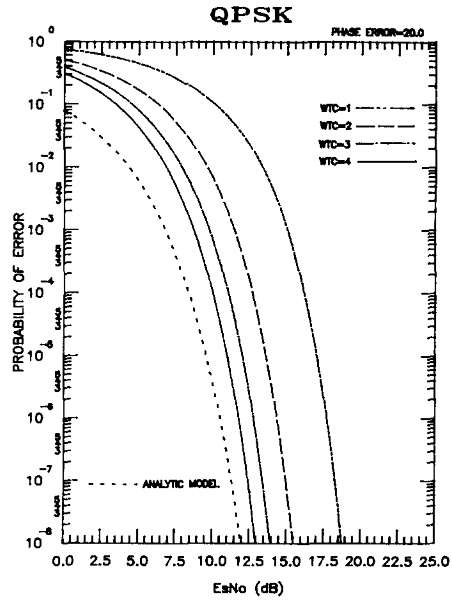


Fig.5. Probability error versus signal-to-noise due various code chip period (Phase error=20).

References

[1] Ziemer, R.E. and R.L. Peterson, Digital Communications and Spread Spectrum Systems, New York: MacMillan, (1985), Chapter 2.

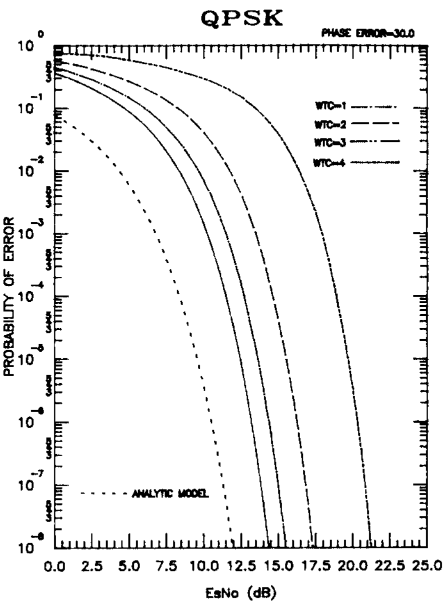


Fig.6. Probability error versus signal-to-noise due various code chip period (Phase error=30).

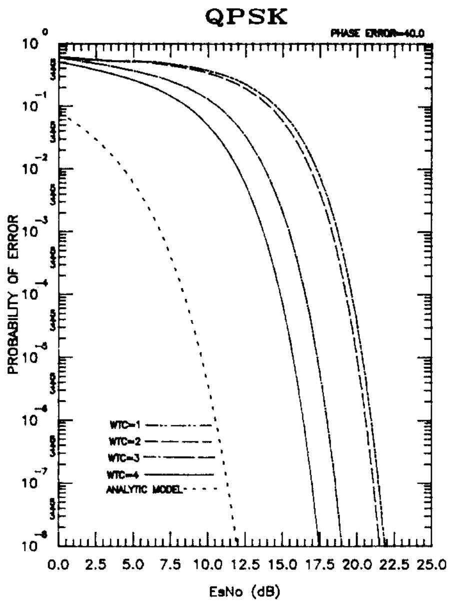


Fig.7. Probability error versus signal-to-noise due various code chip period (Phase error=40).

[2] Ziemer, R.E. and W.H. Tranter, Principles of Communications, Second Edition, Boston: Houghton Mifflin Company (1985) Chapter 7.

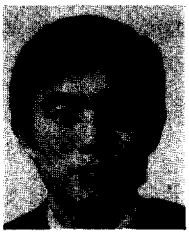
[3] Lunagach, R.S., "Performance of a Direct Sequence Spread-Spectrum System with Long Period and Short Period Code Sequence," *IEEE Trans. on Commun.*, vol. COM-31, pp. 412-419, March 1983.

[4] Milstetein, L.B., S. Davidovici, and D.L. Schilling, "The Effect of Multiple-Tone Interfering Signals on a Direct Sequence Spread Spectrum Communication System," *IEEE Trans. on Commun.*, vol. COM-30, pp. 436-446, March 1982.

[5] Rickholtz, R.L., D.L. Schilling, and L.B. Milstein, "Theory of Spread-Spectrum Communication-A Tutorial," *IEEE Trans. on Commun.*, vol. COM-30, pp. 855-884, May 1982.

[6] Scholtz, R.A., "The Origins of Spread-Spectrum Communications," *IEEE Trans. on Commun.*, vol. COM-30, pp. 822-854, May 1982.

著 者 紹 介



朴 鎭 秀(正會員)
1948年 8月 15日生. 1975年 2月 한양대학교 전자공학과 학사학위 취득. 1977年 2月 한양대학교 전자통신공학과 석사학위 취득. 1985年 2月 한양대학교 전자통신공학과 박사학위 취득. 1987年 1月~1988年 1月 Univ. of Colorado(Post Doc.). 1978年 3月~현재 청주대학교 전자공학과 부교수. 주관심 분야는 디지탈 통신, Spread Spectrum 등임.