

論 文

# 일방향 exploding reflector 개념에 적용한 Fourier 변환기법에 의한 Seismic modeling

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## Seismic Modeling by Fourier Transform Method with One-Way Exploding Reflector Concept

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**要 約** Zero offset section을 얻기 위해서는 Common depth gathering을 CDP stacking하여 얻은 방법이 사용되고 있으나 본 논문에서는 exploding reflector 개념을 사용하였다. 탄성과 파동 방정식으로부터 상방향 파동장만을 표현하는 일방향 파동 방정식을 얻어 비교하였으며 이때 얻어진 일방향 파동 방정식은 경사각 90도까지 표현가능하며 다른 기법에서는 나타나지 않는 경사각 90도 부근의 Signal도 표현 가능했다. 이러한 결과는 공간영역에서 쉽게 구현할 수 없고 오직 Fourier 방법에 의해서만 가능하다. 본 연구 논문은 위의 기법에 의한 결과를 실질적으로 Process 하고 해석하는데 어려움이 많은 overthrust 구조에 대해서 ray tracing 방법과 wave 기법에 의한 결과와 비교하였으며 thrust 구조상의 특성에 의해서 상실되고 수집하기 어려운 diffraction signal 들도 자세하게 나타낼 수 있었다.

**ABSTRACT** Although CDP stacking of common depth gathering is used to get the zero-offset-section, the exploding reflector concept is examined for the modeling of zero source to receiver offset sections in this paper. The acoustic wave equation is compared with a one way wave equation which represents the upgoing wave field only. The one way wave equation used is not derived through an expansion and, therefore, can represent dips up to 90 degrees and may not lose the signals by the dipping angles. There is apparently no simple counterpart of this equation in the space domain and it can be conveniently implemented only by a Fourier method. This paper compares their modeling technique with ray tracing and wave method for over thrust structure which is one of the geological structures are difficult to process and interpret. As the result of modeling much clean and accurate signals, especially, diffractions from the corner and dipping angles can be gathered.

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### I. Over view of the forward modeling by Fourier Transform method

The full acoustic wave equation can be used the first performing a spatial Fast Fourier Trans-

for the forward problem. The 2D seismic response, due to a localized source, over geologic structures with both density and velocity variations can be calculated and compared to theoretical and physical modeling results. When both density and seismic wave velocity are functions of space, the two dimensional acoustic wave equation is:

$$\frac{\partial}{\partial x} \left[ \frac{1}{\rho(x, z)} \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial z} \left[ \frac{1}{\rho(x, z)} \frac{\partial p}{\partial z} \right] = \frac{1}{k(x, z)} \ddot{p} + S \quad (1)$$

where  $\rho(x, z)$  is the density,  $K(x, z)$  is the bulk modulus,  $P(x, z, t)$  is the pressure and  $S(x, z, t)$  is the source term, which equals the divergence of the body force divided by the density.  $\ddot{p}$  in the above equation denotes second order differentiation with respect to time.

A discrete approximation to above equation may be given by

$$Lp^n(i, j) = \frac{1}{k\Delta t^2} [p^{n+1}(i, j) - 2p^n(i, j) + p^{n-1}(i, j)] + s^n(i, j) \quad (2)$$

where  $p^n(i, j)$  and  $s^n(i, j)$  respectively represent the value of pressure and the source at the spatial location  $X=X_0+(i-1)\Delta X, Z=Z_0+(j-1)\Delta Z$  and at time  $t=n\Delta T$ . In the equation (2),  $Lp^n(i, j)$  represents the discrete numerical approximation of the left-hand side of the equation (1)<sup>(1)(2)</sup>.

The term  $Lp^n(i, j)$  in equation (2), which represents the numerical approximation of the left-hand side of the equation(1), is calculated in two separate passes, one for the terms containing X derivatives and one for the terms containing Z derivatives. A differentiation in the space

domain corresponds to a multiplication by  $-i$  in the wave number domain. In the pass for the X derivatives,  $\frac{\partial}{\partial x} \left[ \frac{1}{\rho} \frac{\partial p}{\partial x} \right]$  is calculated for each of the grid lines which parallels the X direction. Along each line  $\frac{\partial p}{\partial x}$  is calculated by the first performing a spatial Fast Fourier Transform (FFT) on P. The result is then complex multiplied by the spatial wave number  $iK$ . This operation is followed by an inverse FFT back into spatial X domain to yield  $\frac{\partial p}{\partial x}$ . In the space-time domain  $\frac{\partial p}{\partial x}$  is thereafter multiplied by  $\frac{1}{\rho(x, z)}$  the spatial derivative of the result,  $\frac{\partial}{\partial x} \left[ \frac{1}{\rho} \frac{\partial p}{\partial x} \right]$  then being approximated by repeating the above mentioned FFT operations.

The calculation of  $\frac{\partial}{\partial x} \left[ \frac{1}{\rho} \frac{\partial p}{\partial x} \right]$  this involves for FFTs and a complex multiplication with  $-iR$  to obtain  $\frac{\partial p}{\partial x}$ , a multiplication of this result by  $\frac{1}{\rho}$  in the spatial domain, and two additional FFTs and a complex multiplication by  $iK$  to obtain  $\frac{\partial}{\partial x} \left[ \frac{1}{\rho} \frac{\partial p}{\partial x} \right]$  When this calculation is completed along all computation grid lines parallel to the X direction, a similar procedure is applied in a second pass through the discretely sampled pressure value to form  $\frac{\partial}{\partial z} \left[ \frac{1}{\rho} \frac{\partial p}{\partial z} \right]$  along all the grid lines in the Z direction. The results of both passes are then simply added together to yield  $Lp^n(i, j)$  in equation (2)<sup>(2)</sup>.

## II. Exploding Reflector Concept

The forward modeling reviewed in the previous chapter can directly simulate a common shot gather by locating the source at a single

point close to the surface. However, if a zero source to receiver offset or a common depth-point-stacked section simulation is desired, then apparently numerous shot gather need to be obtained and combined, which requires large amounts of computer time. For this reason the exploding reflector concept was developed to enable the approximation of the zero offset section in a single computer run with considerable savings in computation. This concept assumes that sources are located at the material boundaries and their strengths are proportional to the reflection coefficients at the same time,  $t=0$ . The time histories at the surface ( $Z=0$ ) approximate a zero offset section. In order to match arrival times, the material velocities must be one-half of the actual velocities. The zero offset section obtained using this concept contains only events which have the same raypath on the way down and up. Events which follow different raypath before and after reflection will not be represented in the exploding reflector model<sup>(1)</sup>.

Approximate zero offset seismic sections associated with this exploding reflector concept can be easily obtained by means of the forward modeling presented in the previous chapter. The source at a single point is turned off and sources are located along the material boundaries instead. The strength of every source point is equal to the reflection coefficient at that location. This is achieved in the previous chapter and then calculating the reflection coefficients for the grid points. The actual material velocities are halved in order to match the arrival times of the reflections with physical results.

### III. Exploding reflector modeling with one-way wave equation

A one way wave equation derived by Gazdag

(1981) is adopted for this modeling algorithm<sup>(1)</sup>. The equation is obtained through the splitting of the acoustic wave equation in the wave number domain, into two one-way wave equations, one for up going and one for down going energy. Derivatives of the velocity field are ignored. For an acoustic medium in which the density is constant, the acoustic wave equation is

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{C^2(x, z)} \frac{\partial^2 p}{\partial t^2} \quad (3)$$

Where  $X$  and  $Z$  are cartesian coordinates,  $p(x, z, t)$  is the pressure and  $C(x, z)$  is the acoustic velocity. Noting that the Laplacian  $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$  corresponds in the wave-number domain to  $-(K_x^2 + K_z^2)$ . Equation (3) is formally rewritten as

$$-(k_x^2 + k_z^2) \tilde{P} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \quad (4)$$

where a  $\sim$  symbol denotes a 2D Fourier Transform with respect to  $X$  and  $Z$ . For homogeneous media ( $c=\text{constant}$ ) it is possible to write the one-way wave equations as follows:

$$\pm i \text{sign}(k_z) (k_x^2 + k_z^2)^{1/2} \tilde{P} = \frac{1}{C} \frac{\partial \tilde{p}}{\partial t} \quad (5)$$

The sign in (5) is chosen depending upon whether one desires an upgoing or a downgoing one-way wave equation<sup>(1)(2)(3)</sup>. This equation has no explicit counterpart in the space domain and therefore can be used conveniently only by a Fourier method.

For the time integration of (5) a second order explicit differencing scheme is chosen. Denoting  $p^n(x, z, t=n\Delta t)$  by  $p^n(x, z)$ , this approximation is

$$\frac{\partial p^n}{\partial t} = \frac{p^{n+1} - p^{n-1}}{2 \Delta t} \quad (6)$$

with this differencing scheme one obtain dispersion and stability relations which resemble the equivalent relations for the full acoustic wave equation. In the second order scheme used for this study, the accuracy of the computations can be controlled by decreasing the time-step size.

#### IV. Seismic Source representation

The source term in equation (2) is usually applied at a single point at the top of the grid in order to simulate a seismic shot. The source as a function of time must be carefully chosen because its frequency band has to be limited to a range which is appropriate for the spatial grid used. If this is not done, erroneous long-period components can enter the solution due to aliasing in a manner which can not be easily remedied by subsequent filtering.

The maximum frequency in the source term will govern the overall resolution of the result. Thus a source frequency band is chosen which is appropriate for the geological model divided by the maximum frequency will determine the shortest spatial wave length that will be used in the modeling. Sampling theory then dictates that half this shortest wavelength must be the maximum spatial sample increment. Thus the formula adopted for the maximum spatial sampling increment become

$$F_{max} = \frac{C_{min}}{2 \cdot \max(\Delta x, \Delta z)} \quad (7)$$

where  $C_{min}$  in the lowest velocity in the input model and  $F_{max}$  is the highest significant frequency component in the source wavelet.

#### V. Test and Interpretation

To generate the zero-offset section through

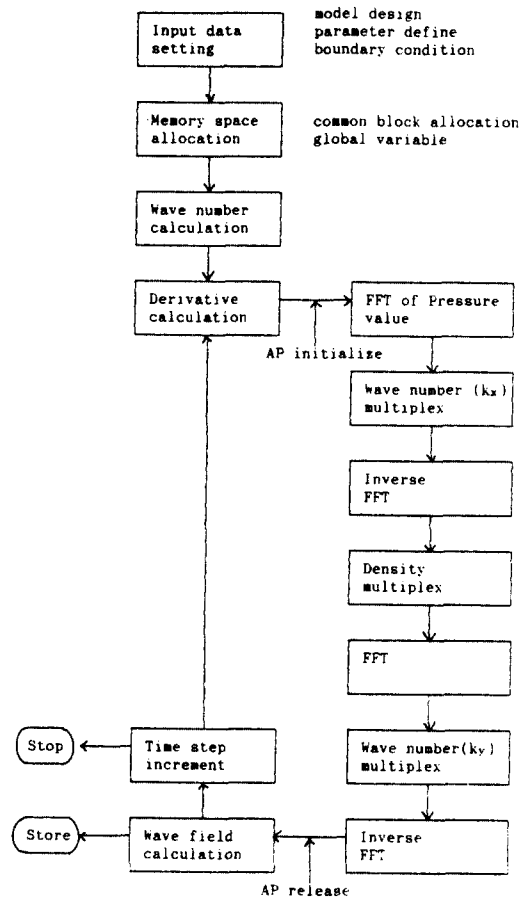


Fig. 1. Flow diagram of the modeling algorithm

this algorithm, we used VAX 780 system with FPS 100 array processor. The flow diagram of the algorithm is shown in Figure 1.

The model we study here is an overthrust structure (Figure 2). Overthrust are spectacular geological features for which large masses or rock are displaced great distances. As shown in the figure 2, line EE' is the thrust fault, in which the right hanging walls A, B, and C move up relatively to left foot wall. The compressive forces deformed and folded the overthrust hanging wall<sup>(5)(6)</sup>. If we draw a vertical line FF' and check the geological section along it we see the geological sections of A, B, and C

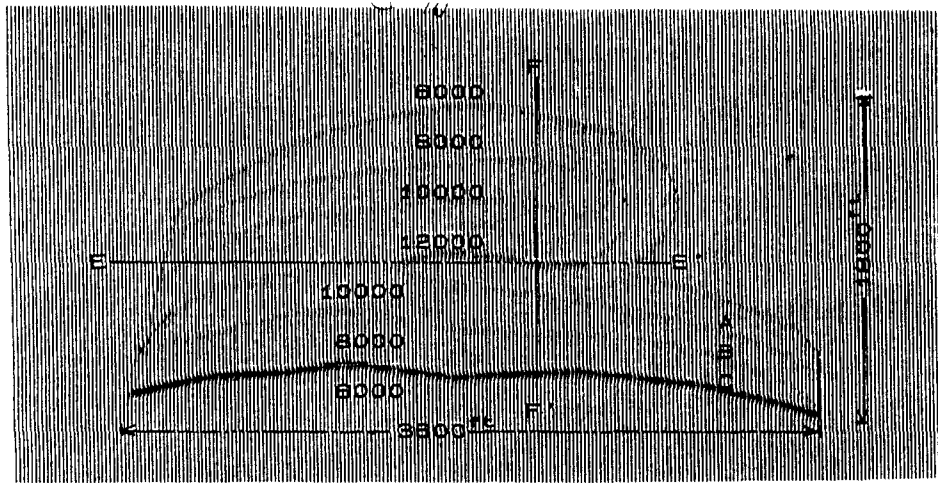


Fig. 2. Overthrust model  
Six layers with four different velocities

repeated. The order formations have overthrust the younger formations. This kind of geological structure has been the subject of infuse investigation recently in the Rocky mountain area. Overthrust form important traps and there delineation a difficult exploration problem.

The velocities increase from 6000 ft/sec to 12000 ft/sec and then decrease to 6000 ft/sec through formations. Scismic surveying over such a structure is difficult because of the relatively complicated folding and faulting, and varied velocities. The overthrust model we

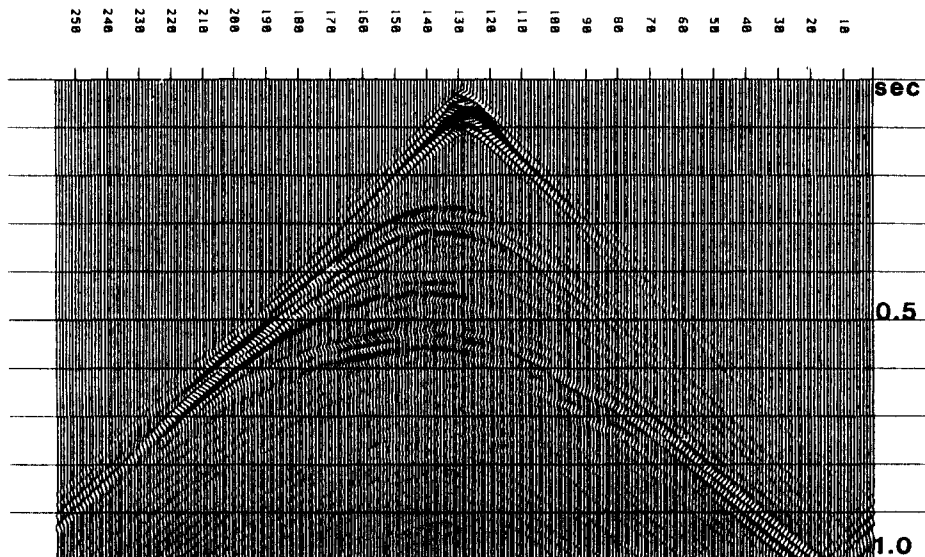


Fig. 3. Common Source Synthetic Time Section  
(Fourier Transform Method)

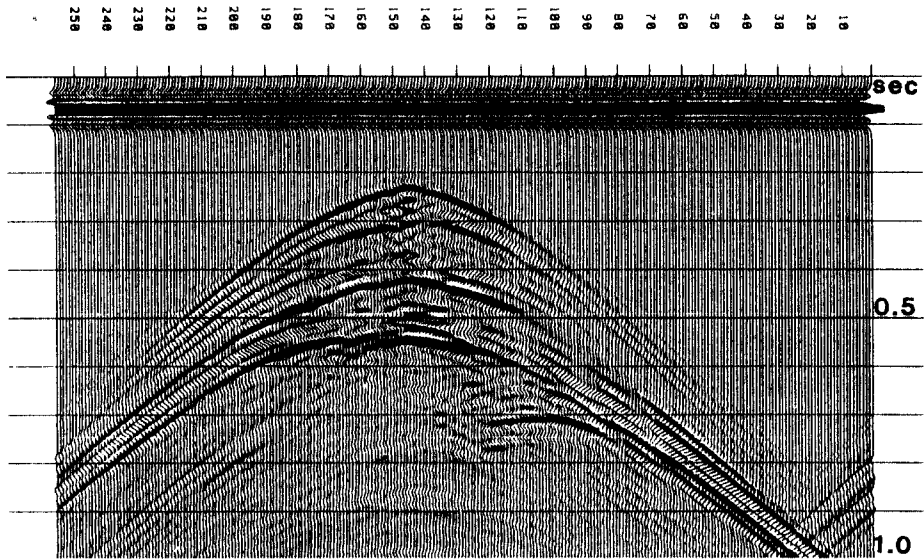


Fig. 4. Plane Wave Synthetic Time Section  
(Fourier Transform Method)

use here includes six layers with four different velocities (Figure 2). The velocities of the layers are 6000 ft/sec, 8000 ft/sec, 10000 ft/sec, and 12000 ft/sec. The density is assumed to be constant. For this relatively complicated structures,

we divided boundaries of the formations into several interfaces as the velocity varied between the top and bottom of the model.

Figure 3, 4, and 5 are the synthetic time sections which were simulated by Fourier for-

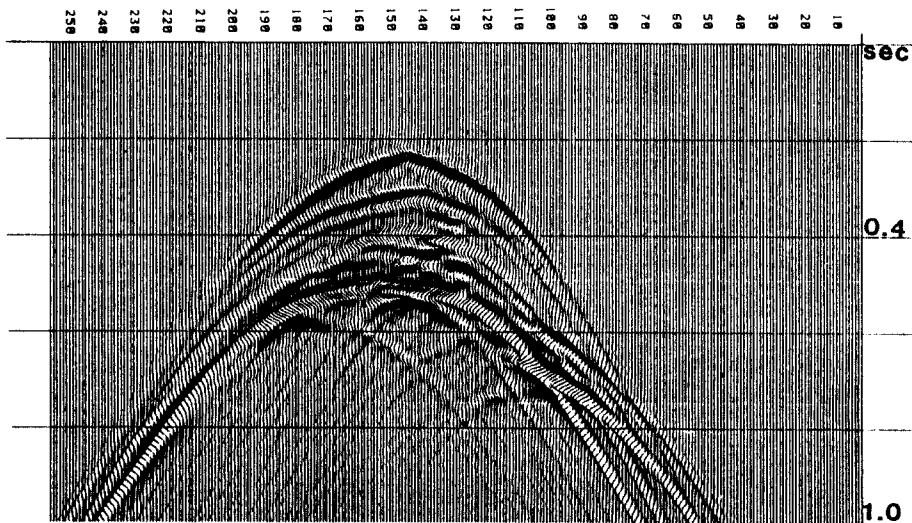


Fig. 5. Zero Offset Synthetic Time Section  
(Fourier Transform Method: One Way Exploding Reflector)

ward modeling; figure 3 for common source shooting, figure 3 for plane waves and figure 5 for a one-way exploding reflector model. As can be seen in the figure 3 and 4, the first events for reflections occurred at times differing by 50 ms, because we used wavelet source of different period. For common source shooting, a symmetric source with 200 ms period was used and for the plane wave 100 ms period symmetric wavelet was used and peak arrives 100 ms and 50 ms latter, respectively. In the plane wave and common source shooting, multiple waves recorded around 700 ms and diffractions from corner interfaces appeared. In the one-way exploding reflector system, there are no multiples and the diffractions are much clear. Figure 6,7 and 8 show the synthetic time sections by ray method and a wave method, respectively. These sections by the ray method and the wave method do not show significant differences. But it looks like the wave method handles the diffracted waves

more accurately. As seen in the figure 3,4,5,6,7, and 8. Fourier transform method shows every events clearly, especially multiples and diffractions. The synthetic section by the one-way exploding reflector (Figure 5) shows very clearly every events from the interfaces.

Figure 6-1 and 6-2 shows snapshots which were generated by a one-way exploding reflector model at times of 120 ms, 220 ms, 320 ms, 400 ms, 500 ms, and 600 ms. Because our overthrust structure is so shallow and the arrival time difference is so short, it is somewhat difficult to pick up every event using snapshots.

## VI. Conclusion

As can be seen in this study, overthrusts are one of the most complicated structures we need to investigate in exploration geophysics. We have developed efficient algorithm using Fourier transform method to simulate synthetic

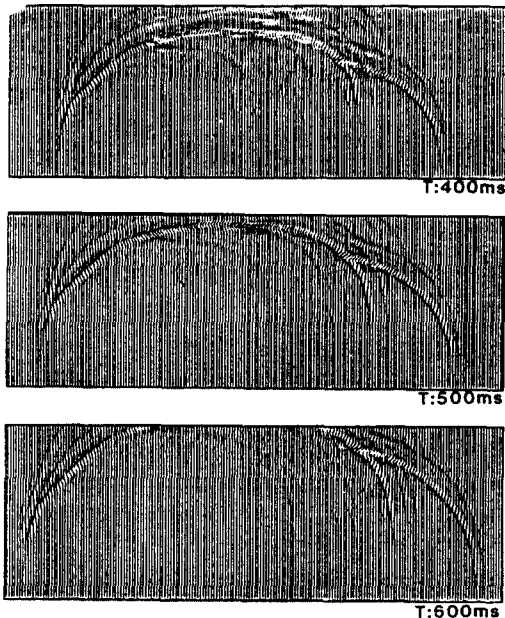


Fig. 6-1. Snapshots  
(One Way Exploding Reflector)

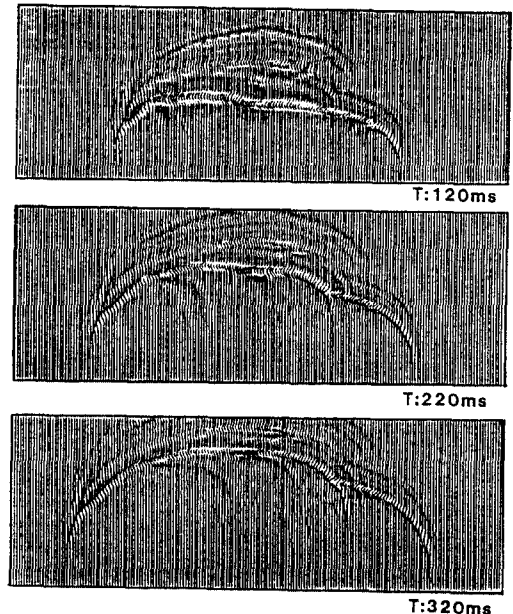


Fig. 6-2. Snapshots  
(One Way Exploding Reflector)

section of promising structures. Synthetic sections by the Fourier transform method show every events clearly, especially multiples and diffractions. The exploding reflector algorithm

was utilized to simulated approximate zero source to receiver offset seismic sections. The use of the acoustic wave equation in the exploding reflector model generates suprious second-

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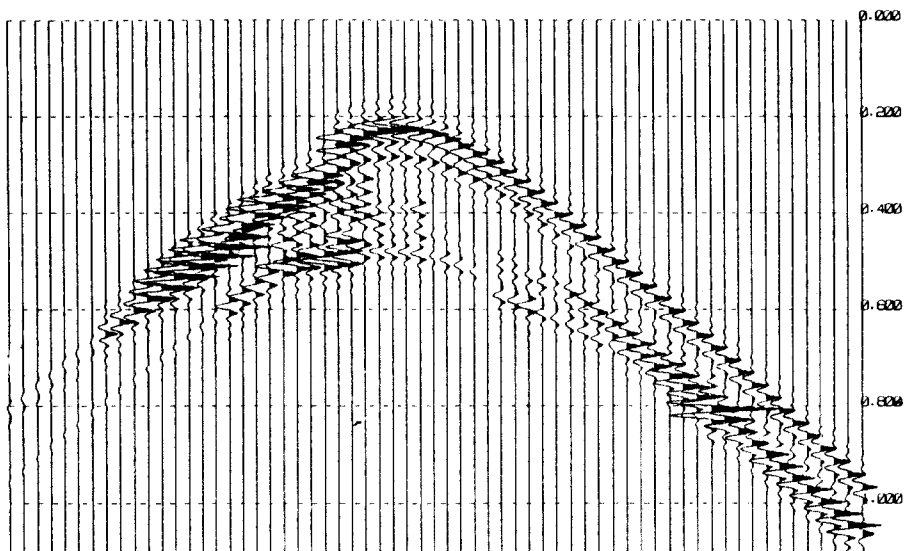


Fig. 7. Common Source Synthetic Time Section (Ray Tracing Method)

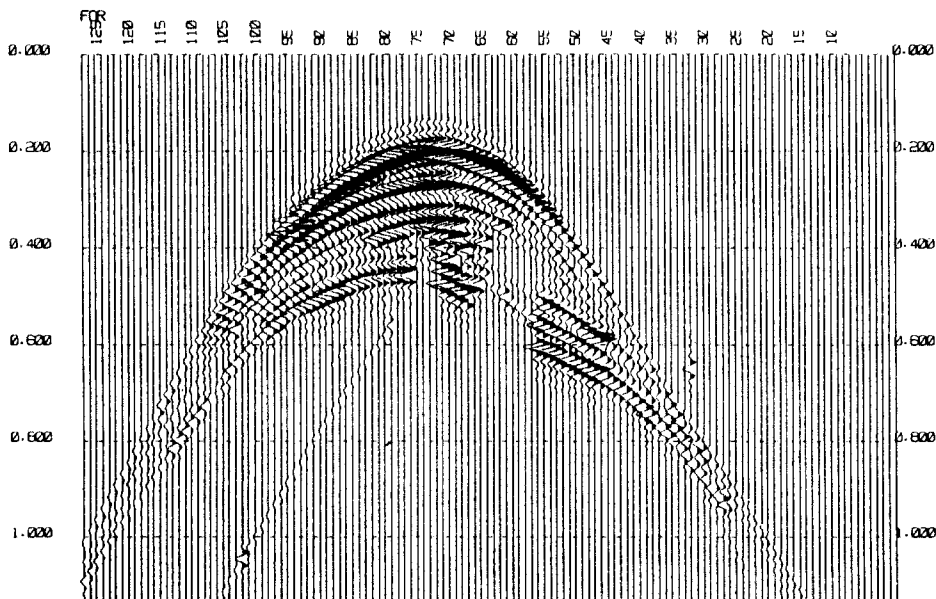


Fig. 8. Zero Offset Synthetic Time Section (Ray Tracing Method)



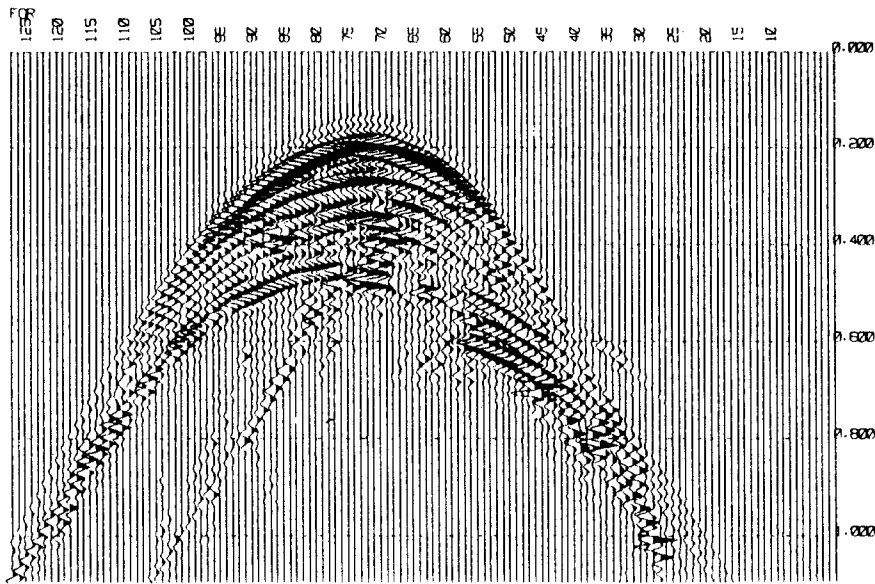


Fig. 9. Zero Offset Synthetic Time Section (Kirchhoff Integration)

dary events in the results. A one-way wave equation is preferable because of its elimination of inappropriate reflections at the material interfaces.

For the further study, we will continue to developed algorithm for inverse problems.

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